The Dynamics of Firm Size Distribution*

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Abstract

The shape and evolution of firm size distribution has been studied in industrial organization and labor economics. The standard hypothesis of Gibrat's law of proportionate effect posits that the rate of firm growth is size-independent. We test Gibrat's law using a new empirical methodology and considering the underinvestigated Brazilian case. Quantile regression is used to estimate the evolution of firm size distribution and to investigate the validity of Gibrat's law in some parts of the conditional distribution, unlike previous studies, which considered the conditional mean dynamics only. An interesting empirical issue is that usual IV/GMM methods are inappropriate under the null but consistent under the alternative hypothesis, while non-IV methods that impose exogeneity are consistent only under the null hypothesis. Results suggest that Gibrat's law is rejected; that smaller firms grow faster; and that there seems to be a strong negative asymmetry in conditional distribution as firm size increases.

Keywords: Gibrat's Law, Quantile Regression, Brazil, Employment.

JEL Codes: L11, C33.

1. Introduction

How firms grow and how markets are shaped is a matter of considerable interest in economics. Estimates of firm size, measured by the number of employees or by net sales are key in macroeconomics and important for the implementation of public policies that allow increasing income and job opportunities and boosting industrial development. One of the first studies on firm size was conducted by Gibrat (1931) – see a recent review in Sutton (1997) –, who found an asymmetric distribution across manufacturing firms in France in the early 20th century and proposed a size-independent firm growth model. This became known as *Gibrat's law of proportionate effect*. A simple stochastic model suggested by Gibrat, with Gaussian shocks, indicates a lognormal firm size distribution. Therefore, a concentration measure can be obtained from dispersion parameters of the distribution.

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Gibrat's law was used by some authors, such as Ijiri and Simon (1977) and Lucas (1978), to construct more sophisticated models of firm size distribution, in an attempt to remedy the shortcomings of the original model, as pointed out by Kalecki (1945) and Sabóia (1977). In particular, Kalecki showed that if the original model were right, size dispersion would grow unconstrained, which is not observed in practice. This occurs because Gibrat's model implies that firm size is a random walk. Sabóia generalizes the model so that growth rates follow an ARIMA model.

However, several international studies suggest that Gibrat's law, even when modified, does not provide any empirical basis. An initial agreement among these authors, especially Evans (1987) and Hall (1987), is that the rate of firm growth decreases as firm size increases. Other researchers state that Gibrat's law may be valid in some cases, with controls for minimum size or problems with the entry of new firms or exit of incumbent firms.

In the Brazilian economic literature, Resende (2005) tested the implications of Gibrat's law for the lognormality of sector firm size distribution and rejected the hypothesis in most economic sectors.

As the hypothesis of Gibrat's law of size-independent growth is extremely popular in the international literature and since it has not been assessed in Brazil, the major aim of the present paper is to test the hypothesis of Gibrat's law in the dynamics of firm size distribution using recent data from the econometric literature in Brazil, *i.e.*, data from the *Pesquisa Industrial Anual – Empresa* (PIA), an annual industrial survey conducted by the Brazilian Institute of Geography and Statistics (IBGE).

Theoretically and empirically speaking, the implicit dynamics of Gibrat's law is related to areas other than industrial organization. His model may be associated with the evolution of income distribution and size (of countries or cities, for instance, Resende (2004) for Brazilian cities), mobility, and with the analysis of labor market dynamics, where the creation and elimination of job opportunities (*i.e.*, changes in employment at the firm level) plays a crucial role in the elucidation of labor market dynamics (see, for example, Davis et al. (1996)).

From an econometric standpoint, the test for the hypothesis of size-independent growth assumed by Gibrat's law contains some important features. At first glance, the hypothesis implies that the evolution of firm size is a random walk, bringing it into the context of the unit root test literature. Nevertheless, the panel data used to test the hypothesis cover a short time horizon, not allowing for the use of tests that rely on an increasing T for asymptotic derivations. Likewise, usual methods for the estimation of dynamic panel data models, such as instrumental variables (IV) or generalized method of moments (GMM), imply stationarity of the data. Thus, estimators that could be consistent become inconsistent under the null to be tested, compromising hypothesis testing.

Studies that contribute towards this debate have been conducted only recently.

Interestingly enough, Bond et al. (2002) and Hall and Mairesse (2005) suggest the use of ordinary least square (OLS) estimators, assuming explanatory variables to be exogenous in a model in which size is a random walk under the null and an autoregressive process with drift under the alternative hypothesis. For the authors, unit root tests based on IV/GMM estimators have not shown better properties in finite samples than the tests based on OLS, in their simulations.

Moreover, an important contribution of this study is the analysis of the evolution of firm size distribution considering the *whole* distribution, instead of focusing on the conditional mean dynamics, through the use of quantile regression (Koenker, 2005). Therefore, our analysis bears close resemblance, for instance, to the assessment of per capita income convergence in a Markov analysis (Quah (1996), *e.g.*) and to transition matrices, as opposed to growth regressions *a la* Barro, popularized by Barro and Sala-i-Martin (1998).[1] That is, we seek to assess whether the independence between growth and size occurs for all firm sizes or only at the local level.

Our results suggest that Gibrat's law is rejected for the conditional mean dynamics and for the highest quantiles of size distribution, with or without the use of instrumental variables in OLS or quantile regression. Smaller firms tend to grow faster than larger ones; the latter of which show asymmetric growth. In other words, there seems to be an upper limit for the growth of larger firms, which may restrict size distribution over time. On the other hand, for relatively smaller firms, growth seems to be size-independent. These facts are not observed in conditional mean models.

The paper is organized as follows. Section 2 introduces the theoretical framework and Section 3 describes the econometric model to be used, *i.e.*, the empirical model and the estimation models. Section 4 provides an overview of the data used and Section 5 displays the empirical results. The last section presents the final remarks.

