Inflation Dynamics in Brazil: The Case of a Small Open Economy*

Waldyr Dutra Areosa**
Marcelo Medeiros***

Abstract

This paper derives and estimates a structural model for inflation in an open economy. The model represents the standard new-Keynesian Phillips curve (NKPC) and the hybrid curve proposed by Woodford (2003) and Galí and Gertler (1999) as special cases. We present two sets of estimates for the Brazilian economy, initially regarded as a closed economy and then as a small open economy. According to the recent literature, the model contemplates indexation to past inflation and a measure of marginal cost as relevant inflation indicators. Some of the results can be summarized as follows: (i) Brazil, when regarded as a closed economy, has a relatively higher level of nominal rigidity than that of the United States and Europe, and a high level of indexation as well; (ii) In an open economy with indexation, nominal exchange rate appreciation plus foreign inflation affects consumer inflation, and this effect becomes more intense with larger economic openness; (iii) There is a small direct impact of the variables associated with economic openness, with the sum of their coefficients being close to zero; (iv) However, the indirect impact is significant, consistently changing the weights associated with lagged inflation and the expected future inflation.

Keywords: Inflation, Phillips Curve, Open Economy, Brazil.

JEL Codes: E31.

*Submitted in February 2006. Revised in April 2007. The opinions expressed in this paper are those of the authors and do not necessarily represent those of Banco Central do Brasil.

**Pontifícia Universidade Católica do Rio de Janeiro and Banco Central do Brasil. E-mail: wdutra@econ.puc-rio.br

***Pontifícia Universidade Católica do Rio de Janeiro. E-mail: mcm@econ.puc-rio.br

1. Introduction

In recent years, a new-Keynesian model has replaced the IS-LM-OA model in monetary policy analysis.[1] The major features of this approach include the following:

1. Microfoundations: The equations on which these models are based derive from microfoundations. Therefore, their parameters have structural interpretations, related to technology and market structure, and may then be calibrated in a more analytical fashion. In other words, the possible array of values that they can assume is limited by the theory itself. The resulting models can be used to test different alternative policies, in an attempt to respond to the "criticisms" over the validity of these comparisons;[2]

2. Imperfect competition: Presumably, there is some kind of imperfect competition for the goods market, instead of perfect competition;

3. Nominal rigidity: The new-Keynesian models include price and/or wage rigidity;

4. Real rigidity: In addition to facts that cause nominal rigidity, the new-Keynesian models also contemplate real rigidity – factors that cause real wage rigidity or relative price rigidity in firms due to changes in aggregate demand.

Following the same line of research, numerous studies have been carried out to assess one of the major issues in macroeconomics: short-run inflation dynamics.[3] This is also one of the most discussed issues, but several questions remain unanswered.

These studies try to capture the characteristics of the price setting process by explicit solving an optimization problem of an individual firm. Consequently, we obtain a relation between short-run inflation and some measure of real activity, in the spirit of a Phillips curve. The explicit use of microfoundations not only makes this relation more robust but also adds some important features that distinguish it from previous ones.

After improvements in theoretical modeling came the econometric analyses of the new-Keynesian Phillips curve (NKPC). These studies provide some interesting conclusions, but they also bring some problems to the existing theory. For instance, it seems that these models have some difficulty in capturing inflation persistence without generating inflation stickiness, which is hard to explicitly encourage in the analyzed countries. Some recent works address this specific problem

[1] For a collection of the major results, see Clarida et al. (1999).
[2] In the sense of Lucas Jr. (1976).
[3] See Goodfriend and King (1997) for a summary.

by deriving a hybrid NKPC, with both forward-looking and backward-looking elements.[4] This example shows that we can adapt the NKPC model to diverse price behaviors when considering the characteristics that may be found in the productive agents' decision-making process, for instance, supply shocks, lagged price changes, indexation, etc.

In the last few years, new elements have been added to the standard new-Keynesian model in order to render it more suitable to empirical observations. Of special interest is economic openness, where the standard closed-economy model is adapted to an open economy. Some interesting examples include Galí and Monacelli (2002), Razin and Yuen (2002), and Campos and Nakane (2003). However, as far as we know, there is no structural econometric paper explicitly based on an open-economy microfounded model. Taking into account trade liberalization in the analysis of the Brazilian economy may have a remarkable impact, both quantitatively and qualitatively. In this context, there is some interest in the validity of the NKPC model for the Brazilian economy, which is regarded as a small open economy. This peculiar aspect should be taken into consideration when constructing the microeconomic price decision-making model of firms so as to generate an NKPC model with structural parameters that suit this reality.

This paper is organized as follows. Section 2 specifies the theoretical model for our open economy. Section 3 lays out the standard price-setting model that generates the NKPC model, introducing the models for a closed and small open economy as special cases. Section 4 presents the model from Section 3 log-linearized around a steady state with zero inflation, which serves as the basis for the construction of several NKPC specifications (closed economy, open economy, small open economy and indexation) performed in Section 5. Section 6 presents the NKPC estimates, both in a closed and in an open economy. These results are analyzed in light of the recent behavior of Brazilian economy and compared with the results available in the literature for the United States and Europe. Section 7 concludes.

2. Economic Model

The framework is based on the models developed by Campos and Nakane (2003) and Galí and Monacelli (2002), who extend the dynamic general equilibrium model proposed by Woodford (2003) to an open economy. Two types of countries should be considered: the domestic country, which produces the goods indexed by H, and the rest of the world, denoted by an asterisk, which produces the goods indexed by F. Each economy is populated by identical representative individuals with infinite life. There is a continuum of differentiated goods, and all these goods are produced by firms that are engaged in monopolistic competition.

[4] See Galí and Gertler (1999) and Galí et al. (2001).

We assume that domestic firms only produce in the domestic economy and that they only employ domestic workers, and that the rest of the world follows a similar behavior. Each type of good is produced with only one type of labor.

2.1 Families

2.1.1 Preferences

Consider an economy populated by a representative agent that seeks to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t; \xi_t\right) - v\left(H_t\left(i\right); \xi_t\right) \right] \right\} \tag{1}$$

where C_t is an aggregate consumption index that takes into account each of the goods provided, whereas $H_t\left(i\right)$ is the amount of labor of type i that is provided. Each of the differentiated goods (indexed by $i \in [0,1]$) uses a type of specialized labor for its production; labor of type i is used to produce differentiated good i. Each agent specializes in the provision of only one type of labor, acting competitively in that specific market. Moreover, ξ_t is an exogenous shock, β is the discount factor $(0 < \beta < 1)$ and E_0 is the expectation operator conditional on time 0.

The function $u\left(C_t; \xi_t\right)$ represents the agent's instantaneous utility of consuming the aggregate consumption index C_t, being an increasing concave function for every possible value of ξ_t. In turn, $v\left(H_t\left(i\right); \xi_t\right)$ represents the disutility of supplying the amount H_t of labor of type i; being an increasing convex function for every possible value of ξ_t.

2.1.2 Consumption indexes

The composite consumption index C_t is defined by:

$$C_t = \left[(1-\delta)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \delta^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where δ is a constant that help us measure economic openness, whereas $C_{H,t}$ and $C_{F,t}$ are the consumption indexes for domestically produced and imported goods defined as constant elasticity of substitution (CES) indexes, as described in Dixit and Stiglitz (1977): $C_{H,t} = [\int_0^1 C_{H,t}(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$ and $C_{F,t} = [\int_0^1 C_{F,t}(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$.

Under this specification, η measures the elasticity of substitution between domestically produced and imported goods. The elasticity of substitution among the goods in each category is given by θ. We assume $\eta > 1$ and $\theta > 1$.[5]

[5]The equation $C_t^* = [(\delta^*)^{\frac{1}{\eta}} \left(C_{H,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\delta^*)^{\frac{1}{\eta}} \left(C_{F,t}^*\right)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$ defines the consumption index for the rest of the world, where $C_{H,t}^*$ and $C_{F,t}^*$ are defined analogously to $C_{H,t}$ and $C_{F,t}$, where, for instance, $C_{H,t}^*$ is the consumption of domestically produced goods abroad.

2.1.3 Price indexes

Price index P_t is defined as the minimum expenditure that buys one unit of the consumption index C_t:

$$P_t \equiv \left[(1-\delta)\left(P_{H,t}\right)^{1-\eta} + \delta\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

where $P_{H,t} \equiv \left[\int_0^1 P_{H,t}\left(i\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}\left(i\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ are the price indexes for the domestically produced and for the imported goods, respectively.[6]

2.1.4 World demand

The optimal allocation of a given expenditure in each category of goods provides the demand functions:

$$C_{H,t}\left(i\right) = \left(\frac{P_{H,t}\left(i\right)}{P_{H,t}}\right)^{-\theta} C_{H,t}; \qquad C_{F,t}\left(i\right) = \left(\frac{P_{F,t}\left(i\right)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

for every $i \in [0,1]$.

The optimal allocation between domestically produced and imported goods implies that:

$$C_{H,t} = (1-\delta)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \qquad C_{F,t} = \delta\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

When the price indexes for domestically produced and imported goods are the same (as in the steady state around which the model is approximated), parameter δ corresponds to the amount of domestic consumption allocated to imported goods. It thus represents a natural parameter of economic openness.[7]

Define the aggregate goods produced at home and those produced abroad by:

$$Y_t = \left[\int_0^1 Y_t\left(i\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \text{ and } Y_t^* = \left[\int_0^1 Y_t^*\left(i\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

[6]The price index for the rest of the world, denominated in foreign currency, is defined by $P_t^* \equiv \left[\delta^*\left(P_{H,t}^*\right)^{1-\eta} + (1-\delta^*)\left(P_{F,t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$, where $P_{H,t}^*$ and $P_{F,t}^*$ are defined analogously to $P_{H,t}$ and $P_{F,t}$.

[7]Analogously, for the rest of the world, optimal allocation is $C_{H,t}^* = \delta^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*$ and $C_{F,t}^* = (1-\delta^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*$, where δ^* represents, in a steady state, the amount of consumption of the rest of the world allocated to domestically produced goods.

Let $C^*_{H,t}(i)$ be the demand of the rest of the world for domestically produced good i. For domestic markets to reach an equilibrium, it is essential that

$$Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} [(1-\delta) C_t + \delta^* (q_t)^{\eta} C^*_t] \tag{3}$$

where $q_t = \frac{e_t P^*_t}{P_t}$ denotes the real exchange rate, *i.e.*, the ratio between consumer price indexes denoted in the same currency, where e_t is the nominal exchange rate (price of foreign currency *vis-à-vis* the domestic currency). In addition, we denote the terms of trade of the economy, *i.e.*, the price of imported goods in comparison with that of domestically produced goods, by $TT_t = P_{F,t}/P_{H,t}$.

