Instability and chaotic dynamics in stock returns

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Abstract

In this paper we examine certain properties of the Dow Jones and the Nikkey indices, investigating the existence of stochastic and deterministic nonlinear structures. Using the detrended fluctuation analysis, we construct a local measurement of randomness which identifies some extreme events and their impact on the randomness of the systems. Our results suggest no evidence of chaos in the data. In fact, GARCH processes explain most of the nonlinear dependence in the Dow Jones daily returns and the estimated Kolmogorov entropy for the Nikkey index diverges, conversely to what one would expect if the data followed a chaotic dynamics.

Resumo

O artigo investiga algumas propriedades dos índices de ações Nikkey e Dow Jones, tais como a existência de estruturas não-lineares determinísticas e estocásticas. Usando o método "detrended fluctuation analysis", construímos uma medida local de aleatoriedade que identifica alguns eventos extremos e seus impactos na aleatoriedade dos sistemas em estudo. Os resultados não sugerem evidência de caos nos dados. De fato, processos GARCH explicam a maior parte da dependência não-linear nos retornos diários do Dow Jones enquanto que a entropia de Kolmogorov estimada para o índice Nikkey diverge, evidenciando assim um sistema estocástico para este índice.

Key Words: Chaos, Kolmogorov entropy, detrended fluctuation analysis, correlation dimension and stock returns .

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1. Introduction.

During the past three decades a large number of papers have been published on the source of the instability in financial time series. Numerous studies have investigated their stochastic properties. The empirical evidence suggests that stock returns are not normally distributed, have fat tails, display nonlinear dependence and non-periodic cycles. Leptokurtic distributions have been observed in stocks and indices by analyzing both high-frequency and daily data. The origin of the observed leptokurtosis is still in debate. To account for nonlinear dependence, several researchers have been tried to explain it in terms of stochastic models that are nonlinear in variance such as ARCH and stochastic volatility models. The phenomenon of non-periodic cycles in financial markets was firstly studied by Mandelbrot (1971) who considered the possibility and implications of persistent financial time series. It is well known that such series are characterized by distinct but non-periodic cyclical patterns. Since then, several empirical studies have lent further support to Mandelbrot's findings. In fact, the most studies have investigated these properties with an underlying stochastic system. The interest is to recover the natural notion of instability or randomness multiplication in data generating processes.

Since the 1980s it has been recognized in the physical sciences that unpredictable time series and stochastic processes are not synonymous. Specifically, chaos theory has shown that unpredictable time series can arise from deterministic nonlinear systems. This observation raises the question concerning the mechanisms that generate observed, apparently stochastic, financial time series. In this way, an important reason for the interest in chaotic behavior is that it can potentially explain fluctuations in financial markets which appear to be random. So there is need to test for the presence of chaos.

As for the main conclusions of the literature, there is a broad consensus of support for the preposition that stock returns are characterized by a pattern of nonlinear dependence. On the other hand, the evidence on chaos is more mixed. Scheinkman and LeBaron (1989) analyzed stock markets, and Vaidynanathan and Krehbiel (1992) investigated S&P 500 index, all finding evidence of chaos. Mayfield and Mizrach (1992) find evidence of chaos in high-frequency returns in the S&P 500 cash index. Abhyankar et al. (1995) examine the behavior of the U.K Financial Times Stock Exchange 100 (FTSE-100) index over the first six months of 1993 (using 1-, 5-, 15-, 30-, and 60-minute returns. They find evidence of nonlinearity, but not of chaos. Abhyankar et al. (1997) test for nonlinear dependence and chaos in real-time returns on the world's four most important stock market indexes – FTSE-100, S&P 500, the Nikkey 225 Stock Average and the Deutscher Aktienindex (DAX). Using the BDS and the NEGM test, they reject the hypothesis of independence in favor of a nonlinear structure for all series, but find no evidence of chaos. More recently, Barkoulas and Travlos (1998) test for chaos in an emerging stock market, the Athens stock exchange (Greece), and find weak evidence in support of a nonlinear deterministic data generating processes or chaos. In fact, much disagreement and controversy have arisen about the available results, not only in stock returns but also in general economic data. Results may be difficult to find that are consistent across variations in sample size, testing approach and aggregation.