2. Theoretical Framework

The analysis of firm size distribution in an economy has always aroused the interest of economists. Perhaps the most representative hypothesis is the so-called *Gibrat's law* (Sutton, 1997). In the 1930s, Gibrat found out that the firm size distribution (either measured by the number of employees, sales, or added value) was well approximated by an outstandingly asymmetric lognormal distribution. Gibrat intended to construct a model for the dynamics of firm size that could be consistent with the stylized fact (or "law") of lognormal distribution. He proposed that the size of a firm i in a given period t, Y_{it}, was a function of its size in the preceding period (Y_{it-1}) multiplied by the exponential random term ($v'_{it} = v_{it} + a_i$)

[1] For the use of quantile regression in growth equations, see Porto Jr. and Ribeiro (2001) and, more recently, Mello and Perrelli (2003).

with mean a_i (we assume the expected value of v_{it} to be zero for normalization), *i.e.*,

$$Y_{it} = Y_{it-1} exp(v_{it} + a_i)$$

Thus, the behavior of the log of firm size ($y_{it} = lnY_{it}$) can be described as

$$y_{it} = y_{it-1} + a_i + v_{it} \tag{1}$$

Term a_i can be similar across firms and its value can be zero. Suppose that random terms a_i and v_{it} are Gaussian, then firm size will have a lognormal distribution.

This model has some interesting characteristics. First, note that the model, when we omit i, is a random walk (or AR(1) with unit root). Second, by rewriting the equation in differences of y_{it}, *i.e.*, $\Delta y_{it} = (y_{it} - y_{it-1})$, we have

$$\Delta y_{it} = a_i + v_{it} \tag{2}$$

i.e., the growth rates for firm size (approximated by $ln(Y_{it}/Y_{it-1})$) follow a random walk, regardless of the firm's previous size. For this reason, Gibrat's law is known as *law of proportionate effect*.

Theoretically, the model has some flaws, especially because it implies that variance (and the mean for $a_i \neq 0$) of the log-size distribution grows unconstrained over time. However, the evidence for some countries suggests that size distribution remains constant over time or changes only slightly, lending very little credence to this form of the "law".

According to Kalecki (1945), this requires an alternative representation of the firm size generating process, which can be written as

$$y_{it} = \beta y_{it-1} + a_i + v_{it} \tag{3}$$

where $|\beta|<1$, *i.e.*, a stationary AR(1) model.[2] The rate of firm growth is then inversely proportional to firm size, which can be observed by rewriting equation (3) as

$$\Delta y_{it} = (\beta - 1) y_{it-1} + a_i + v_{it} \tag{4}$$

The distribution of firms is still lognormal, but variance (and mean) does not grow unconstrained over time.[3]

[2]Sabóia (1977) generalizes the model for an ARMA(p,q) process. I am grateful to a referee for this reference.

[3]Note that in a stationary ARMA model, the process mean is fixed. By using lognormal distribution properties, the expected value of firm size (instead of taking the log-size into account) is given by $E(Y_{it}) = exp(a_i/(1-\beta) + \frac{1}{2}\sigma_v^2/(1-\beta^2)) = exp(a_i + \frac{1}{2}\sigma_v^2/(1+\beta))exp(\beta - 1)$. If the variance of random shocks v changes over time, the average size of firms can change, but the log of the expected size remains unchanged.

As can be observed in the recent review by Wit (2005), several authors raised alternative hypotheses to those of Kalecki to warrant that the model of firm size evolution could be compatible with a long-run stationary distribution. Different hypotheses lead to different long-run distributions, such as lognormal, Pareto, Yule, and Waring, power laws (quite common in econophysics) and the so-called *Zipf's law*.

Most authors consider the entry of new firms and the exit of incumbent firms to be a limiting factor for the increase in firm size, *e.g.*, Simon and Bonini (1958), Kwasnicki (1998). Other authors, such as Levy and Solomon (1997), *apud* Wit (2005) or Mansfield (1962), consider the implications of minimum firm size and its relationship with stability for a fixed number of firms. Summarizing the debate at that time, Mansfield had already stated that Gibrat's law could be valid in different situations: (i) for all firms, (ii) for all surviving firms (*i.e.*, only for those firms where Y_{it-1} and $Y_{it} > 0$), or (iii) for "large" firms above a minimum efficient scale. Of the three situations pointed out by Mansfield, the most thriving and less problematic[4] for empirical application is that the law may be valid for "large" firms (Simon and Bonini, 1958). This will be the form we will use herein together with the PIA data for Brazil.

Furthermore, due to the sample design, the selected sample includes a fixed number of ("large") firms. In this regard, Richiardi (2003) raises the hypothesis that the expected increase in firm size is a random variable for which the dynamic process is sometimes a random walk and sometimes a stationary process. The fact is that this type of model is compatible with a stationary firm size distribution. The context of variable coefficients allows the dynamic model of Gibrat's law to be valid for some firm sizes but not for others.

The available literature on the empirical validity of (1) relative to (3) is quite extensive. In several cases, model (1) is rejected in favor of (3), as in Evans (1987) and Hall (1987). These two studies show that, in the case of the U.S.A., small firms grow faster than larger ones. The hypothesis of homoskedastic errors was clearly rejected (the variance of errors decreases with firm size). The authors also conclude that changes in employment depend inversely on firm ages, *ceteris paribus*. Finally, the authors seek to control for the fact that the group of firms assessed is not the same over time, due to the entry and exit of firms in and out

[4]The development of a study on the dynamics of firm size distribution (Gibrat's law) is less problematic for a fixed number of firms (surviving firms and those without entry or exit in and out of the market) since: (a) does not require any knowledge about the distributions of the rates of potential growth, which is unknown, (b) does not require sample selection modeling, which was tested by Hall (1987) and Evans (1987) without satisfactory results. In particular, for Heckman correction, as used by the above-mentioned authors, it is necessary to identify characteristics that affect the possibility of survival of a firm, but not its size. As the bankruptcy process may be understood as a bad draw of the distribution of v_{it}, finding such variable does not seem plausible. The above-mentioned authors explored nonlinearities of firm size in the selection process. However, Johnston and DiNardo (1995) strongly advocate that this identification strategy is weak.

of the market, and they conclude that such exit is inversely proportional to firm size.

3. Empirical Model and Estimation Methods

The model to be estimated is based on equation (3). The estimation can be made by observing firms throughout two time periods, at least. However, there are two major problems regarding the estimation.

First of all, we have the endogeneity of lagged employment in relation to the individual random term, a_i. This term combines all time-invariant factors that allow for a distinction between firms, for instance, the technology used, the owners' management ability, the quality of equipment, and so on. These factors have a permanent effect on firm size. Thus, the estimation of a regression model that does not take into account the correlation of this term with lagged employment may yield inconsistent estimates.