Analogously, for the rest of the world, we have

$$Y^*_t(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\theta} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} [\delta C_t + (1-\delta^*)(q_t)^{\eta} C^*_t] \tag{4}$$

Using (3) and (4) to calculate the aggregates in (2) we have

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} [(1-\delta) C_t + \delta^* (q_t)^{\eta} C^*_t] \tag{5a}$$

$$Y^*_t = \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} [\delta C_t + (1-\delta^*)(q_t)^{\eta} C^*_t] \tag{5b}$$

then

$$Y_t(i) = Y_t \left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\theta} \text{ and } Y^*_t(i) = Y^*_t \left[\frac{P_{F,t}(i)}{P_{F,t}}\right]^{-\theta} \tag{6}$$

2.1.5 Consumer's budget constraint

The maximization of (1) is subject to a range of budget constraints of the form

$$\int_0^1 [P_{H,t}(i) C_{H,t}(i) + P_{F,t}(i) C_{F,t}(i)]\, di + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t(i) H_t(i) + T_t$$

for $t = 0, 1, 2, ...$, where $P_{H,t}(i)$ and $P_{F,t}(i)$ denote the prices of domestically produced and imported good i, D_{t+1} is the nominal payoff at $t+1$ of the portfolio set up at the end of t (which includes participations in firms), W_t is the nominal wage and T_t denotes lump-sum transfers. All of these variables are denominated in domestic currency. $Q_{t,t+1}$ is the stochastic discount factor for nominal payoffs.

Families have access to a whole set of contingent assets that are negotiated abroad. Note that money is not present in the budget constraint or in the utility function. This modeling strategy is used in most of recent studies on monetary policy where this policy is specified according to an interest rate rule, without the need of explicitly introducing money in the model.[8]

After considering consumption indexes and aggregate prices, the budget constraint can be rewritten as:

$$P_t C_t + E_t \{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t(i) H_t(i) + T_t \tag{7}$$

Therefore, the first-order conditions resulting from the family's problem represented by (1) subject to (7) are:

$$\frac{v_h(H_t(i); \xi_t)}{u_c(C_t; \xi_t)} = \frac{W_t}{P_t} \tag{8}$$

which represents the labor supply, and:

$$\beta \left(\frac{u_c(C_{t+1}; \xi_{t+1})}{u_c(C_t; \xi_t)} \right) \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \tag{9}$$

that stands for consumption allocation.

By applying the conditional expectation operator on both sides of (9) and by rearranging the terms, we obtain a conventional stochastic Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{u_c(C_{t+1}; \xi_{t+1})}{u_c(C_t; \xi_t)} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{10}$$

where $R_t^{-1} = E_t \{Q_{t,t+1}\}$ is the price of a riskless short-term (one period) asset (denominated in domestic currency) and, therefore, R_t is its gross return.

Considering complete financial markets, a first-order condition analogous to (10) is also valid for foreign consumers:

$$\beta \left(\frac{u_c(C^*_{t+1}; \xi^*_{t+1})}{u_c(C^*_t; \xi^*_t)} \right) \left(\frac{P^*_t}{P^*_{t+1}} \right) \left(\frac{e_t}{e_{t+1}} \right) = Q_{t,t+1} \tag{11}$$

By combining (10) and (11), together with the definition of real exchange rate and under the assumption of a stationary solution, we obtain:

$$\begin{aligned} \left(\frac{u_c(C_t; \xi_t)}{u_c(C^*_t; \xi^*_t)} \right) q_t &= E_t \left\{ \left(\frac{u_c(C_{t+1}; \xi_{t+1})}{u_c(C^*_{t+1}; \xi^*_{t+1})} \right) q_{t+1} \right\} \\ &= E_t \left\{ \left(\frac{u_c(C_{t+j}; \xi_{t+j})}{u_c(C^*_{t+j}; \xi^*_{t+j})} \right) q_{t+j} \right\} = \vartheta \end{aligned}$$

[8] For further details, see Woodford (2003).

where $\vartheta \equiv \left(\frac{u_c(\bar{C})}{u_c(\bar{C}^*)} \right) \bar{q}$ and, therefore:

$$u_c\left(C_t;\xi_t\right) = \vartheta \frac{u_c\left(C_t^*;\xi_t^*\right)}{q_t} \tag{12}$$

for every t.

2.2 Firms

There is a continuum of differentiated goods in which each good is produced by only one firm under monopolistic competition. The imperfect competition environment has two implications: (i) firms are price setters; and (ii) the equilibrium output is lower than a socially optimal one.[9]

The aim of every producer is to maximize profits by setting a price for the produced good:

$$\begin{cases} \underset{P_{H,t}(i)}{Max}\, \Pi_t\left(i\right) = Y_t\left(i\right) P_{H,t}\left(i\right) - P_{H,t} CV_t\left(i\right) \\ s.a. \quad Y_t\left(i\right) = Y_t \left[\frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\theta} \end{cases}$$

where $Y_t\left(i\right)$ is the demand for good i given by (6) and $CV_t\left(i\right)$ is the real variable cost (in terms of the consumption basket $C_{H,t}$) for producing $Y_t\left(i\right)$ units of i, which is described next.

The production technology for good i is given by $Y_t\left(i\right) = A_t f\left(H_t\left(i\right)\right)$, where $A_t > 0$ is a time-varying exogenous factor and $f\left(\cdot\right)$ is an increasing concave function.

The nominal variable cost of amount $Y_t\left(i\right)$ of good i is given by:

$$CV_t^n\left(i\right) = W_t\left(i\right) H_t\left(i\right) = W_t\left(i\right) f^{-1}\left(Y_t\left(i\right)/A_t\right) \tag{13}$$

Differentiating (13), we obtain the nominal marginal cost of supplying good i:

$$CM_t^n\left(i\right) = \frac{W_t\left(i\right)}{A_t} \Psi\left(Y_t\left(i\right)/A_t\right) \tag{14}$$

where $\Psi\left(Y_t\left(i\right)\right) = \frac{1}{f'\left(f^{-1}\left(Y_t(i)\right)\right)}$ is a positive increasing function.

By substituting the labor supply function (8) into (14), we obtain:

$$CM_t^n\left(i\right) \equiv P_t \frac{v_h\left(Y_t\left(i\right)/A_t;\xi_t\right)}{A_t u_c\left(C_t;\xi_t\right)} \Psi\left(Y_t\left(i\right)/A_t\right)$$

Thus, the real variable cost in terms of the consumption basket $C_{H,t}$ of good i can be expressed as:

[9] The socially optimal output is the one that a central planner would choose to be produced by every firm in order to maximize welfare, without budget constraints, and subject only to the relationships between consumption, working hours, and aggregate output.

$$CV_t(i) = \frac{W_t(i)}{P_{H,t}} H_t(i) = \left(\frac{P_t}{P_{H,t}}\right) CV\left(Y_t(i), C_t; \tilde{\xi}_t\right) \tag{15}$$

where $CV\left(Y_t(i), C_t; \tilde{\xi}_t\right) = CV_t^n(i)/P_t$ is the real variable cost function in terms of consumption basket C_t and $\tilde{\xi}_t$ represents the effect from shocks $\{\xi_t, A_t\}$.

The real marginal cost in terms of the consumption basket $C_{H,t}$ is given by:

$$CM_t(i) = \frac{P_t}{P_{H,t}} \frac{v_h\left(Y_t(i)/A_t; \xi_t\right)}{A_t u_c\left(C_t; \xi_t\right)} \Psi\left(Y_t(i)/A_t\right) = \frac{P_t}{P_{H,t}} s_t\left(Y_t(i), C_t; \tilde{\xi}_t\right)$$

where $s_t\left(Y_t(i), C_t; \tilde{\xi}_t\right)$ is the real marginal cost function in terms of consumption basket C_t.

By using expression (15), it is possible to rewrite the firm's problem as:

$$\underset{P_{H,t}(i)}{Max}\, \Pi_t(i) = Y_t\left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\theta} P_{H,t}(i) - P_t CV\left(Y_t\left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\theta}, C_t, ; \tilde{\xi}_t\right) \tag{16}$$

whose first-order condition yields:

$$P_{H,t}^f(i) = \mu CM_t^n(i) \tag{17}$$

The optimal price when prices are flexible, $P_{H,t}^f(i)$, is obtained by applying a mark-up $\mu \equiv \frac{\theta}{\theta-1} > 1$ to the nominal marginal cost $CM_t^n(i) \equiv P_{H,t} CM_t(i)$. Note that the lower the degree of substitutability among domestic goods (*i.e.*, the lower θ is), the higher the mark-up.

2.3 Price setting: Calvo's model

Now we consider a situation in which prices are not completely flexible in each time period. An approach regarded as standard for the modeling of nominal price rigidity, due to its simplicity, is the one developed by Calvo (1983). In this model, a fraction $0 < \alpha < 1$ of prices remain fixed every period, whereas new prices are chosen for the other $1-\alpha$ goods. Thus, the likelihood of a given price being adjusted in any period is $1-\alpha$, regardless of the time elapsed since its last update and independently of its current value.

This assumption, albeit unrealistic, remarkably simplifies the analysis of inflation dynamics. In effect, as each firm, by choosing a new price for its product at t, has to deal with the same problem, the optimal price $P_{H,t}^c$ is same for each one of them and, in equilibrium, all prices chosen at t have the same value $P_{H,t}^c$. Formally, let $P_{H,t}^c(i)$ be the price set by firm i while adjusting its price at t. Under Calvo's price setting model, $P_{H,t+k}(i) = P_{H,t}^c(i)$ with probability α^k for $k = 0, 1, 2, \ldots$.

Note that the average length of time during which a given price remains valid is given by $(1-\alpha)\sum_{k=0}^{\infty} k\alpha^{k-1} = (1-\alpha)^{-1}$.

Based on the facts shown above, by choosing a new price at t, firm i seeks to maximize the current value of its profit flow, conditional on the choice of the price:

$$\begin{cases} \underset{P^c_{H,t}}{Max} \sum_{k=0}^{\infty} \alpha^k E_t \left\{ Q_{t,t+k} \Pi^c_{t+k}(i) \right\} \\ s.a.\ Y_{t+k}(i) = Y_{t+k} \left[\frac{P^c_{H,t}}{P_{H,t+k}} \right]^{-\theta} \end{cases}$$

where $\Pi^c_{t+k}(i)$ is defined in (16) but with $P_{H,t+k}(i) = P^c_{H,t}(i)$.

Since all firms that choose their prices in a given period choose the same price, we should overlook subscript i. Therefore, the first-order condition of the problem above indicates that the firm should fix $P^c_{H,t}$ so as to comply with the following:

$$P^c_{H,t} = \mu \frac{E_t \left\{ \sum_{k=0}^{\infty} \Omega_{t+k} CM^n_{t+k} \right\}}{E_t \left\{ \sum_{k=0}^{\infty} \Omega_{t+k} \right\}} \tag{18}$$

Now, the optimal price is obtained by applying a mark-up on a **weighted average of all marginal costs expected in the future**, with weighting factor $\Omega_{t+k} \equiv \alpha^k Q_{t,t+k} \left(P_{H,t+k}\right)^{\theta} Y_{t+k}$. Observe that in the absence of nominal price rigidity ($\alpha = 0$) equation (18) becomes (17), demonstrating that when the firm can adjust its prices in each period, future economic conditions become less important in setting the current price.