The primary goal of this paper is to investigate how much of the apparent randomness in the time series pattern of returns is explicable by a chaotic process. In order to investigate the intrinsic randomness of the system, we present here a method to sort out temporal correlations in financial time series within the *detrended fluctuation analysis* (DFA) statistical method proposed by Peng et al. (1994). This method has demonstrated its usefulness in the investigations of long-range dependence in DNA nucleotide sequences and heartbeat time series. We employ the concepts of correlation dimension and Kolmogorov entropy to search for a chaotic structure in the Dow Jones and the Nikkey daily returns. This study is important for two reasons. First, since there is much controversy on the evidence of chaos in stock returns, we revised this question investigating two important indices that were not investigated with the methods used in this paper. In fact, it is important to investigate the dynamics of two indices that represent different economies of great importance. Second, we present here a method commonly used in statistical physics in order to investigate the underlying randomness from a time series and its sensibility to local trends. This method can be useful to understand large fluctuations in financial markets. The next section describes deterministic chaos and its diagnostics. Section 3 discusses some randomness tests and reports their results. Section 4 reports the results from the chaos analysis. The last section summarizes the findings and discusses the significance and implications of the research.

2. Presence of Chaos.

Chaos deals with the irregular behavior of solutions to deterministic equations of motion. The equations must be nonlinear to generate chaotic solutions, but apart from that can be remarkably simple. Chaotic solutions are only accurate for a length of time governed by the errors on initial conditions. In many cases chaotic solutions relax on to a strange attractor which has a fractal structure and typically a non-integral dimension.

According to Brock et al. (1991), the time series $\{x_t\}$ has a C^2 deterministic explanation if there exist a system (h, F, w_0) such that $x_t = h(w_t)$, $w_{t+1} = F(w_t)$, $w(0) = w_0$, where $h : \Re^n \rightarrow \Re^1$, $F : \Re^n \rightarrow \Re^n$ are both in the C^2 class. Furthermore, we require that

F have an ergodic invariant measure ω that is absolutely continuous with respect to the Lebesgue measure. The interest lies in recovering the market dynamics by analyzing the time series $\{x_t\}$. The dimension of the attractors of systems with many degrees of freedom may increase rather rapidly after the transition to chaos. There is the problem of how to estimate the dimension of the strange attractors which underlies an irregular time series generated by a system with many degrees of freedom. Dimension estimation is an important problem because, if the time series exhibits low dimensional behavior, then it should be possible to model the underlying system accurately and hence obtain insight into the behavior of the system.

Techniques of state space reconstruction were introduced by Packard et al. (1980) and Takens (1981), which show that it is possible to address this problem of dimension estimation as follows. Suppose that a scalar time series $\{x_t\}$ is generated by an N – dimensional attractor of a deterministic dynamic system with n degrees of freedom and define an m-dimensional vector constructed from the observed time series

$$x_t^m = (x_t, \ldots, x_{t+m-1}) = \left(h(w_t), \ldots, h\left(F^{m-1}(w_t)\right)\right) \equiv I_m, \quad (1)$$

where F^{m-1} is the composition of F with itself $m-1$ times in equation (1). Since the true system that generated the time series is n-dimensional, Takens (1981) proved that for smooth pairs (h, F) the map $I_m : \Re^n \rightarrow \Re^m$ will be an embedding map for $m \geq 2n+1$. Taken's theorem guarantees that if the embedding dimension m is sufficiently large with respect to the dimension of the manifold on which the attractor lies, the m-dimensional image of the attractor provides a correct topological picture of its dynamics. In particular, the dimension of the reconstructed attractor is N, regardless of the value of m, and is invariant with respect to the measurement function h.