According to Angrist and Krueger (1999), there are many ways to ascertain that the estimates of β be consistent. One way to avoid omitted variable bias is to include variables that proxy a_i. Therefore, it is necessary to include characteristics of the firms (*e.g.*: age) and of the manufacturing sector into which the firm is inserted, in (3), in order to guarantee that the random term have the necessary properties required for consistency and inference. Thus, the estimated model would be

$$y_{it} = \beta y_{it-1} + \mathbf{x}'_{it}\delta + v_{it} \tag{5}$$

where $\mathbf{x}_{it}$ is a $k \times 1$ vector of controls and δ is a $k \times 1$ vector of coefficients, including a constant.

Unfortunately, there is no guarantee that these "proxies" for a_i, represented in $\mathbf{x}'_{it}$, will succeed in fully "ridding" the random term of the regression of the correlation with lagged size. This problem is analogous to the estimation of dynamic panel data models.[5] An alternative estimation method for equation (4) which directly eliminates endogeneity is to neutralize the effect of the random term by using first differences over time,

$$\Delta y_{it} = (y_{it} - y_{it-1}) = \beta \Delta y_{it-1} + (a_i - a_i) + v_{it} - v_{it-1}$$

i.e.,

$$\Delta y_{it} = \beta \Delta y_{it-1} + e_{it} \tag{6}$$

where, $e_{it} = v_{it} - v_{it-1}$.

[5] Our presentation follows Arellano and Bover (1990), Wooldridge (2002) and Baltagi (2005), among others.

It is possible to demonstrate that the estimates by OLS are inconsistent due to the correlation between Δy_{it-1} and the random term, e_{it}. In large samples, the inconsistency of the OLS estimator b of β is such that the regression of (6) underestimates the real parameter value. On the other hand, the estimation of (3) by OLS yields estimates that overestimate the coefficient β in large samples. The degree of overestimation depends on the ratio between the variances of a_i and the random term v_{it}.

To obtain a consistent estimator for (6), one may use all lags of y_{it} greater than two periods, *i.e.*, y_{it-2}, y_{it-3}, etc., as instruments, since the error is an MA(1) process.[6] As the model is overidentified, a potentially more efficient estimator than IV is the GMM proposed by Arellano and Bover (1990), in which all overidentifying restrictions of the lagged dependent variables and their first differences are taken into consideration.[7]

The second problem with regard to estimation and the hypothesis test for Gibrat's law concerns the fact that, under the null hypothesis, H_0:β=1, there is no relationship between Δy_{it-1} and the instruments, since y_{it} turns into a random walk. Therefore, the coefficients estimated by instrumental variables are not identified. Given that there exists an underestimation bias in the first-stage estimate, *i.e.*, the regression of Δy_{it-1} as a function of y_{it-2} or Δy_{it-2} and remaining lags, hypothesis tests might not indicate that there is no relationship between the endogenous explanatory variable and the instruments.

Finally, the third problem concerns the fact that, whereas the test for Gibrat's law can be presented as a unit root test for y_{it}, the data on firm size are often presented as short panels, not allowing the use of estimators and tests that rely on asymptotic results in the time dimension, for instance, unit root tests for panel data, described in Maddala and Kim (1998).

Hall and Mairesse (2005) and Bond et al. (2002) applied Monte Carlo simulations to estimate an autoregressive model for short panels with the autoregressive coefficient possibly equal to 1. The model takes the following form

$$y_{it} = \beta y_{it-1} + (1-\beta)a_i + v_{it} \tag{7}$$

Note that the model is a pure *random walk* if $\beta = 1$, and an autoregressive model with heterogeneity (a_i) if $|\beta| < 1$. The OLS estimator is consistent under the null of $\beta = 1$. A test for this hypothesis based on a simple t statistic has the correct size, with a better performance than other estimators or customized tests. In the case of a false null hypothesis, the power of the test is very good if $V(a_i)$ is low *vis-à-vis* $V(v_{it})$. Under the alternative hypothesis, the IV/GMM estimators

[6]Note that the smallest sample for the estimation of the model is a panel with three time series observations.

[7]Kiviet (1995) indicates that this instrumental variables estimator (also known as Anderson-Hsiao) can have a better performance in small samples than the theoretically more efficient GMM estimator proposed by Arellano and Bond.

are biased downward in small samples. Thus, the best way to test the hypothesis of Gibrat's law, *i.e.*, $\beta = 1$, seems to be the use of OLS, and afterwards, if $|\beta| < 1$, modeling using instrumental variables.

Hall and Mairesse (2005) take a step further and state that, based on their estimates and simulations, "*short panels of firm data are better described as having highly varied and persistent initial conditions rather than permanent unobserved firm effects*"(p. 24–25), implying that the variance of a_i is smaller than the variance of the pure random term. Therefore, estimators that suppose weak exogeneity of y_{it-1}, with OLS can be used without the potential for incorrect results.

Anyway, both methods discussed above (least squares or instrumental variables) are used to estimate the conditional mean of the dependent variable. As the mean does not fully represent a univariate distribution, it may be interesting to estimate conditional quantiles. This is more relevant in our case when one knows, based on the literature, that the conditional distribution is heterogeneous in variance and when one tries to find the dynamics of data distribution. Using the conditional mean to identify the dynamics of a distribution may yield spurious results, as underscored by Friedman (1992) in the literature discussion on growth regressions *a la* Barro.

The study of the whole conditional distribution can be easily carried out using quantile regression (Koenker and Basset (1978) and Koenker (2005) for a more recent and comprehensive presentation, which was used as a source for the presentation below). Quantile regression allows estimating the conditional percentiles of the dependent variable distribution (in our case, the log of firm size), providing more information than that obtained from conditional mean estimators, under relatively fewer hypotheses about the behavior of conditional percentiles.[8]

To understand the quantile regression estimator, consider y_i as the variable of interest ($i = 1, \ldots, n$). The model assumes that the θ'th percentile of the conditional distribution of y, given a vector of explanatory variables (x_i), is linear in x_i, *i.e.*,

$$Q_\theta(y|x) \equiv inf\{y|F_i(y|x) \geq \theta\} = x_i'\beta_\theta$$

[8] A linear relationship of percentiles is assumed for different explanatory variable values. The study with estimates based on conditional means (their respective variances) requires stronger hypotheses about the conditional distribution for demanding, for instance, that the percentiles of the conditional distribution be parameterized by mean and variance only. Alternatively, we have the study of cells, which is nonparametric, but feasible only in case of discrete regressors (Buchinsky, 1984).

where $F_i(\ )$ is the conditional distribution function, $Q_\theta(y|x) = F^{-1}(\theta)$ is the respective percentile function (quantile function) and β_θ is an unknown vector of coefficients, whose estimation for different values of θ on the interval (0,1) is the objective of the problem. For example, for x_i as a vector of 1's, we have $u_i = y_i - \beta_0$, and if F_u is symmetric at zero, $Q_{0.5}(y) = 0$, *i.e.*, the median of y will be β_0.