3. Linearized Model

3.1 Steady state

The next step for NKPC derivation is to log-linearize the main equations in our model. In other words, let us suppose an environment where any exogenous disturbance encompasses only small variances over time. Let us log-linearize this model around a steady state with the following characteristics: $\bar{P}^c_H = \bar{P}_H = \bar{P}_F = \bar{P}$, $\overline{TT} = \bar{q} = 1$, $\bar{C} = \bar{Y}$, $\bar{C}^* = \bar{Y}^*$, where variable $\hat{X}_t$ represents the percentage deviation of variable X_t from its steady state value $\bar{X}$; *i.e.* $\hat{X}_t \equiv \frac{X_t - \bar{X}}{\bar{X}}$.[10]

3.1.1 Some identities: domestic inflation, consumer inflation, real exchange rate and terms of trade

By log-linearizing the consumer price index equation around our steady state with $P_{H,t} = P_{F,t}$ we obtain:

[10] For small variances, $\hat{X}_t \approx \log\left(X_t/\bar{X}\right)$.

$$\hat{P}_t \equiv (1-\delta)\,\hat{P}_{H,t} + \delta \hat{P}_{F,t} = \hat{P}_{H,t} + \delta \widehat{TT}_t \tag{19}$$

where $\widehat{TT}_t \equiv \hat{P}_{F,t} - \hat{P}_{H,t}$.

It then follows that domestic inflation - defined as the rate of change in domestic prices, *i.e.*, $\hat{\pi}_{H,t} \equiv \hat{P}_{H,t} - \hat{P}_{H,t-1}$ – and consumer inflation $\hat{\pi}_t$ are related accordingly to:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \delta \widehat{\Delta TT}_t \tag{20}$$

making the difference between the two inflation measures proportional to the percentage variation in the terms of trade, with the coefficient of proportionality given by the trade liberalization parameter δ.

Purchasing power parity (PPP) holds, implying that:

$$\begin{aligned} P_{H,t}(i) &= e_t P^*_{H,t}(i) \,\forall\, i \in [0,1] \Longrightarrow P_{H,t} = e_t P^*_{H,t}, \\ P_{F,t}(i) &= e_t P^*_{F,t}(i) \,\forall\, i \in [0,1] \Longrightarrow P_{F,t} = e_t P^*_{F,t} \end{aligned}$$

By log-linearizing these expressions, we obtain:

$$\hat{P}_{H,t} = \hat{e}_t + \hat{P}^*_{H,t} \quad \text{and} \quad \hat{P}_{F,t} = \hat{e}_t + \hat{P}^*_{F,t}$$

By combining these results, we can rewrite the terms of trade as follows:

$$\widehat{TT}_t \equiv \hat{P}_{F,t} - \hat{P}_{H,t} = \hat{e}_t + \hat{P}^*_{F,t} - \hat{P}_{H,t}. \tag{21}$$

Likewise, for the rest of the world, we have:

$$\hat{P}^*_t \equiv \delta^* \hat{P}^*_{H,t} + (1-\delta^*)\,\hat{P}^*_{F,t} = \hat{P}^*_{F,t} - \delta^* \widehat{TT}_t \tag{22}$$

By combining (21) and (22):

$$(1-\delta^*)\,\widehat{TT}_t = \hat{e}_t + \hat{P}^*_t - \hat{P}_{H,t} \tag{23}$$

By log-linearizing the real exchange rate equation and substituting the result in (23), we have:

$$\hat{q}_t = (1-\delta^*)\,\widehat{TT}_t + \hat{P}_{H,t} - \hat{P}_t = (1-\delta-\delta^*)\,\widehat{TT}_t, \tag{24}$$

where the latter equality can be obtained by (19).

In other words, the real exchange rate (in log differences) is proportional to the terms of trade (in log differences), being the coefficient of proportionality an inverse function of the domestic and foreign economic openness. Note that, even though the PPP is considered separately for each good, the real exchange rate can still fluctuate over time due to relative price movements between the domestic and foreign consumption baskets, which usually differ as to their compositions.

3.2 Price setting by the firm

Define $p^c_{H,t} \equiv \frac{P^c_{H,t}}{P_{H,t}}$. By log-linearizing condition (18) around our steady state, we get:

$$\hat{p}^c_{H,t} = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left\{\widehat{CM}_{t+k} + \sum_{j=1}^{k}\hat{\pi}_{H,t+j}\right\} \tag{25}$$

where $\widehat{CM}_{t+k}$ is the log-linear approximation of the marginal cost to its steady state value μ^{-1}.

The expression (25) can be rewritten in a more concise fashion. By $\hat{p}^c_{H,t} - (\alpha\beta)\, E_t\left\{\hat{p}^c_{H,t+1}\right\}$, we obtain:

$$\hat{p}^c_{H,t} = (1-\alpha\beta)\,\widehat{CM}_t + (\alpha\beta)\, E_t\left\{\hat{\pi}_{H,t+1}\right\} + (\alpha\beta)\, E_t\left\{\hat{p}^c_{H,t+1}\right\} \tag{26}$$

3.3 Price level dynamics

Under the nominal price rigidity model proposed by Calvo, all the prices chosen at a given time t have the same value denoted by $P^c_{H,t}$ when in equilibrium. The remaining fraction α of prices charged at t is simply a subset of prices charged at $t-1$, where each price appears in the distribution of unchanged prices at t with the same relative frequency of price distribution of $t-1$ (it is crucial that each price have the same probability of being adjusted in a given period). Therefore, the dynamics of the Dixit-Stiglitz domestic price index at t can be described by equation:

$$P_{H,t} \equiv \left[\alpha\left(P_{H,t-1}\right)^{1-\theta} + (1-\alpha)\left(P^c_{H,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{27}$$

To determine the behavior of this price index, only its initial value and optimal price $P^c_{H,t}$ chosen in each period are necessary. The determination of $P^c_{H,t}$, however, relies on the current and future demand for good i, but (6) implies that other prices affect the demand curve for good i only through the value of price index P_t. Thus, it is possible to determine the equilibrium value of index P_t as a function of its value in the previous period, of the expected future path of this index, and of the current and future values of the aggregate real variables. In short, there is no need for additional information on past prices.

The log-linearized version of equation (27) takes the form:

$$\hat{p}^c_{H,t} = \left(\frac{\alpha}{1-\alpha}\right)\hat{\pi}_{H,t} \tag{28}$$

3.4 New-Keynesian Phillips curve as a function of marginal cost

By substituting (28) into (26), we get:

$$\hat{\pi}_{H,t} = \kappa^n \widehat{CM}_t + \beta E_t \left\{\hat{\pi}_{H,t+1}\right\} \tag{29}$$

where the coefficient $\kappa^n \equiv (1-\alpha\beta)(1-\alpha)/\alpha$ is a function of the adjustment frequency of prices α and of the intertemporal discount factor β and indicates the level of **nominal rigidity** of the economy.

As a traditional Phillips curve, inflation depends positively on the determination of the level of activity and on a term that reflects the effect of expected inflation. A key difference is that the relevant expectation is $E_t\{\hat{\pi}_{H,t+1}\}$ instead of $E_{t-1}\{\hat{\pi}_{H,t}\}$. Consequently, inflation depends exclusively on the expected future path of the marginal cost, as can be more easily observed by iterating (29) forward, thus obtaining:

$$\hat{\pi}_{H,t} = \kappa^n \sum_{k=0}^{\infty} \beta^k E_t \left\{\widehat{CM}_{t+k}\right\}$$

expliciting that the price decision in this model (as in a closed economy) is forward-looking; inflation depends not only on the current marginal cost, but also on future marginal costs, although such dependence decreases as we look further to the future.

The reason, as previously described in (18), is simple: firms adjusting their prices in a given time period acknowledge that the chosen price remains unchanged for a random number of periods. As a result, the firms choose the price as a mark-up over a weighted average of future marginal costs, instead of considering the current marginal cost only.

3.5 Domestic consumption and world production

In a closed economy, there is a direct relationship between consumption and production, with $C_t = Y_t$, $\forall t$. In an open economy, however, this relationship is not direct, as can be confirmed in (5a). We want a (log-linear) relationship between domestic consumption and the levels of domestic and foreign production. By log-linearizing (12), we have:

$$\hat{C}_t = \hat{C}_t^* + \sigma \hat{q}_t \tag{30}$$

where σ^{-1} is the elasticity of substitution of consumption for both countries.

The log-linear approximation of (5a) yields:

$$\hat{Y}_t = (1-\delta)\hat{C}_t + \delta \hat{C}_t^* + \eta\left(\hat{P}_t - \hat{P}_{H,t}\right) + \eta\delta\hat{q}_t$$

which combined with (30) results in:

$$\hat{Y}_t = \hat{C}_t + \delta\left(\eta - \sigma\right)\hat{q}_t + \eta\left(\hat{P}_t - \hat{P}_{H,t}\right) \tag{31}$$

Likewise, we have the following expression for the foreign output:

$$\hat{Y}_t^* = \hat{C}_t + \left(1 - \delta^*\right)\left(\eta - \sigma\right)\hat{q}_t + \eta\left(\hat{P}_t - \hat{P}_{F,t}\right) \tag{32}$$

To establish a relation between $\hat{P}_t - \hat{P}_{H,t}$ and the domestic output, we first combine (24) with (19) to obtain:

$$\hat{q}_t = \left(\frac{1-\delta-\delta^*}{\delta}\right)\left(\hat{P}_t - \hat{P}_{H,t}\right) \quad \text{and} \quad \hat{q}_t = \left(\frac{1-\delta-\delta^*}{\delta - 1}\right)\left(\hat{P}_t - \hat{P}_{F,t}\right) \tag{33}$$

By substituting the equations above into (31) and (32):

$$\hat{Y}_t = \hat{C}_t + \left[\eta + \left(\eta - \sigma\right)\left(1 - \delta - \delta^*\right)\right]\left(\hat{P}_t - \hat{P}_{H,t}\right) \tag{34}$$

$$\hat{Y}_t^* = \hat{C}_t + \left[\eta\left(\frac{\delta - 1}{\delta}\right) + \left(1 - \delta^*\right)\left(\eta - \sigma\right)\left(\frac{1-\delta-\delta^*}{\delta}\right)\right]\left(\hat{P}_t - \hat{P}_{H,t}\right) \tag{35}$$

By subtracting the second equation from the first one, we have:

$$\hat{P}_t - \hat{P}_{H,t} = \varsigma\left[\hat{Y}_t - \hat{Y}_t^*\right] \tag{36}$$

where $\varsigma \equiv \delta/\eta - \left(\eta - \sigma\right)\left(1 - \delta - \delta^*\right)^2$.

Finally, by substituting the latter equation into (34), we have a relation between domestic consumption and domestic and foreign output:

$$\hat{C}_t = \left(1 - \varsigma\chi\right)\hat{Y}_t + \varsigma\chi\hat{Y}_t^* \tag{37}$$

where $\chi \equiv \eta + \left(\eta - \sigma\right)\left(1 - \delta - \delta^*\right)$.