2.1 Correlation Dimension.

There are a number of procedures to distinguish a deterministic chaotic process from a truly random process. According to Grassberger and Procaccia (1983a, 1984), one of the efficient ways to test for chaos is to consider the correlation integral, which is a measure of spatial correlation of scattered points or particles in m-dimensional space, using the statistic

$$C_{m,T}(\varepsilon) = \sum_{T<S} I_{\varepsilon}\left(x_t^m, x_s^m\right) \times \left[2/T_m\left(T_{m-1}\right)\right], \tag{2}$$

where $T_m = T - (m-1)$, $x_t^m = (x_t, \ldots, x_{t+m-1})$, x_t being a time series, and $I_{\varepsilon}\left(x_t^m, x_s^m\right)$ an indicator function which equals 1 if $\|x_t^m - x_s^m\| < \varepsilon$ and 0 otherwise. Here, $\{x_t\}$ is a scalar time series under scrutiny for randomness. In order to use (2) to measure intertemporal local correlation and other kinds of dependence, one imbeds $\{x_t\}$ in an m-dimensional space by forming m-vectors $x_t^m = (x_t, \ldots, x_{t+m-1})$ starting at each date t.

For a deterministic chaotic system, as $T \to \infty$

$$C_{m,T}(\varepsilon) \to C_m(\varepsilon) \equiv Pr\left\{\|x_t^m - x_s^m\| < \varepsilon\right\}, \tag{3}$$

for almost all initial conditions. The quantity in (3) is called the correlation integral which is the probability that two points are within a certain distance from one another. As Grassberger and Procaccia (1983a, 1984) showed with $\varepsilon \to 0, C_m(\varepsilon) \sim \varepsilon^v$, where v is the correlation exponent. The definition of correlation dimension in an embedding dimension m is

$$d_m = \lim_{\varepsilon \to 0} \lim_{T \to \infty} \log \left[C_{m,T}(\varepsilon) \right] / \log(\varepsilon). \tag{4}$$

The correlation dimension itself is given by

$$d = \lim_{m \to \infty} d_m, \tag{5}$$

and Grassberger and Procaccia (1983a, 1984) showed that

$$\log_2 C_m(\varepsilon) = \log_2 A + v \log_2 \varepsilon, \tag{6}$$

where A is a constant. The estimate of v as $m \to \infty$ provides the correlation dimension estimate of the dynamic system. For $m \geq 2n+1$, Brock (1986) showed that the correlation exponent is independent of both the norm used and the embedding m.

The dimension of the dynamic system is determined by first estimating the slope of the regression line of $\log_2 C_m(\varepsilon)$ on $\log_2 \varepsilon$ and an intercept for each embedding dimension m. If as m increases v continues to rise, then the system is stochastic. If, however, the data are generated by a deterministic process consistent with chaotic behavior, then v reaches a finite saturation limit beyond some relatively small m. In principle, we can detect any form of deterministic chaos, given enough data. However, in practice there is never enough data to detect high-dimensional chaos, so the best we can do is to detect low-dimensional chaos.

Consider again

$$C_{m,T}(\varepsilon) = \sum_{T<S} I_\varepsilon \left(x_t^m, x_s^m \right) \times \left[2/T_m \left(T_{m-1} \right) \right], \tag{7}$$

where $T_m \equiv T - (m-1)$. The limit of (7) exists almost surely under mild stationarity and ergodicity assumptions. Denote this limit by $C_m(\varepsilon)$. If the process $\{x_t\}$ is *i.i.d.*, it can be shown that $C_m(\varepsilon) = [C_1(\varepsilon)^m]$ for all m and ε. Brock, Dechert, LeBaron and Scheinkman (1996) proposed the BDS test, which is based on the following statistic:

$$W_{m,T}(\varepsilon) = \frac{\sqrt{T}\left[C_{m,T}(\varepsilon) - C_{1,T}(\varepsilon)^m\right]}{\hat{\sigma}_{m,T}(\varepsilon)}, \tag{8}$$

where $\hat{\sigma}_{m,T}(\varepsilon)$ is an estimate of the standard deviation under the *i.i.d.* null hypothesis. They showed that, under the null hypothesis of *i.i.d.*, $W_{m,T}(\varepsilon) \xrightarrow{d} N(0,1)$ as $T \to \infty$. However, the asymptotic distribution is not appropriate when the test statistic is computed using standardized residuals of ARCH models.