The vector of coefficients can be estimated by the minimization of

$$\Sigma_i \rho_\theta(y_i - x_i'\beta_\theta), \quad \text{or} \quad \Sigma_i \rho_\theta(u_i)$$

with

$$\rho_\theta = [\theta u_i I(u_i \geq 0) + (\theta. - 1)u_i I(u_i < 0)]$$

where $I(\ )$ is the index function.

Note that errors u_i are expressed in absolute values and are unequally weighted, except for $\theta = 1/2$, when the conditional median is estimated. The minimization problem above does not have a closed-form solution, but it can be estimated by linear programming methods (Koenker and Basset, 1978). Under very weak hypotheses about the random term, one may demonstrate that the vector of coefficients is consistent and asymptotically normal. Just like the sample median, in many cases, the estimator is more efficient than the least squares method.

Some important properties of the estimator should be considered. First, the estimator is robust to outliers in y_i. Secondly, the coefficients are equivariant with respect to monotonic transformations (such as log), unlike the mean. Thirdly and most importantly, in case of heterogeneous data (including also heteroskedasticity), the conditional percentiles may vary across the distribution and thus the effect of explanatory variables will be different at different points of the distribution. Let us suppose a model with *iid* errors, that is, the conditional distribution does not vary as to its characteristics for different values of the explanatory variable, except for its center (conditional mean), which follows a linear relationship with the explanatory variables. Likewise, the quantile regression coefficients will be constant for different values of x except for the constant, whose coefficient relies on the quantile of the error term for the selected percentile,

$$Q_\theta(y|x) = \{\beta_0 + Q_\theta(u)\} + x'\beta$$

where x does not include a column of 1's. However, if the percentiles of the conditional distribution systematically change with the explanatory variables, the slope coefficients will be different for different quantiles, *i.e.*,

$$Q_\theta(y|x) = \beta_0 + x'\{\beta_\theta + Q_\theta(u)\}$$

For instance, Hendricks and Koenker (1992) estimated that the average price effect throughout the day for household consumers of electric power in Chicago is

statistically zero for small consumers and strong for large consumers. The average effect conceals remarkable heterogeneity in price sensitivity.

Machado and Mata (2000) have recently applied quantile regression to the problem of firm size distribution. The authors used quantile regression and estimated that the effect of sectoral attributes on location measures, scale, kurtosis and asymmetric distribution of firm size is significant, with different quantile regression coefficients across the conditional distribution.

As previously pointed out, it is necessary to use instrumental variables. Their use for the estimation of conditional quantile parameters was assessed by Powell (1983) and Chen and Portnoy (1996). An application of this method can be seen in Ribeiro (2001). Similarly to the two-stage least squares (2SLS) regression, the endogenous explanatory variable is predicted in the first stage, by least squares (or quantile) regression, using instruments; in the second stage, the prediction is used as explanatory variable in a model estimated by quantile regression. This two-step estimator is known in the literature as two-stage quantile regression (2SQR). As with the 2SLS estimator, the variance-covariance matrix should be calculated with caution.

The interpretation of the coefficients estimated by quantile regression for models (3) – with the dependent variable in the level –, and (4) – with the first-differenced variable – should be made carefully. Although the autoregressive coefficient for the conditional mean of (3) is the same as that for model (4), the coefficients may differ in terms of quantile regression between (3) and (4) for the same quantile. One should not categorically state that if the coefficient of Δy_{it-1} in (4) is equal to zero for $\theta = 10\%$, the coefficient of y_{it-1} for $\theta = 10\%$ will be equal to 1 in (3). As the dependent variable of (4) is a function of other random variables ($z_{it} = \Delta y_{it} = y_{it} - y_{it-1}$), according to the convolution theorem it is not possible to obtain the percentiles ofy_{it} from the percentiles of z_{it}. Anyway, note that the basic hypothesis under analysis herein, *i.e.*, the validity of Gibrat's law, can be tested in (4), with the hypothesis that the autoregressive coefficient is equal to zero, or in (3) with the hypothesis that the coefficient is equal to 1.

4. Database

The study uses data from the annual industrial survey (PIA) conducted by IBGE for 1996-1999. The time frame was restricted due to changes in sampling methodology between 1995 and 1996. In order to minimize the heterogeneity of the sectors and to allow comparisons with the available literature, firm size was calculated according to the number of employees on December 31 of each year.[9] The mining and quarrying sectors were omitted due to their nature, and sector CNAE37 was excluded for confidentiality reasons.

[9]The study was replicated using the annual rate of average employment as firm size parameter, without statistically significant differences in results. The results are available from the author upon request.

The sampling pattern of PIA, along with the theoretical background above, indicate the use of surviving firms. Only medium-sized and large firms were are included, with at least 100 employees. Confidentiality issues led to the exclusion of firms with average rate of employment greater than 5,000 employees. Sampling adjustments and the theoretical background make it clear that the results do not represent the whole population of manufacturing firms in Brazil. This would require the assessment of entries and exits of firms in and out of the market, but this is not empirically trivial in terms of data sampling (Bivar and Rodrigues, 2001), given the type of data used (Hall (1987) and Evans (1987)).[10]

The descriptive statistics are shown in Table 1. Average employment ranged from 418.5 employees in 1996 to 379.9 in 1999. The distribution is clearly asymmetric to the right with a median between 218 employees in 1996 and 206 in 1999. The asymmetric log-size distribution can be observed in the asymmetry coefficient. This is shown in Figure 1, with the distributions of 1996 and 1999, the initial and final years of the sample. There is a shift of firm size distributions in 1999 to the left comparatively to 1996, which indicates a decrease at all levels of manufacturing employment. This result can be seen in further detail in Figure 3 that present the difference in the quatiles of the 1996 and 1999 distributions. Between 1996 and 1999, except for the large difference in the smaller quantiles, the rate of employment in 1999 was 5 to 13% lower than that for 1996, in1996, in several quantiles.