3.6 Marginal cost

From the equation for the marginal cost and (6), we obtain the following equation for the real marginal cost in terms of the consumption basket $C_{H,t}$ for the firms randomly selected to adjust their prices at t:

$$CM_{t+k} = \left(\frac{P_{t+k}}{P_{H,t+k}}\right) s_t\left(\left(\frac{P_{H,t}^c}{P_{H,t+k}}\right)^{-\theta} Y_{t+k}, C_{t+k}; \tilde{\xi}_{t+k}\right)$$

By log-linearizing and using (19), this equation becomes:

$$\widehat{CM}_{t+k} = \omega\hat{Y}_{t+k} + \sigma^{-1}\hat{C}_{t+k} - \omega\theta\left[\hat{p}_{H,t}^c - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] + \lambda\tilde{\xi}_{t+k} + \delta\widehat{TT}_{t+k} \tag{38}$$

where $\omega > 0$ is the elasticity of the real marginal cost in terms of the aggregate consumption basket relative to the firm's production, $\sigma^{-1} > 0$ is the elasticity of the real marginal cost in terms of the aggregate consumption basket relative to the level of aggregate output of the economy, and $\lambda > 0$ is the elasticity of the marginal cost in terms of the aggregate consumption basket relative to $\tilde{\xi}_t$, all of them considered to be in steady state.

By substituting (37) into (38), we obtain a relation between the variation in marginal cost and the variations in domestic output, in foreign output, and in the terms of trade:

$$\begin{aligned} \widehat{CM}_{t+k} &= \left(\omega + \sigma^{-1}\left[1 - \varsigma\chi\right]\right)\hat{Y}_{t+k} + \left(\sigma^{-1}\varsigma\chi\right)\hat{Y}^*_{t+k} + \delta\widehat{TT}_{t+k} \\ &- \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] + \lambda\tilde{\xi}_{t+k} \end{aligned} \tag{39}$$

Marginal cost is affected by the terms of trade and foreign output. Both variables wind up affecting the real wage due to their effect on labor supply resulting from their impact on domestic consumption. The influence of technology (by means of its direct effect on labor productivity) and of domestic output (by means of its effect on employment and real wage) is analogous to that which is observed in a closed economy, with exception of the factor $\varsigma\chi$ (function of parameters δ, δ^*, η and σ) directly related to economic openness. The higher this factor, the lesser/greater the impact of the variation in domestic/foreign output on the variation of the marginal cost.

From (24), (33) and (36) we have:

$$\delta\widehat{TT}_{t+k} = \varsigma\left[\hat{Y}_{t+k} - \hat{Y}^*_{t+k}\right] \tag{40}$$

By substituting (40) into (39), we obtain a relation between the real marginal cost of firms randomly selected to adjust their prices at t and the domestic and foreign output:

$$\begin{aligned} \widehat{CM}_t &= \left(\omega + \sigma^{-1} - \varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}_t + \left(\varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}^*_t \\ &- \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] + \lambda\tilde{\xi}_t \end{aligned}$$

4. New-Keynesian Phillips Curve (NKPC)

4.1 Closed economy

In a closed economy, we have $\delta = \delta^* = 0$ and therefore $\pi_t = \pi_{H,t}$ and $\varsigma = 0$. The Phillips curve as a function of the marginal cost and the equation for the marginal cost become:

$$\begin{aligned}\hat{\pi}_t &= \kappa^n \widehat{CM}_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\} \\ \widehat{CM}_t &= \left(\omega + \sigma^{-1}\right)\hat{Y}_t - \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] + \lambda\tilde{\xi}_t\end{aligned}$$

4.1.1 Potential output and output gap

Define the output gap x_t as the deviation (in log differences) of output $\hat{Y}_t$ from its natural level or potential output $\hat{Y}^n_t$, where the latter is defined as the equilibrium level of output in the absence of nominal rigidity. Formally: $x_t \equiv \hat{Y}_t - \hat{Y}^n_t$.

The natural level of output can be obtained after imposing the steady state value on the marginal cost, *i.e.*, $CM\left(Y^n_t, Y^n_t; \tilde{\xi}_t\right) = \mu^{-1}$ for every t, by log-linearizing this relation in order to obtain:

$$\left(\omega + \sigma^{-1}\right)\hat{Y}^n_t = -\lambda\tilde{\xi}_t$$

Then, the equation for marginal cost becomes:

$$\widehat{CM}_t = \left(\omega + \sigma^{-1}\right)x_t - \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right]$$

By substituting this relation into (25), letting $\hat{p}^c_{H,t} - (\alpha\beta)\, E_t\left\{\hat{p}^c_{H,t+1}\right\}$ and by substituting (28) in the resulting equation, we obtain the renowned **closed-economy new-Keynesian Phillips curve**:

$$\hat{\pi}_t = \kappa x_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\} \tag{41}$$

where $\kappa \equiv \kappa^n\kappa^r$, $\kappa^n \equiv (1-\alpha\beta)(1-\alpha)/\alpha$ and $\kappa^r \equiv \left(\omega + \sigma^{-1}\right)(1+\omega\theta)^{-1}$.

The coefficient κ now depends on two factors: κ^n and κ^r. The first one, as we have already seen, indicates the level of nominal rigidity of the economy. The second one, function of the price elasticity of demand θ and of elasticities (in steady state) of the real marginal cost relative to the firm's production, ω, and relative to the level of output, σ^{-1}, measures the level of **real rigidity** of the economy, which is related to **strategic complementarity/substitutability** among various producers.[11]

The existence or not of strategic complementarity is that determines whether the fraction of producers with sticky prices has some disproportionate effect on the level of adjustment of aggregate prices.

[11] Price decisions are strategic complements if an increase in the prices charged for other goods increases the optimal price by a given producer. For further details, see Woodford (2003).

4.2 Open economy

In an open economy, we have:

$$\begin{aligned}
\hat{\pi}_{H,t} &= \kappa^n \widehat{CM}_t + \beta E_t\left\{\hat{\pi}_{H,t+1}\right\} && (42)\\
\widehat{CM}_t &= \left(\omega + \sigma^{-1} - \varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}_t + \left(\varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}_t^* \\
&- \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] + \lambda\tilde{\xi}_t && (43)\\
\hat{\pi}_t &= \hat{\pi}_{H,t} + \left(\frac{\delta}{1-\delta-\delta^*}\right)\Delta\hat{q}_t && (44)
\end{aligned}$$

4.2.1 Potential output and output gap

Define again the output gap x_t as the deviation (in log differences) of output $\hat{Y}_t$ from its natural level or potential output $\hat{Y}_t^n$, where the latter is now defined as the equilibrium level of output in the absence of nominal rigidity **both in the domestic economy and in the rest of the world**. Formally:

$$\left(\omega + \sigma^{-1} - \varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}_t^n + \left(\varsigma\left[\sigma^{-1}\chi - 1\right]\right)\hat{Y}_t^{*n} + \lambda\tilde{\xi}_t = 0$$

Then, the equation for the marginal cost becomes:

$$\widehat{CM}_t = \left(\omega + \sigma^{-1} - \varsigma\left[\sigma^{-1}\chi - 1\right]\right)x_t + \left(\varsigma\left[\sigma^{-1}\chi - 1\right]\right)x_t^* - \omega\theta\left[\hat{p}^c_{H,t} - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right] \quad (45)$$

By substituting this relation into (25), letting $\hat{p}^c_{H,t} - (\alpha\beta)\,E_t\left\{\hat{p}^c_{H,t+1}\right\}$ and by substituting (28) in the resulting equation, we have our open-economy new-Keynesian Phillips curve for domestic inflation:

$$\hat{\pi}_{H,t} = \kappa^{(\varsigma,\chi)}x_t + \kappa^{(\varsigma,\chi)*}x_t^* + \beta E_t\left\{\hat{\pi}_{H,t+1}\right\} \quad (46)$$

where $\kappa^{(\varsigma,\chi)} \equiv (1-\alpha\beta)(1-\alpha)\left(\omega + \sigma^{-1} - \varsigma\left[\sigma^{-1}\chi - 1\right]\right)/\alpha\,(1+\omega\theta)$ and $\kappa^{(\varsigma,\chi)*}$ $\equiv (1-\alpha\beta)(1-\alpha)\left(\varsigma\left[\sigma^{-1}\chi - 1\right]\right)/\alpha\,(1+\omega\theta)$.

Finally, by substituting (44) into (46), we have the **open-economy NKPC for consumer inflation**:

$$\hat{\pi}_t = \kappa^{(\varsigma,\chi)}x_t + \kappa^{(\varsigma,\chi)*}x_t^* + \beta E_t\left\{\hat{\pi}_{t+1}\right\} + \psi^{(\delta,\delta^*)}\left[\Delta\hat{q}_t - \beta E_t\left\{\Delta\hat{q}_{t+1}\right\}\right] \quad (47)$$

where $\psi^{(\delta,\delta^*)} \equiv \delta/(1-\delta-\delta^*)$.

Now, in addition to the inclusion of a term for the foreign output gap, we may observe that economic openness, represented by parameters ς and χ, affects domestic output gap. Also observe the impact of the real exchange rate on domestic inflation, which can be more easily demonstrated after we iterate equation:

$$\Gamma_t = \Upsilon_t + E_t \{\Gamma_{t+1}\} \tag{48}$$

obtaining:

$$\hat{\pi}_t = \psi^{(\delta,\delta^*)} \Delta \hat{q}_t + \sum_{k=0}^{\infty} \beta^k E_t \{\Upsilon_{t+k}\} \tag{49}$$

where $\Gamma_t \equiv \hat{\pi}_t - \psi^{(\delta,\delta^*)} \Delta \hat{q}_t$ and $\Upsilon_t \equiv \kappa^{(\varsigma,\chi)} x_t + \kappa^{(\varsigma,\chi)*} x_t^*$.

In an open economy, stabilization of inflation is related to the stabilization of the marginal cost (now affected by foreign output, and not liable to be controlled by the monetary authority) and to the stabilization of real exchange movements. Note that the impact of these two factors is closely related to the degree of openness of the domestic economy and of the foreign economy as well, by way of $\psi^{(\delta,\delta^*)}$, $\kappa^{(\varsigma,\chi)}$ and $\kappa^{(\varsigma,\chi)*}$.

4.2.2 Small open economy

If the domestic economy is regarded as small by the international market, it is possible to treat the rest of the world as a closed economy; *i.e.*, $\delta^* \to 0$ and therefore $\hat{Y}_t^* = \hat{C}_t^*$ and $\hat{P}_t^* = \hat{P}_{F,t}^*$.[12] Then:

$$\hat{\pi}_t = \kappa^{\delta} x_t + \kappa^{*\delta} x_t^* + \beta E_t \{\hat{\pi}_{t+1}\} + \psi^{\delta} \left[\Delta \hat{q}_t - \beta E_t \{\Delta \hat{q}_{t+1}\}\right] \tag{50}$$

where

$$\begin{aligned}
\kappa^{\delta} &= (1-\alpha\beta)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\omega + \sigma^{-1} - \varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta} - 1\right]}{1+\omega\theta}\right) \\
\kappa^{*\delta} &= \left(\frac{\varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta} - 1\right]}{1+\omega\theta}\right)\psi^{\delta} = \frac{\delta}{1-\delta} \\
\chi^{\delta} &\equiv \eta + (\eta - \sigma)(1-\delta), \qquad \varsigma^{\delta} \equiv \frac{\delta}{\eta - (\eta-\sigma)(1-\delta)^2}
\end{aligned}$$

Note that in this case the foreign output gap is exogenous to our economy. However, there exists another specification based on Galí and Monacelli (2002).