Table 1

Table 1a – Descriptive Statistics for Firm Size (number of employees on Dec 31)

Year	1st Quart.	Median	Mean	3rd Quart.	SD	Asymm	Kurt.
1999 (N99)	133,0	206,0	379,9	376.0	90.0	0,3992	2,8683
1998 (N98)	134,0	205,0	381,5	380.0	90.0	0,4228	2,7602
1997 (N97)	141,0	216,0	406,8	409.0	90.0	0,4403	2,7537
1996 (N96)	141,0	218,0	418,5	415.0	90.0	0,4380	2,7847

Table 1b – Descriptive Statistics for Log Firm Size (number of employees on Dec 31)

Year	1st Quart.	Median	Mean	3rd Quart.	SD	Asymm	Kurt.
1999 (lnN99)	4,890	5,328	5,481	5,930	0,873	0,1585	1,9880
1998 (lnN98)	4,898	5,323	5,503	5,940	0,829	0,184	2 1,9217
1997 (lnN97)	4,949	5,375	5,567	6,014	0,821	0,199	5 1,9092
1996 (lnN96)	4,949	5,384	5,571	6,029	0,850	0,193	6 1,9244

Source: author estimates based on PIA/IBGE data. $N = 5745$.
Note: Statistics used: Asymm.=$(Q.75 + Q.25 - 2^{*}Q.50)/(Q.75 - Q.25)$, Kurt.= $(Q.90 - Q.10)/(Q.75 - Q.25)$, where $Q.z$ is such that $F(x) = z$. For Normal distribution, Asymm.= 0 and Kurt= 1.9.

[10] As previously commented and also pointed out by one of the referees, the selection of only firms whose size is greater than a given value (selection conditional on the explanatory variable) may produce selection biases in the assessment of the conditional distribution of all firms. Given the difficulties regarding the feasibility of a model for selection bias correction, as in Buchinsky (1998), we make it clear that the results are conditional on the type of firms selected.

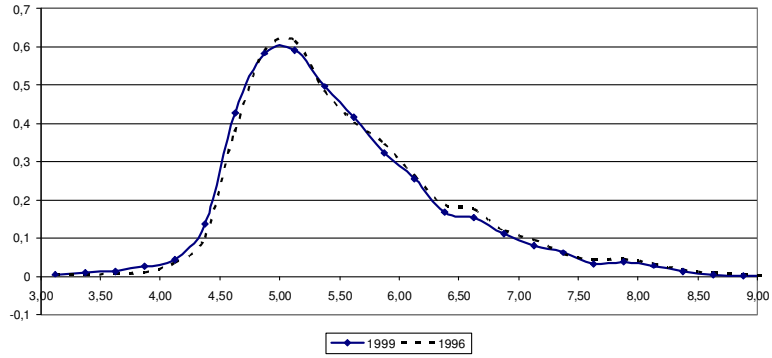


Figure 1
Firm size distribution (log employment) – 1999 and 1996

Let us now have a look at the results of the regression analysis.

5. Empirical Results

The results of the regression analysis using a simple model (3), without the use of instrumental variables, are shown in Table 2 and Figure 3. This model yields good results under the null of the validity of Gibrat's law and a small variance in individual heterogeneity. On the other hand, we expect coefficients to be inconsistent and overestimated if Gibrat's law does not apply.

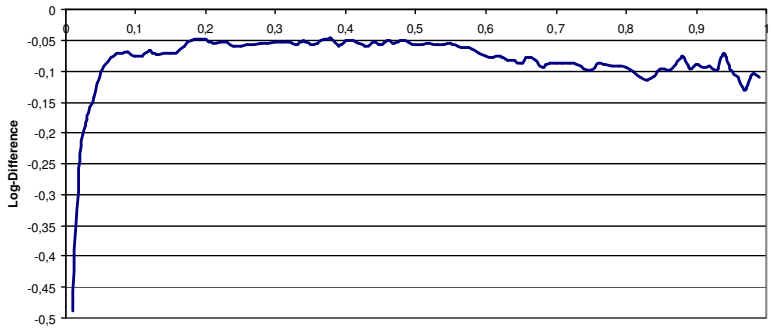


Figure 2
Differences in quantiles of firm log-size distribution – 1996–1999

In the three sampled years, R^2 is high and the autoregressive coefficient is statistically smaller than 1, even though it approaches 1 during the study period. Specification tests indicate the presence of heteroskedasticity, except for 1997. The possibility of heteroskedastic errors is common in the literature. By interpreting the coefficient according to equation (4), we can see that, on average, smaller firms grow faster, suggesting that Gibrat's law was not valid for Brazilian firms during the study period.

Table 2
Autoregressive model for firm size – OLS

Variable	lnN99	lnN98	lnN97
Constant	0.1998*	0.3202*	0.5576*
	(0.048)	(0.029)	(0.037)
lnN98	0.9597*		
	(0.009)		
lnN97		0.9311*	
		(0.005)	
lnN96			0.8991*
			(0.006)
F statistic-p-value	0	0	0
R-squared	0.83	0.8515	0.8662
Het test (p-value)	0.0005	0.0133	0.1717

Source: author estimates based on PIA/IBGE data. $N = 5745$.
Notes: * – significant at 5%. Het test (p-value) – p value for Koenker (1981) heteroskedasticity test.
Standard deviations robust to heteroskedasticity in brackets.
5745 observations.

However, the average effect conceals some heterogeneity that had already been detected by the heteroskedasticity test. Table 3 shows the quantile regression estimates for the main percentiles and Figure 3 shows the autoregressive coefficients for the 5th to the 95th percentiles, adn their 90% confidence intervals. As previously mentioned, we would not expect significant differences in the autoregressive coefficient for different percentiles if there was no heteroskedasticity nor a relationship between firm size and the random term, as pointed out by Koenker (2005). But in Figure 4, we note that the effect of firm size on growth in 1999 (calculated as the coefficient minus 1) varies across the distribution. The autoregressive coefficient decreases as percentiles increase. Up to the 25th percentile we cannot reject the hypothesis that firm size does not affect growth, since the autoregressive coefficient is not statistically different from 1. For percentiles greater than 25%, the autoregressive coefficient decreases to less than 1, with very narrow intervals, suggesting significant changes in the coefficient, reaching less than 0.89 for the 95th percentile.

Table 3
Autoregressive model for firm size – quantile regression

Panel a – 1999					
Variable	tau = 0,1	tau = 0,25	tau = ,05	tau = ,075	tau = 0,9
Constant	-0,2681	-0,0658	0,0710	0,3096	0,6528
	(0,064)	(0,022)	(0,012)	(0,020)	(0,038)
lnN98	0,9980	0,9921	0,9862	0,9623	0,9273
	(0,012)	(0,004)	(0,002)	(0,003)	(0,006)
R1	0,4772	0,6207	0,7089	0,7322	0,7130
Panel b – 1998					
Variable	tau = 0,1	tau = 0,25	tau = ,05	tau = ,075	tau = 0,9
Constant	0,0634	0,0635	0,1419	0,2963	0,6127
	(0,071)	(0,030)	(0,017)	(0,017)	(0,027)
lnN97	0,9253	0,9603	0,9694	0,9586	0,9254
	(0,013)	(0,005)	(0,003)	(0,003)	(0,004)
R1	0,4145	0,5899	0,7053	0,7468	0,7398
Panel c – 1997					
Variable	tau = 0,1	tau = 0,25	tau = ,05	tau = ,075	tau = 0,9
Constant	0,3547	0,2597	0,2252	0,4147	0,7940
	(0,039)	(0,027)	(0,015)	(0,019)	(0,036)
lnN96	0,8864	0,9322	0,9585	0,9445	0,9035
	(0,008)	(0,005)	(0,003)	(0,003)	(0,006)
R1	0,4893	0,6296	0,7124	0,7314	0,7063

Source: author estimates based on PIA/IBGE data 1996-1999.
Notes: * – significant at 5%. R1 – Quality measure for the adjustment of the regression in the quantile of Machado and Koenker (1999).
Standard deviations robust to heteroskedasticity in brackets.
5745 observations.

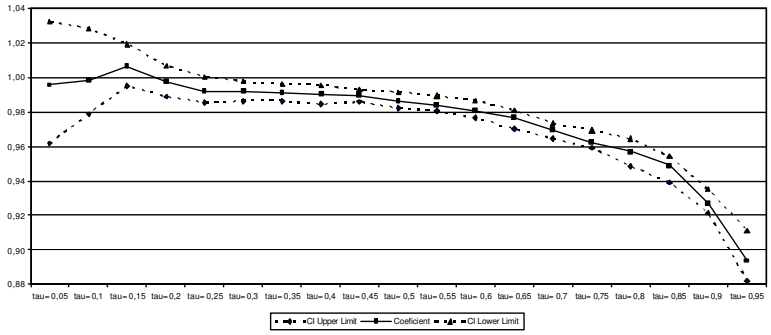


Figure 3
Autoregressive coefficient estimates and confidence interval (CI) for the firm size model 1999-1998

The use of quantile regression allows us to assess the dynamics of employment in a unique fashion. By estimating the coefficients for each percentile, we may estimate the conditional distribution for each value of the conditioning variable using a semiparametric approach. For 1999, for instance, this is shown in Figure 4. To help with the interpretation of results, we added a 45^o line to the figure. What median quantile regression and 10th and 90th percentile straight lines show is that the conditional distribution is concentrated below the 45^o straight line as firm size increases.

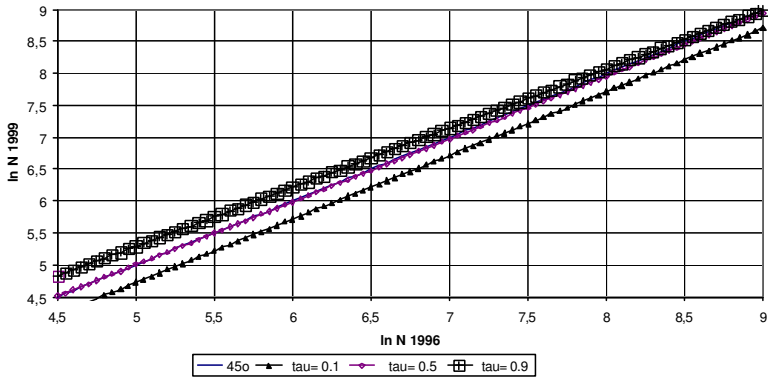


Figure 4
Autoregressive coefficient via quantile regression for firm log-size – 1999-1996

Figure 5 shows this effect more clearly, with the difference in size estimated on the quantile regression straight lines of Figure 4, conditional on the firm's previous size. For example, as the median regression line intersects the zero point when the log-size is equal to 4.5, we may say that 50% of such firms increase their size. Likewise, for firms with log-size of 4.5, the probability of growing more than 0.30 is 10% only.

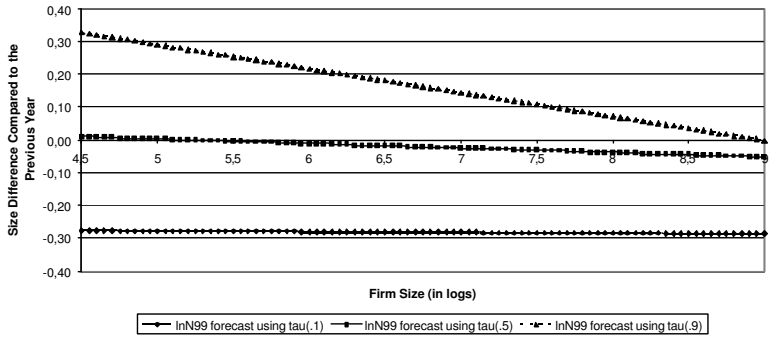


Figure 5
Conditional distributions of employment size compared to the previous period – 1999

The figure also shows some important asymmetry in the rate of growth expected for different firm sizes. Larger firms have fewer chances of growing than smaller ones, since the 90th decile line of the conditional distribution of growth rates decreases systematically. However, the probability of sharp size reductions in size is very similar for different-sized firms, as shown by the virtually horizontal straight line of the 10th decile.