[12] Note that this procedure generates only one approximation for a small economy, not affecting the results in a considerable manner. However, a more careful approach would be to build a small world economy as a continuum of small open economies or to control the size (number of products) of each economy.

Define the output gap $\tilde{x}_t$ as the deviation (in log differences) of output $\hat{Y}_t$ from its natural level $\tilde{Y}_t^n$, where the latter is defined as the equilibrium level of output in the absence of nominal rigidity and **conditional on the output from the rest of the world**. Formally:

$$\left(\omega + \sigma^{-1} - \varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta} - 1\right]\right)\tilde{Y}_t^n + \left(\varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta} - 1\right]\right)\hat{Y}_t^* + \lambda\tilde{\xi}_t = 0$$

Therefore, the equation (45) for the marginal cost can be rewritten as

$$\widehat{CM}_t = \left(\omega + \sigma^{-1} - \varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta} - 1\right]\right)\tilde{x}_t - \omega\theta\left[\hat{p}_{H,t}^c - \sum_{i=1}^{k}\hat{\pi}_{H,t+k}\right]$$

yielding

$$\hat{\pi}_t = \kappa^{\delta}\tilde{x}_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\} + \psi^{\delta}\left[\Delta\hat{q}_t - \beta E_t\left\{\Delta\hat{q}_{t+1}\right\}\right] \tag{51}$$

Equation (51) is our **NKPC for a small open economy**.

4.3 Extension: Indexation

One of the problems caused by the totally forward-looking specification of the new-Keynesian Phillips curve is that it cannot explain the empirically observed inflationary inertia. To get around this problem, suppose now that each firm continues with fixed probability $1-\alpha$ of adjusting its prices in each period and, consequently, the probability α of keeping its price unchanged. Nevertheless, when not randomly selected, the firm adjusts its prices based on the past domestic inflation and on a coefficient of indexation $\gamma \in [0,1)$. So, if $\gamma \simeq 1$ indexation is almost complete, but if $\gamma = 0$ there is no indexation. In this case, the firm's problem becomes:

$$\begin{cases} \underset{P_{H,t}^c}{Max} \sum_{k=0}^{\infty} \alpha^k E_t\left\{Q_{t,t+k}\Pi_{t+k}^{\gamma}(i)\right\} \\ s.a.\ Y_{t+k}(i) = Y_{t+k}\left[\frac{P_{H,t}^c}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma}\right]^{-\theta} \end{cases}$$

where $\Pi_{t+k}^{\gamma}(i) = Y_{t+k}^c(i)\,P_{H,t}^c\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma} - P_{H,t}CV_t(i)$.

The first-order condition of the problem above indicates that the firm should fix $P_{H,t}^c$ so as to comply with the following:

$$P_{H,t}^c = \mu E_t\left\{\sum_{k=0}^{\infty}\Xi_{t+k}CM_{t+k}^n\right\}/E_t\left\{\sum_{k=0}^{\infty}\Xi_{t+k}.\right\}$$

Now, the optimal price is obtained by applying a mark-up over a weighted average of all marginal costs of production expected in the future, with weighting factor $\Xi_{t+k} \equiv \alpha^k\ Q_{t,t+k}\ (P_{H,t+k})^{\theta}\ Y_{t+k}\ \left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma(1-\theta)}$, where the nominal

marginal cost is equal to $CM^n_{t+k} = P_{H,t+k}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma} CM_{t+k}$. By log-linearizing this equation, we have:

$$\hat{p}^c_{H,t} = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left\{\widehat{CM}_{t+k} + \sum_{j=1}^{k}\left[\hat{\pi}_{H,t+j} - \gamma\hat{\pi}_{H,t+j-1}\right]\right\}$$

The dynamics of the domestic price index can be described by equation:

$$P_{H,t} \equiv \left[\alpha\left(P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\gamma}\right)^{1-\theta} + (1-\alpha)\left(P^c_{H,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

which can be log-linearized in order to yield:

$$\hat{p}^c_{H,t} = \left(\frac{\alpha}{1-\alpha}\right)\left[\hat{\pi}_{H,t} - \gamma\hat{\pi}_{H,t-1}\right]$$

By solving it as previously presented, we get:

$$\hat{\pi}_{H,t} - \gamma\hat{\pi}_{H,t-1} = \kappa^n\widehat{CM}_t + \beta E_t\left\{\hat{\pi}_{H,t+1} - \gamma\hat{\pi}_{H,t}\right\}$$

By combining it with (20) we have a Phillips curve relative to the marginal cost with indexation:

$$\Lambda_t = \kappa^n\widehat{CM}_t + \beta E_t\left\{\Lambda_{t+1}\right\} \tag{52}$$

where $\Lambda_t \equiv \hat{\pi}_t - \psi^{(\delta,\delta^*)}\Delta\hat{q}_t - \gamma\left[\hat{\pi}_{t-1} - \psi^{(\delta,\delta^*)}\Delta\hat{q}_{t-1}\right]$.

By iterating this equation, we have:

$$\hat{\pi}_t - \gamma\hat{\pi}_{t-1} = \psi^{(\delta,\delta^*)}\left[\Delta\hat{q}_t - \gamma\Delta\hat{q}_{t-1}\right] + \kappa^n\sum_{k=0}^{\infty}\beta^k E_t\left\{\widehat{CM}_{t+k}\right\}$$

Just as in a closed economy, we can observe **inflationary inertia**, represented by component $\gamma\hat{\pi}_{t-1}$. However, economic openness allows (for $\gamma \simeq 1$) the acceleration of prices to be influenced by exchange rate acceleration, an effect that expands as the liberalization of domestic and foreign economies increase.

Note that (52) is a generalization of some models described in the literature. By closing the economy ($\delta, \delta^* \to 0$), we obtain the hybrid NKPC developed by Woodford (2003) and Galí and Gertler (1999):[13]

$$\hat{\pi}_t - \gamma\hat{\pi}_{t-1} = \kappa^n\widehat{CM}_t + \beta E_t\left\{\hat{\pi}_{t+1} - \gamma\hat{\pi}_t\right\}$$

[13] Formally, the hybrid NKPC proposed by Galí and Gertler (1999) is slightly different, with the degree of price rigidity affecting the composite parameter associated to past inflation.

Additionally, by considering the total absence of indexation ($\gamma = 0$) we get the standard NKPC:

$$\hat{\pi}_t = \kappa^n \widehat{CM}_t + \beta E_t \{\hat{\pi}_{t+1}\}$$

Thus, we may use (52) as a general econometric specification, testing several special cases. This is shown in the subsequent section.

5. Estimation of NKPC for Brazil

This section estimates the model parameters, in reduced and structural forms. First, we consider Brazil a closed economy. This specification has two interesting characteristics: (i) it is used as a benchmark for comparison with open-economy models and (ii) it allows a more straightforward comparison with the results described in the literature that use this specification, especially for the United States and Europe. Therefore, our interest is to estimate:

$$\begin{aligned} \hat{\pi}_t &= \phi_1 \hat{\pi}_{t-1} + \tilde{\phi}_2 \widehat{CM}_t + \phi_3 E_t \{\hat{\pi}_{t+1}\} \\ \hat{\pi}_t &= \phi_1 \hat{\pi}_{t-1} + \tilde{\phi}_7 x_t + \phi_3 E_t \{\hat{\pi}_{t+1}\} \end{aligned}$$

where $\phi' s$ are functions of the six structural parameters: $\alpha, \beta, \gamma, \theta, \sigma^{-1}, \omega$.[14]

In what follows, we consider Brazil a small open economy. So, we are interested in estimating the following relations, as a function of nominal exchange rate and foreign inflation where $\tilde{q}_t \equiv \hat{e}_t + \hat{p}_t^*$:

$$\begin{aligned} \hat{\pi}_t &= \phi_1 \hat{\pi}_{t-1} + \phi_2 \widehat{CM}_t + \phi_3 E_t \{\hat{\pi}_{t+1}\} + \phi_4 \Delta \tilde{q}_{t-1} + \phi_5 \Delta \tilde{q}_t + \phi_6 E_t \{\Delta \tilde{q}_{t+1}\} \\ \hat{\pi}_t &= \phi_1 \hat{\pi}_{t-1} + \phi_7 \tilde{x}_t + \phi_3 E_t \{\hat{\pi}_{t+1}\} + \phi_4 \Delta \tilde{q}_{t-1} + \phi_5 \Delta \tilde{q}_t + \phi_6 E_t \{\Delta \tilde{q}_{t+1}\} \end{aligned}$$

where now the $\phi' s$ are functions of the previous six structural parameters, α, $\beta, \gamma, \theta, \sigma^{-1}, \omega$, and of the open-economy parameter δ.[15]

To make the comparison with the results obtained for the U.S and European economies easier, we use the same econometric strategy employed by Galí and Gertler (1999) and Galí et al. (2001). The basic idea is that, since under rational expectations the forecast error of Λ_{t+1} in (52) is uncorrelated with the information available up to t, we have:

[14] Formally, $\phi_1 \equiv \frac{\gamma}{1+\beta\gamma}$, $\phi_2 \equiv \left(\frac{1}{1+\beta\gamma}\right)\left(\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right)$, $\phi_3 \equiv \frac{\beta}{1+\beta\gamma}$, and $\phi_7 \equiv \left(\frac{1}{1+\beta\gamma}\right)\left(\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right)\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)$.

[15] Formally, $\phi_2 \equiv \left(\frac{1-\delta}{1+\beta\gamma}\right)\left(\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right)$, $\phi_4 \equiv -\left(\frac{\gamma\delta}{1+\beta\gamma}\right)$, $\phi_5 \equiv \delta$, $\phi_6 \equiv -\left(\frac{\beta\delta}{1+\beta\gamma}\right)$, and $\phi_7 \equiv \left(\frac{1-\delta}{1+\beta\gamma}\right)\left(\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right)\left(\frac{\omega+\sigma^{-1}-\varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta}-1\right]}{1+\omega\theta}\right)$. Parameters ϕ_1 and ϕ_3 are the same ones defined for the closed economy.

$$E_t \left\{ \left(\beta \left[\Lambda_{t+1}^e - \Lambda_{t+1} \right] \right) z_t \right\} = 0 \tag{53}$$

where z_t is a vector of variables for the information set at time t or in the previous period, and is orthogonal to the surprise in Λ at $t+1$. The orthogonality condition (53) is the basis for the estimation of the model, using the Generalized Method of Moments (GMM). In this study, instruments of period $t-1$ and those preceding it are used instead of instruments of time t and before it since, as argued by Galí et al. (2001), such instruments can tackle measurement errors in the marginal cost more adequately, and besides, variables dated at t are not always available when the agents form their expectations in that period. The instruments used are inflation measured by the broad consumer price index , known as IPCA (five lags), share of labor in production[16] as a measure of the marginal cost, output gap,[17] and wage inflation (two lags for each variable). Moreover, the open-economy model uses the nominal exchange rate movements plus the foreign U.S. inflation (five lags) as an additional instrument. The estimates are based on monthly data obtained from IPEAData, using a sample for the 1995:01 – 2003/09 period.[18]

Now we present the regression results. Initially, we are going to work with a closed-economy model, showing first the results in reduced form, and then the results in structural form. The estimates of a small open-economy model are subsequently displayed. The standard errors of parameters and J statistic and its respective p-value are reported for all estimations.[19] The heteroskedasticity and autocorrelation consistent (HAC) covariance matrix with Bartlett kernel and Newey-West fixed bandwidth are used for all regressions.