Now changing our focus to model (4), *i.e.*, the model of firm growth (first differences) in Table 4, we have the OLS estimates, and in Table 5, we have the results for the consistently estimated model (using instrumental variables). Table 4 allows us to have a better understanding of the effect of first differences on the rejection of the random walk hypothesis of firms' log-size. As expected, the coefficients are negative and imply theoretical values of 0.80 to 0.85 for the autoregressive coefficient, according to the asymptotic bias described in Arellano and Bover (1990). The results suggest that firm growth rates are negatively correlated with their size.

Table 4
Autoregressive model for firm size – First differences and OLS

Variable	ΔlnN99	ΔlnN98
Constant	-0,0285*	-0,0639*
	(0,005)	(0,004)
ΔlnN98	-0,1016*	
	(0,045)	
ΔlnN97		-0,0848*
		(0,029)
F statistic-p-value	0	0
R-squared	0,0083	0,0067
Het test (p-value)	0	0,0807

Source: author estimates based on PIA/IBGE data 1996-1999.
Notes: *–significant at 5%. Het test (p-value)–p value for Koenker (1981) heteroskedasticity test. Standard deviations robust to heteroskedasticity in brackets.
5745 observations.

Looking at the estimates obtained by the instrumental variables (or by 2SLS), in Table 5, we first observe the quality of the instruments. Although R^2s are low, the F tests of the regressions with each endogenous explanatory variable in relation to instrumental variables suggest that the instruments may be valid (Bound et al., 1995), indicating the rejection of the hypothesis of independence between growth and firm size.

Table 5
Autoregressive model for firm size – First differences and instrumental variables

Variable	ΔlnN99	ΔlnN99	ΔlnN98
Constant	0.0011	-0.0054	-0.0611*
	(0,008)	(0,008)	(0,005)
ΔlnN98a	0.3659*		
	(0,115)		
ΔlnN98a		0.2624*	
		(0,104)	
ΔlnN97a			0.524*
			(0,075)
R-squared	0,0033	0,0019	0,0192
Instruments	lnN97	lnN97,lnN96	lnN96
F test – 1st stage	180.6*	103.4*	468.6*
R-squared – 1st stage	0,0305	0,0348	0,0754

Source: author estimates based on PIA/IBGE data 1996-1999.
Notes: * – significant at 5%. a – variable predicted by the indicated instruments.
Standard deviations robust to heteroskedasticity in brackets.
5745 observations

The estimates of the autoregressive coefficients are significant, positive and much lower than the estimates shown in Table 2 and with a different sign from that observed in Table 4, as expected. Once again, by interpreting the coefficient as in model (4), we conclude that smaller firms grow faster.

With regard to quantile regression estimates using instrumental variables, the results in Table 6 demonstrate large variability of coefficients across the percentiles. This may be due to the effect of weak (but relevant) instruments. For 1999, the autoregressive coefficient at the 10th percentile is not significant. The higher the decile, the higher the coefficient, reaching up to 0.75 for the 90th percentile. The result is different for 1998, where the percentiles of the autoregressive coefficient are U-shaped, even though the confidence intervals are wide at the ends. By looking at the coefficients of several percentiles in Figures 6–7, we confirm the results shown by panels (a) and (c) in Table 6.[11]

Table 6
Autoregressive model for firm size – First differences and quantile regression with instrumental variables

			Panel a – ΔlnN99		
Variable	tau = 0,1	tau = 0,25	tau = 0,5	tau = 0,75	tau = 0,9
Constant	-0,2620	-0,0970	0,0087	0,1323	0,2955
	(0,0266)	(0,0066)	(0,0034)	(0,0077)	(0,2530)
ΔlnN98a	0,3520	0,1931	0,2084	0,4940	0,7571
	(0,3132)	(0,0770)	(0,0403)	(0,0901)	(0,2890)
Instruments	lnN97				
			Panel b – ΔlnN99		
Variable	tau = 0,1	tau = 0,25	tau = 0,5	tau = 0,75	tau = 0,9
Constant	-0,2668	-0,1011	0,0049	0,1246	0,2877
	(0,0257)	(0,0063)	(0,0033)	(0,0073)	(0,0238)
ΔlnN98a	0,2126	0,1394	0,1564	0,3812	0,6302
	(0,2932)	(0,0713)	(0,0379)	(0,0828)	(0,2713)
Instruments	lnN97, lnN96				
			Panel c – ΔlnN98		
Variable	tau = 0,1	tau = 0,25	tau = 0,5	tau = 0,75	tau = 0,9
Constant	-0,3494	-0,1549	-0,0279	0,0662	0,1980
	(0,0198)	(0,0055)	(0,0026)	(0,0042)	(0,0147)
ΔlnN97a	0,5832	0,3726	0,2874	0,3660	0,6636
	(0,2308)	(0,0641)	(0,0302)	(0,0487)	(0,1712)
Instruments	lnN96				

Source: author estimates based on PIA/IBGE data 1996-1999.
Notes: * – significant at 5%. a – variable predicted by the indicated instruments.
Standard deviations robust to heteroskedasticity in brackets.
5745 observations.

[11]The results in panel (b) are very close to those in panel (a).

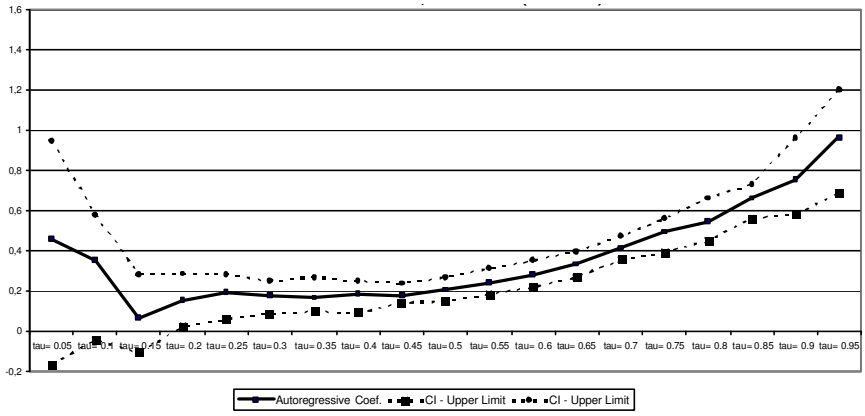


Figure 6
Autoregressive coefficient estimates and confidence interval (CI) for the firm size model – 1999-1998 (Table 6a)