5.1 Closed economy

5.1.1 Reduced form

The reduced-form regression of Phillips curve seeks to obtain the coefficient values directly associated with the model variables. The coefficients were obtained for the specification where the marginal cost is used as a measure of real activity and for the specification with output gap. The following orthogonality conditions are used:

[16] This measure is obtained by multiplying the series of nominal personal income and working population, both from IPEAData, and by dividing the result by the monthly GDP series of Banco Central do Brasil. This measure can be justified for a production function of type $Y_t = A_t H_t$. So $CM_t \simeq L_t$, where $L_t = \frac{W_t H_t}{P_{H,t} Y_t}$.Then we have $\widehat{CM}_t \simeq \hat{L}_t$.

[17] Obtained as a residual of IBGE's industrial production regression on 11 seasonal dummy variables and a linear trend. Similar results can be obtained through the use of a Hodrick-Prescott filter.

[18] www.ipeadata.gov.br.

[19] J Statistic is used to test the validity of additional restrictions when the number of instruments is larger than the number of parameters to be estimated. Under the null hypothesis that additional restrictions are satisfied, statistic J times the number of observations in the regression is asymptotically χ^2 with degrees of freedom equal to the number of additional restrictions.

$$E_t\left\{\left(\hat{\pi}_t - \phi_1\hat{\pi}_{t-1} - \tilde{\phi}_2\widehat{CM}_t - \phi_3\hat{\pi}_{t+1}\right)z_t\right\} = 0 \quad (54a)$$

$$E_t\left\{\left(\hat{\pi}_t - \phi_1\hat{\pi}_{t-1} - \tilde{\phi}_7 x_t - \phi_3\hat{\pi}_{t+1}\right)z_t\right\} = 0 \quad (54b)$$

The results of these two specifications are shown in Table 1.

Table 1
NKPC in a closed economy – reduced form

	Reduced parameters			Test
	ϕ_1	ϕ_2/ϕ_7	ϕ_3	J
$\widehat{CM}_t$	0.444	0.001	0.538	7.637
	(0.062)	(0.002)	(0.073)	(0.571)
x_t	0.458	-0.007	0.520	7.442
	(0.058)	(0.009)	(0.070)	(0.591)

Note: The values in brackets represent the standard errors of parameters or p-values of the J test.

In general, the estimates are consistent with theory. Furthermore, the results for the parameters associated with inflation are reasonably consistent, regardless of the variable used to represent the level of real activity.

However, there are some remarkable differences. First, a comparison between the specification that use the marginal cost with the one that use the output gap indicates a problem previously described in the literature; the negative sign of the specification of the output gap contrasts with theory. On the other hand, the estimate associated with the marginal cost is not statistically significant, even though it has the expected sign.

The empirical studies on Phillips curves often use some measure of the output gap as a relevant indicator of the level of real economic activity. Nevertheless, as highlighted by Fuhrer and Moore (1995), the theoretical new-Keynesian Phillips curve model suggests that inflation should foresee the output gap in the business cycle, as can be observed by iterating equation (41):

$$\hat{\pi}_t = \kappa\sum_{k=0}^{\infty}\beta^k E_t\left\{x_{t+k}\right\}$$

This equation shows us that an increase in current inflation should represent a subsequent increase in the output gap. Figure 1 shows the cross correlation between output gap x_t and inflation $\hat{\pi}_t$.

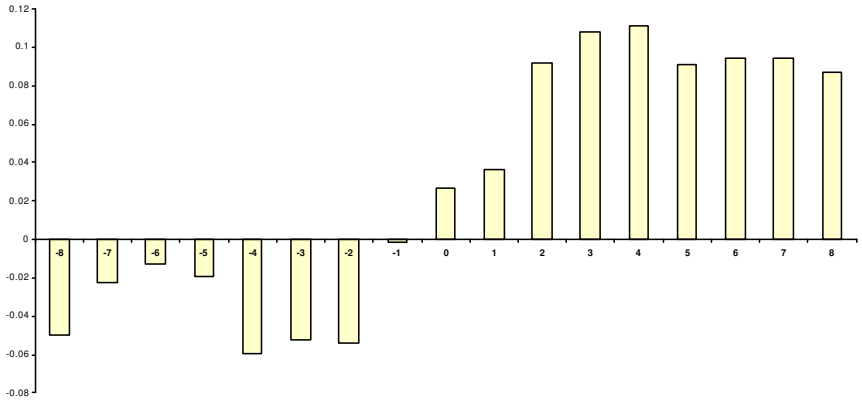


Figure 1
Output gap (t), inflation $(t+k)$

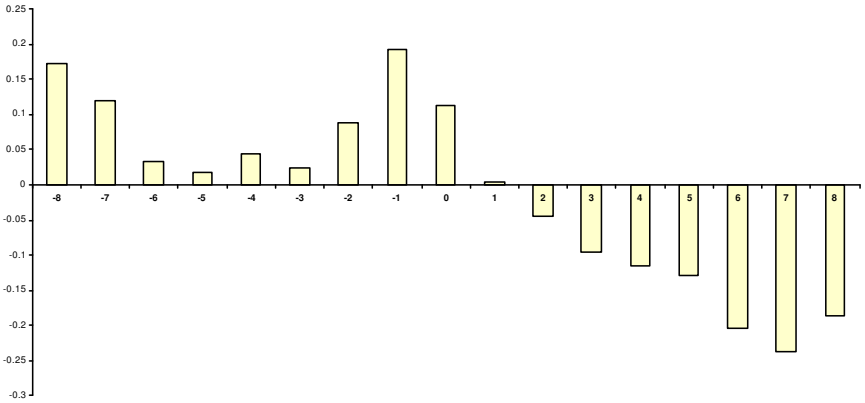


Figure 2
Marginal cost (t), inflation $(t+k)$

As shown in Figure 1, the output gap has a positive correlation with future inflation and a negative one with past inflation. In other words, the data show exactly the opposite behavior expected by the theory.

In order to avoid this problem, we follow Galí and Gertler (1999). The authors use a marginal cost measure in their estimation of the Phillips curve for the U.S.

economy.[20] The series we use to measure $\widehat{CM}_t$ is the share of labor in production. The correlogram of the marginal cost with inflation shows a behavior that is more consistent with the theoretical model, as shown in Figure 2:

Figures 3 and 4 show the behavior of the specification with marginal cost and output gap for a closed economy.

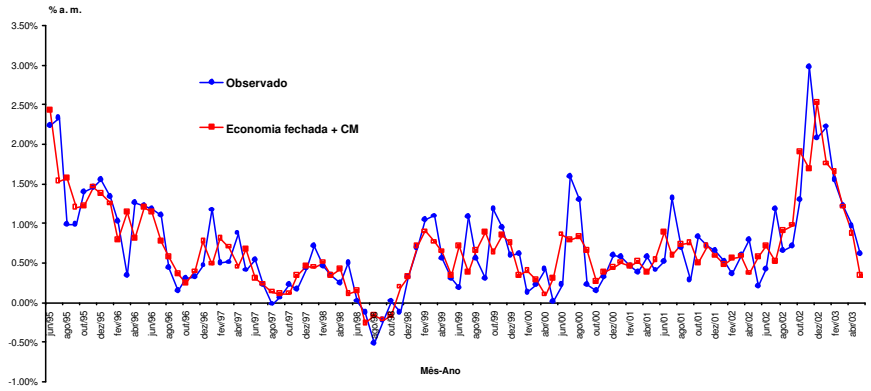


Figure 3
NKPC: Closed economy + marginal cost

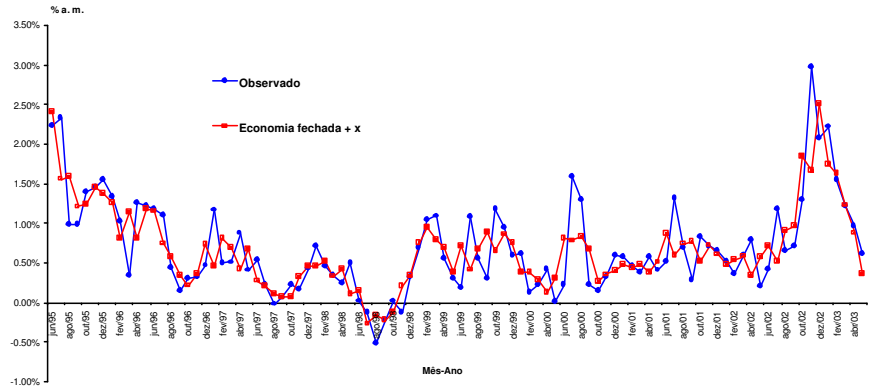


Figure 4
NKPC: Closed economy + output gap

[20] This measure is the labor income share.

5.1.2 Structural form

Now let us redo the previous exercise in such a way that we can directly obtain the structural parameters α, β and γ of our model. We don't present the estimates of parameters θ, σ^{-1}, and ω for two reasons: (1) As observed in the reduced-form estimation, the estimate that uses the output gap is not in line with theory, due to the difficulty in properly constructing this series, and (2) perhaps because of that, it was not possible to identify the composite parameter $\left(\frac{\omega+\sigma^{-1}-\varsigma^{\delta}\left[\sigma^{-1}\chi^{\delta}-1\right]}{1+\omega\theta}\right)$.

Again, we use the GMM with the same set of instruments. However, the nonlinear estimation via GMM in small samples is sensitive to the way by which orthogonality conditions are normalized.[21] For this reason, we use two alternative specifications.