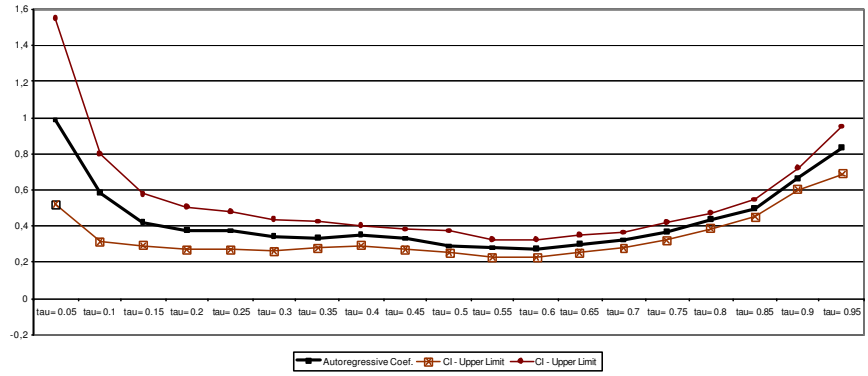


Figure 7
Autoregressive coefficient estimates and confidence interval (CI) for the firm size model – 1998-1997 (Table 6c)

To better understand the conditional distribution across quartiles, have a look at the quantile regression straight lines for the 10th, 50th and 90th percentiles of conditional distributions in Figure 8. This figure shows an interesting data behavior. Note that as we move along the horizontal axis (firm size change), the behavior of tails is different from that of the center of the distribution, measured

by the median, implying a shift in the mode of the distribution. Firms with strong size reductions show a slower decrease, since the major part of the distribution lies above the 45^o straight line, in the negative-negative quadrant. On the other hand, firms with a substantial growth (large positive value on the horizontal axis) have most of their growth rates distributed below the 45^o straight line in conformity with the distribution observed in the negative quadrant. This shift in the mode, or the peak of the density, from positive values, when growth is negative, to negative values, when growth is positive is intuitively expected, as it guarantees that the distribution of firm size will have finite variance.

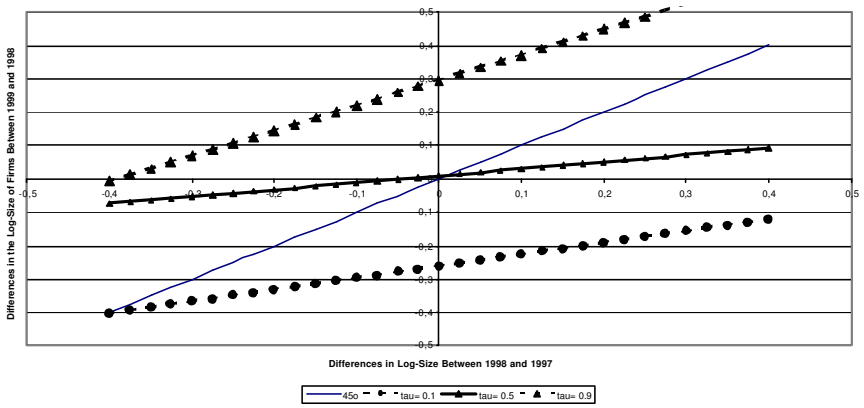


Figure 8
Conditional quantiles of the changes in firm size – 1999

In short, the results of the estimates with or without instrumental variables for the conditional mean or for the quantile regression suggest that the autoregressive coefficient of the generalized model of Gibrat's law (equation 4) is less than one and that smaller firms grow faster (although firms with substantial growth are less likely to sustain this growth in the future). This remarkable symmetry may set a limit to firm size, preventing the variance of size distribution from growing unconstrained. With regard to small firms, Gibrat's law can properly describe their behavior.

6. Final Remarks

The aim of this paper was to assess the dynamics of the distribution of firm size in Brazil, empirically analyzing Gibrat's law for the second half of the 1990s. The distribution of firm size (measured by the number of employees at the end of each year) is asymmetric and remains so even after logarithmic transformation, lending little credence to the assumption that the distribution could be lognormal. In terms of dynamics, we can see that manufacturing employment decreased between 1996 and 1999, with the size distribution shifting to the left almost everywhere.

The hypothesis of Gibrat's law is difficult to be tested because those estimators that are consistent under the null are inconsistent under the alternative hypothesis. In addition, the tests are based on short panel data, where the identification of the parameter in time depends on whether the cross-section sample grows asymptotically instead of temporally, not allowing for the use of unit root tests for the panels.

On the other hand, recent studies have shown that a test based on OLS estimates has an adequate size and is quite powerful. Thus, we initially used OLS to test the hypothesis and then IV variables to estimate the dynamic panel data model.

We also added the use of quantile regression to this literature, which allows estimating the whole conditional distribution using a semiparametric approach. We used quantile regression in an instrumental variable framework in order to identify the model's coefficients.

The OLS regression analysis indicates the rejection of Gibrat's law, except for relatively smaller firms. The quantile regression estimates suggest variable asymmetry and a reduction in the variance of the conditional distribution as firm size increases. The reduction in data dispersion with the increase in size had already been described in the literature as a stylized fact, but the present study seems to be the first one to detect an asymmetric change in dispersion. The augmenting asymmetry in the conditional distribution suggests that there is a mechanism that reduces the possibility of growth for large firms, rendering size distribution constant over time. In other words, the autoregressive coefficient is smaller than one due to an asymmetric effect on the growth of firms and a change in the mode of conditional distribution of growth rates.

The instrumental variables estimates suggest that the autoregressive coefficient of firm size ranges from 0.3 to 0.6 for the estimates of the conditional mean or for the quantile regression, except for larger percentiles, when the coefficient reaches 0.75. In short, Gibrat's law for Brazil seems to be clearly rejected for medium-sized and large firms throughout the study period.

The limitations of this study point to the necessity of further research. First, the result is valid only for surviving firms, *i.e.*, those that continue to operate throughout time. Adequately modeling the firm entry in and exit out of the market is an interesting challenge. Secondly, the panel used is admittedly short

(only 4 years) and covers only manufacturing firms. A wider coverage of sectors and a longer time horizon may enhance the reliability of the data, since there are studies in the literature which support Gibrat's law in the service sector, but not in the manufacturing sector.

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