The calibration of parameter β is also considered. According to (10), the steady-state real interest rate is equal to $R_t = \beta^{-1}$. By applying different alternative methods, Miranda and Muinhos (2003) estimate the equilibrium interest rate to range between 11 and 14% *p.a.* for the Brazilian economy. These values imply a calibration range around 0.99 for parameter β. Thus, two options for the value of parameter β were used in each of the alternative specifications for the orthogonality condition: in the first one, there is no restriction on the possible value of β, whereas in the second one, $\beta = 1$. The orthogonality conditions (1) and (2) are:

$$E_t\left\{\left(\begin{array}{c}\hat{\pi}_t - \left[\frac{\gamma}{1+\beta\gamma}\right]\hat{\pi}_{t-1} \\ -\left[\frac{1}{1+\beta\gamma}\right]\left[\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\widehat{CM}_t - \left[\frac{\beta}{1+\beta\gamma}\right]\hat{\pi}_{t+1}\end{array}\right) z_t\right\} = 0$$

$$E_t\left\{\left(\begin{array}{c}\left[1+\beta\gamma\right]\hat{\pi}_t - \gamma\hat{\pi}_{t-1} \\ -\left[\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\widehat{CM}_t - \beta\hat{\pi}_{t+1}\end{array}\right) z_t\right\} = 0$$

The results are shown in Table 2.

[21] See, for instance, Fuhrer et al. (1995) for further details on this issue.

Table 2
NKPC in a closed economy – structural form

	Structural parameters			Reduced parameters			Test
	α	β	γ	ϕ_1	ϕ_2	ϕ_3	J
(1)	0.990	0.889	0.733	0.444	0.001	0.538	7.638
	(0.035)	(0.149)	(0.134)	[0.444]	[0.001]	[0.538]	(0.571)
(2)	0.995	0.813	0.700	0.446	0.001	0.518	7.746
	(0.021)	(0.136)	(0.110)	[0.444]	[0.001]	[0.538]	(0.560)
$\beta = 1$							
(1)	0.957	1.000	0.731	0.422	0.001	0.578	7.972
	(0.048)	-	(0.187)	[0.444]	[0.001]	[0.538]	(0.632)
(2)	0.945	1.000	0.469	0.319	0.002	0.681	8.493
	(0.035)	-	(0.155)	[0.444]	[0.001]	[0.538]	(0.581)

Note: The values in brackets represent the standard errors of parameters or p-values of the J test.
The values in braces are the reduced-form estimates presented in Table 1.

The estimates of the structural parameters in a closed economy are relatively consistent, both between the two specifications for the orthogonality condition and between the results for restricted versus unrestricted β. However, some comments are necessary.

With method (1) we have $\alpha = 0.99$ with standard error equal to 0.04, which means that prices are set for approximately 8 years! This period is certainly too long and does not seem to correctly represent the expected behavior of the Brazilian economy. Nevertheless, note that small changes may remarkably affect this time horizon, since the average length of time during which a price stays unchanged is given by $1/(1-\alpha)$. By having a standard deviation, say, $\alpha = 0.95$, we have an average time of at least 2 years which, albeit long, is much closer to the expected value. The results improve a little bit when we work with $\beta = 1$. In this case, we have an average time between 1.5 and 2 years. However, these periods are still too long for Brazilian standards, especially when compared to those obtained in an open economy, as we will see later.

Now consider the indexation rate of the economy. Except for method (2) in case of $\beta = 1$, we have $\gamma \simeq 0.70$ with standard error between 0.11 and 0.20, which means that firms transfer nearly 3/4 of the past inflation. This information leads to an indexation rate that is no longer observed in Brazilian economic practice.

Finally, we may observe that the estimation of β is similar in both specifications, around 0.80. In fact, values closer to 0.90 should be expected. However, it is essential to perceive that restricting the value of β to 1 has little effect on the other parameters, except at level α of price rigidity, due to the sensitivity of this parameter to slight changes in its value. Even so, restricting β to values that are more consistent with theory does not affect the results in a significant manner.

Let us now observe the reduced parameters that originated from the estimation of the structural parameters. In general, the results are consistent with those obtained directly, and briefly inform that: (i) the backward-looking component is not negligible, with consistent estimates around 0.45, but (ii) the forward-looking

component is dominant, with values around 0.53, and (iii) the impact on the marginal cost is not statistically significant and has a negligible effect, standing in contrast to theory.

5.1.3 International comparison

In what follows, we compare our results with those obtained in the literature. Galí et al. (2001) presents estimates for the United States and Europe. These values are summarized in Table 3.

Table 3
International parameters: United States and Europe

	Structural parameters			Reduced parameters		
	α	β	γ	ϕ_1	ϕ_2	ϕ_3
United States						
(1)	0.827	0.898	0.451	0.364	0.020	0.599
(2)	0.818	0.878	0.400	0.347	0.026	0.610
Europe						
(1)	0.922	0.920	0.335	0.272	0.006	0.689
(2)	0.907	0.897	0.024	0.025	0.018	0.877

Some comments are necessary before performing the comparative analysis. The results for the United States and Europe are based on quarterly data, where inflation is measured by the GDP deflator. As we will see in the results for the open economy model, an index that includes producer prices has a reasonably different behavior from a consumer price index.

In general, the implications of these results for Europe, the United States, and Brazil are summarized in Table 4.

Table 4
International comparison: Brazil, United States and Europe

	Structural parameters		
	Average period (years)[a]	Real interests (% p.a.)[b]	Indexation (%)[c]
Brazil			
(1)	8.3	310	73.3
(2)	16.7	1099	70.0
United States			
(1)	1.4	54	45.1
(2)	1.4	68	40.0
Europe			
(1)	3.2	40	33.5
(2)	2.7	54	2.4

a. Average period during which a term remains unchanged: $(1-\alpha)^{-1}$.
b. Equilibrium real interest rate: β^{-1}.
c. Percentage of inflation from the previous period transferred to prices: γ.

With regard to structural parameters, Brazil shows high levels of rigidity and indexation compared to the United States and Europe. Despite unrealistic values, the discount parameter also implies a higher equilibrium real interest rate than that observed for the United States and Europe.

Consequently, the resulting reduced parameters have a heavier weight on lagged inflation to the detriment of expected future inflation, rendering the backward-looking component more important. In addition, the impact of the marginal cost is considerably lower in the Brazilian economy.

5.2 Open economy

5.2.1 Reduced form

We redo the previous exercise for the case of a small open economy. Again, the marginal cost and the output gap are used to measure the level of activity. The orthogonality conditions used are:

$$E_t\left\{\begin{pmatrix} \hat{\pi}_t - \phi_1\hat{\pi}_{t-1} - \phi_2\widehat{CM}_t - \phi_3\hat{\pi}_{t+1} \\ -\phi_4\Delta\tilde{q}_{t-1} - \phi_5\Delta\tilde{q}_t - \phi_6\Delta\tilde{q}_{t+1} \end{pmatrix} z_t\right\} = 0$$

$$E_t\left\{\begin{pmatrix} \hat{\pi}_t - \phi_1\hat{\pi}_{t-1} - \phi_2\tilde{x}_t - \phi_3\hat{\pi}_{t+1} \\ -\phi_4\Delta\tilde{q}_{t-1} - \phi_5\Delta\tilde{q}_t - \phi_6\Delta\tilde{q}_{t+1} \end{pmatrix} z_t\right\} = 0$$

where we add five lags of the nominal exchange rate movements plus the U.S. inflation into the information set represented by vector z_t.

The results of both specifications are shown in Table 5.

Table 5
NKPC in an open economy – Reduced form

	Parameters						Test
$\widehat{CM}_t$	0.280	0.003	0.768	-0.029	0.040	-0.036	10.792
	(0.094)	(0.003)	(0.128)	(0.020)	(0.024)	(0.017)	(0.461)
$\tilde{x}_t$	0.276	-0.018	0.777	-0.021	0.025	-0.024	11.652
	(0.081)	(0.010)	(0.132)	(0.017)	(0.019)	(0.014)	(0.390)

Note: The values in brackets represent the standard errors of parameters or p-values of the J test.

Just as in the case of the closed economy, the estimates are consistent with theory. Moreover, the results for the parameters associated with inflation and foreign sector are reasonably consistent, regardless of the variable used to represent the level of real activity.

Nevertheless, there are some important differences. Again, the negative sign in the specification of the output gap contrasts with theory. However, although the impact of the estimate associated with the marginal cost is negligible, it is statistically significant.

Another important aspect concerns the impact of economic openness. In short, even though the variables associated with the foreign sector have a small direct impact, their presence exerts important indirect effects, consistently changing the weights related to the lagged inflation and the expected future inflation.

Figures 5 and 6 show the behavior of the specification with the marginal cost and output gap for an open economy.

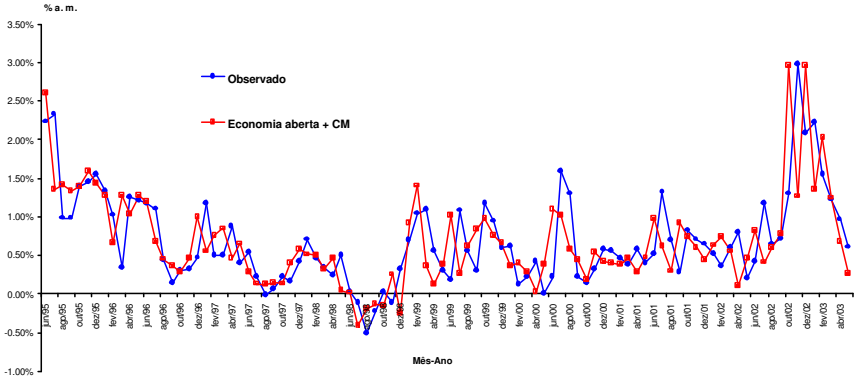


Figure 5
NKPC: Open economy + marginal cost

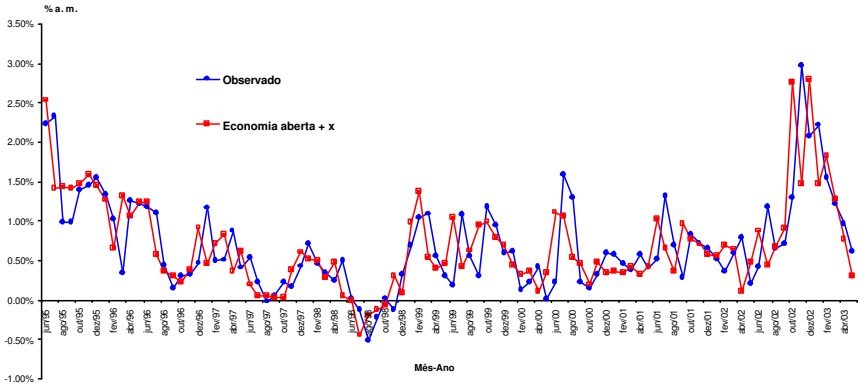


Figure 6
NKPC: Open economy + output gap

5.2.2 Structural form

Again, let us redo the previous exercise in such a way that allows directly obtaining structural parameters α, β, γ and δ in our open-economy model. The GMM is used with the same set of instruments, with the same alternative orthogonality conditions, and with the two options for the value of parameter β. In this case, orthogonality conditions (1) and (2) are:

$$E_t\left\{\begin{pmatrix} \hat{\pi}_t - \left[\frac{\gamma}{1+\beta\gamma}\right]\hat{\pi}_{t-1} \\ -\left[\frac{1-\delta}{1+\beta\gamma}\right]\left[\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\widehat{CM}_t - \left[\frac{\beta}{1+\beta\gamma}\right]\hat{\pi}_{t+1} \\ +\left[\frac{\gamma\delta}{1+\beta\gamma}\right]\Delta\tilde{q}_{t-1} - \delta\Delta\tilde{q}_t + \left[\frac{\beta\delta}{1+\beta\gamma}\right]\Delta\tilde{q}_{t+1} \end{pmatrix} z_t\right\} = 0$$

$$E_t\left\{\begin{pmatrix} [1+\beta\gamma]\,\hat{\pi}_t - \gamma\hat{\pi}_{t-1} \\ -[1-\delta]\left[\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\widehat{CM}_t - \beta\hat{\pi}_{t+1} \\ +[\gamma\delta]\,\Delta\tilde{q}_{t-1} - \delta\,[1+\beta\gamma]\,\Delta\tilde{q}_t + [\beta\delta]\,\Delta\tilde{q}_{t+1} \end{pmatrix} z_t\right\} = 0$$

The results are shown in Table 6.

The values of the structural parameters obtained in an open economy tell us basically the same story of the results obtained in reduced form; the results are relatively consistent between the two specifications for the orthogonality conditions and between the results for restricted versus unrestricted β.

Let us look into the structural parameters in more detail in this case. Parameter α has values between 0.94 and 0.97, implying an average period of 1 to 3 years for prices. These values are more consistent with what is expected for the Brazilian economy than the results obtained for a closed economy, especially if we take into account the standard errors observed, resulting in a minimum parameter value close to 0.90, which gives us an average period of 10 months.

With regard to the level of indexation of the economy, we may observe that the open-economy model is also more coherent with reality. Albeit sensitive to the specification of the orthogonality condition (but not to the restriction on the values of β), the estimates range from 0.24 to 0.58, indicating that approximately 1/4 to 1/2 of past inflation is transferred to prices.

In an open economy, the estimate values of β are higher, ranging from 0.94 to 0.97. These values are closer to those associated with the equilibrium interest rate estimated for Brazil and also to those commonly used in most of the literature (something like 0.99). However, it should be underscored that almost all the international empirical studies have a quarterly periodicity. Thus, while in quarterly models $\beta \simeq 0.99$ implies an equilibrium interest rate around 4% *p.a.*, in monthly models such as the one herein, the value of the equilibrium interest rate is approximately 13% *p.a.* Again, we may note that the restriction of the value of β to 1 does not affect the results obtained for other parameters in a significant fashion.

Table 6
NKPC in an open economy – Structural form

	Structural parameters				Reduced parameters						Test
	α	β	γ	δ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	J
(1)	0.974	0.975	0.575	0.021	0.369	0.001	0.625	-0.008	0.021	-0.013	13.746
	(0.062)	(0.102)	(0.124)	(0.012)	[0.280]	[0.003]	[0.768]	[-0.029]	[0.040]	[-0.036]	(0.392)
(2)	0.954	0.943	0.303	0.019	0.236	0.004	0.734	-0.004	0.019	-0.014	14.224
	(0.031)	(0.061)	(0.105)	(0.012)	[0.280]	[0.003]	[0.768]	[0.029]	[0.040]	[-0.036]	(0.358)
$\beta = 1$											
(1)	0.966	1.000	0.577	0.021	0.366	0.001	0.634	-0.008	0.021	-0.013	13.648
	(0.065)	–	(0.127)	(0.012)	[0.280]	[0.003]	[0.768]	[-0.029]	[0.040]	[-0.036]	(0.476)
(2)	0.938	1.000	0.236	0.015	0.191	0.003	0.809	-0.003	0.015	-0.012	13.895
	(0.033)	–	(0.113)	(0.010)	[0.280]	[0.003]	[0.768]	[-0.029]	[0.040]	[-0.036]	(0.458)

Note: The values in brackets represent the standard errors of parameters or p-values of the J test.
The values in braces are the reduced-form estimates presented in Table 5.

Let us now assess the values obtained for the parameter that results directly from the use of an open-economy model. Parameter δ has a surprising stability in all specifications and is statistically significant, with values around 0.02 and with a standard error of 0.01. The values obtained, which corroborate the modeling strategy for Brazil as a small open economy, imply that only about 2% of domestic consumption is allocated to imported goods. A possible explanation to this relatively small value may not be related to the level of economic openness. We use a inflation measure based on a consumer price index (IPCA). Consider Figure 7, which shows the IPCA and IGP-DI (general price index-internal availability). The latter is further divided into two components: IPC and IPA-DI, which stand for the consumer price index and the producer price index, respectively.

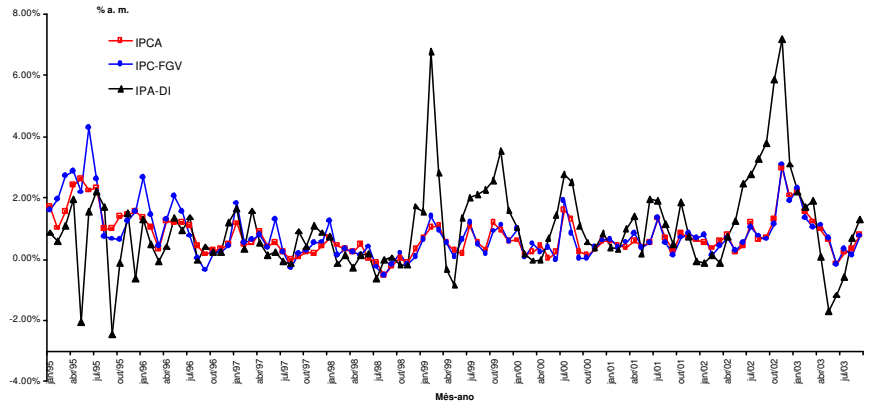


Figure 7
NKPC: Open economy + output gap

There is a vast literature on the small level of transfer of exchange rate depreciation to consumer prices comparatively to its transfer to producer prices.[22] This phenomenon can be explained as follows. First, the final price of imported goods includes some domestic inputs, such as transportation and trade. Another relevant aspect concerns the structure of imports. The lesser the presence of final goods and the greater the presence of intermediate goods, the milder the transfer of exchange rate movements to consumer prices. None of these aspects is analyzed in detail in this paper.

The impact of economic openness is also present in the resulting reduced parameters. Despite a slight sensitivity to the orthogonality condition used, the following major characteristics are observed: (i) the backward-looking component

[22]See, for instance, Bacchetta and Wincoop (2002) and Burstein et al. (2002).

has smaller values than that of a closed economy, between 0.10 and 0.37, (ii) the forward-looking component shows higher values than that of a closed economy, between 0.63 and 0.81, (iii) the impact of the marginal cost, despite being a little irrelevant, is statistically significant, and (iv) the values associated with the foreign sector, albeit small, are statistically significant.

In short, the use of an open-economy model has a small direct impact on open economy variables, with the sum of the parameters associated with $\Delta\tilde{q}_{t+j}$ being near zero. However, there is a considerable indirect effect on the remaining variables, increasing the weight of the forward-looking component of the Phillips curve. Curiously enough, the sum of the parameters associated with the lagged inflation and the expected future inflation is consistently close to 1, as described in the literature.[23]

6. Conclusions

The results of this study suggest that economic openness has important effects on the new-Keynesian Phillips curve (NKPC) parameters. In a model that considers price rigidity according to Calvo (1983) and has inflationary inertia in a way similar to Woodford (2003) and Galí and Gertler (1999), the use of an open-economy model has direct and indirect impacts on inflation dynamics.

The direct impact can be easily observed. As in the case of standard NKPC, inflation is related to a measure of marginal cost and to the expected future inflation. However, in an open economy, inflation is also influenced by contemporaneous real exchange rate movements and by its future expectations. Nevertheless, the impact of exchange rate movements, albeit statistically significant, is not quantitatively important.

On the other hand, the indirect effects of an open-economy model appear to be relevant. Compared to the "hybrid" curve proposed by Woodford (2003) and Galí and Gertler (1999), the open-economy model strengthens the forward-looking component to the detriment of the backward-looking one. This effect, coupled with the small quantitative impact of exchange rate movement on inflation, can render the exchange rate as an informative variable related to the behavior of marginal costs. The increase in the relative weight of expected future inflation compared to the past inflation may mirror an increase in the future path of marginal costs and this increase may be related to the exchange rate pressure on producer prices. Thus, the main role of exchange rate in this model is to "inform" about the behavior of future costs.

As part of future research, it is essential to assess the robustness of results using different estimation methods. Moreover, due to the obvious importance of marginal cost in price behavior, its composition and relationship with the output

[23] As, for example, Buiter and Jewitt (1985), Fuhrer and Moore (1995) and Jondeau and Bihan (2001).

gap should be further clarified. This knowledge may certainly contribute towards shedding some light on the short-run inflation dynamics.

References

Bacchetta, P. & Wincoop, E. V. (2002). Why do consumer prices react less than import prices to exchange rates? NBER Working Paper 9352.

Buiter, W. & Jewitt, I. (1985). Staggered wage setting with real wage relativities: Variations on a theme of Taylor. In *Macroeconomic Theory and Stabilization Policy*. University of Michigan Press, Ann Arbor.

Burstein, A., Eichenbaum, M., & Rebelo, S. (2002). Why are rates of inflation so low after large devaluations? NBER Working Paper 8748.

Calvo, G. A. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, 12:383–398.

Campos, C. F. S. & Nakane, M. I. (2003). Phillips curve and the effects of nominal shocks in open economies: The role of price setting. Mimeo.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? In Taylor, J. B. & Woodford, M., editors, *Handbook of Macroeconomics*. North-Holland.

Clarida, R., Galí, J., & Gertler, M. (1999). The science of monetary policy: A new Keynesian perspective. *Journal of Economic Literature*, 37:1661–1707.

Dixit, A. & Stiglitz, J. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67.

Fuhrer, J. C. & Moore, G. R. (1995). Inflation persistence. *Quarterly Journal of Economics*, 440.

Fuhrer, J. C., Moore, G. R., & Schuh, S. (1995). Estimating the linear-quadratic inventory: Maximum likelihood versus generalized method of moments. *Journal of Monetary Economics*, 35.

Galí, J. & Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, 44:127–159.

Galí, J., Gertler, M., & Lopes-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, 45:1237–1270.

Galí, J. & Monacelli, T. (2002). Monetary policy and exchange rate volatility in a small open economy. Mimeo.

Goodfriend, M. & King, R. (1997). The new neoclassical synthesis and the role of monetary policy. NBER Macroeconomics Annual.

Jondeau, E. & Bihan, H. L. (2001). Testing for a forward-looking Phillips curve. Additional evidence from European and US data. Mimeo, Universite Paris XII Val de Marne.

Lucas Jr., R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:29–46.

Miranda, P. C. & Muinhos, M. K. (2003). A taxa de juros de equilíbrio: Uma abordagem múltipla. Série de Trabalhos para Discussão 66, Banco Central do Brasil.

Razin, A. & Yuen, C. (2002). The "new Keynesian" Phillips curve: Closed economy versus open economy. *Economics Letters*, 75(1):1–9.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.