The BDS statistic appears to have good power against simple nonlinear deterministic systems as well as nonlinear stochastic processes. It is important to point out that additional diagnostic tests are needed to determine the source of the rejection of the null hypothesis. It should be emphasized that this test is not capable of distinguishing nonlinear stochastic dynamics from deterministic chaotic dynamics, although the rejection of the null hypothesis may, of course, motivate the investigation of chaotic models.

2.2 Kolmogorov Entropy.

In addition to estimating dimension, we can estimate the Kolmogorov entropy K for the system, which measures the mean rate of creation of information. The condition of sensitivity to initial conditions that is characteristic of chaotic systems implies divergence of initially adjacent dynamic states. While an initial state of the system may be known with a high but finite degree of precision, the ability to predict later states diminishes because of trajectory divergence. Information is lost, or conversely, more information is required to specify the system with the original precision; the entropy has increased.

Grassberger and Procaccia (1984) showed that the vertical change in the position of the invariant portion of the correlation integral over the scaling region in ε is a lower bound estimate of Kolmogorov entropy. They defined

$$K_{2,m}(\varepsilon) = \frac{1}{\tau} \ln \frac{C_m(\varepsilon)}{C_{m+1}(\varepsilon)}, \tag{9}$$

where $C_m(\varepsilon)$ is defined as before and τ is the delay time between observations, and showed that

$$\lim_{m \to \infty, \varepsilon \to 0} K_{2,m}(\varepsilon) \sim K_2, \tag{10}$$

where K_2 is order-2 Renyi entropy, which is a lower bound estimate of Kolmogorov's entropy ($K_2 \leq K$). Grassberger and Procaccia (1983b) suggest that K_2 is preferable to K. However, there are some practical limitations on the implementation of K_2. The fact that the length of the time series is finite is perhaps the most serious difficulty. Due to the finiteness of the time series, as $m \to \infty$, the correlation integral will be reduced to counting only the points themselves. Accordingly $C_m(\varepsilon)$ and $C_{m+1}(\varepsilon)$ will each converge to the same value, and then $\ln 1 = 0$. Consequently, if one examines too large an embedding dimension then the estimated value of K_2 will be biased towards zero.

Kolmogorov entropy provides a way of categorizing the motion of dynamic systems. If $K_2 = 0$, then the motion is regular (periodic, quasiperiodic, or stationary). If $K_2 > 0$, then the motion is chaotic, and if $K_2 = \infty$, then the motion is stochastic.

3. Randomness Tests.

We employ some randomness tests in order to investigate the nature of the underlying randomness of the system. It should be noted that if the series exhibit long memory, then there is persistent temporal dependence between distant observations. Such series are characterized by distinct but non-periodic cyclical patterns like chaotic processes. The presence of long memory dynamics, which is a special form of nonlinear dynamics, leads to nonlinear dependence in the first moment of the distribution, and hence to a potentially predictable component in the series dynamics.

At the outset, we consider one of the best-known methods, the R/S analysis. This method, proposed by Mandelbrot and Wallis (1968) and based on previous hydrological studies of Hurst (1951), allows the calculation of the self-similarity parameter H. The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. The resulting ratio is known as the rescaled range R/S. In fact, the R/S statistic asymptotically follows the relation

$$(R/S)_n \approx C_{n^H}, (0 < H < 1, C \in IR \in) \tag{11}$$

where $H = 0.5$ indicates short memory in the series, $H > 0.5$ indicates persistence and $H < 0.5$ indicates anti-persistence.

The second method we have used to measure randomness is the detrended fluctuation analysis (DFA) proposed by Peng et al. (1994) for sorting out temporal correlations in DNA sequences in biological studies. Later, several studies have been performed in heartbeat time series and also in many areas of statistical physics [see Blesic et al. 1999, Absil et al. (1999) and Ivanova and Ausloos (1999)]. Recently, a growing number of physicists have attempted to analyze the fluctuation in financial markets using the DFA method [see Ausloos (2000) and János et. Al (1999)]. The method can be summarized as follows. Consider a time series of returns $x(t), t = 1, \ldots, T$. First, the integrate time series $y(t')$ is obtained, $y(t') = \sum_{t=1}^{t'} x(t)$. Next, we divide the $y(t')$, into windows of equal length m. It is not possible in general to divide exactly the T points of the series into windows of length m. For each value of m, we call $\bar{T}$ the larger multiple of m inferior or equal to T. A least-squares line is fitted to the data in each window. The y coordinate of the straight line segments is denoted by $y_m^{(t')}$. Next, the root mean square fluctuation of the integrated and detrended time series is calculated:

$$G(m) = \left[\frac{1}{\bar{T}} \sum_{t'=1}^{\bar{T}} \left[y(t') - y_m(t') \right]^2 \right]^{1/2} . \tag{12}$$

This calculation is repeated over all intervals. A linear relationship on a double log graph of $G(m)$ and the interval size (window) m indicates the presence of a power-law scaling. Similarly to the Hurst analysis, if there is no correlation or only short memory $G(m) \approx m^{\alpha}$, with $\alpha = 1/2$. An exponent value $\alpha > 1/2$ is usually associated with persistence and $\alpha < 1/2$ with anti-persistence. Some of the DFA main advantages over other techniques like Fourier transform or R/S methods are that (i) local and large-scale trends are avoided at all scales m in equation (12), and (ii) local correlations can be easily proved.

3.1 Fluctuation Analysis and Randomness.

Our data consists of the Dow Jones and the Nikkey daily returns. The time period ranges from December 28, 1986 to October 20, 2000. Table 1 reports summary statistics for the two series. All series have fat tails. The kurtosis coefficients are all substantially larger than that of the standard normal distribution. Jarque-Bera's χ^2 statistic, which summarizes the joint deviation of the third and fourth moments from those of a normal distribution, is strongly significant as a result. The significant deviation from normality can be an indication of nonlinear dynamics.

Table 1
Descriptive Statistics of Daily Returns

	Dow Jones	*Nikkey*
Mean Daily return	0.0002	-2.73E-05
Standard Deviation	0.0048	0.0066
Skewness	-3.655	0.158
Kurtosis	83.80	11.77
Jarque-Bera Test (P-Value)	0.000	0.000

Table 2 gives the results of the traditional R/S analysis and DFA results. For the Dow Jones daily returns the estimated Hurst exponent is 0.5, which means that there is no evidence of long-range correlations. However, the estimated Hurst exponent for the Nikkey index is 0.66 representing a long-range correlation property. The results of the DFA analysis represented by the α exponent exhibit always smaller exponents than the ones obtained with the traditional R/S analysis. The obtained exponents indicate anti-persistence behavior in both indices.

Table 2
Long-Term Memory Analysis

Daily Returns	H	α
Dow Jones	0.50	0.45
Nikkey	0.66	0.47

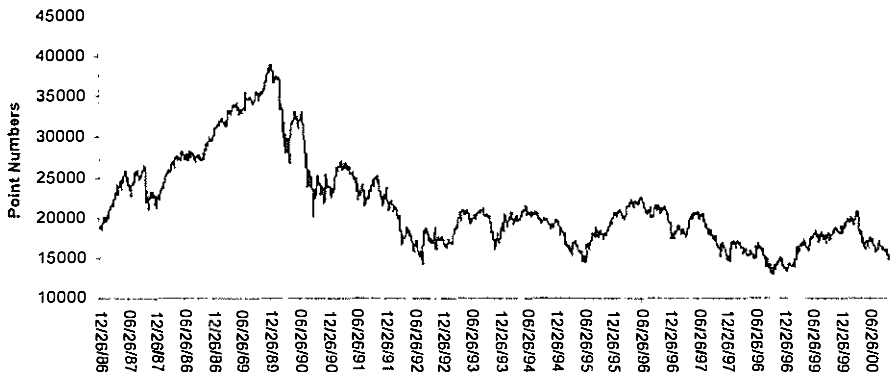
Notes: H is the Hurst exponent in the equation (11) and α is the exponent from the DFA analysis.

In order to investigate the α exponent over time and some visible "random" pattern in these markets, we first constructed a "window observation" of length 6 months placed at the beginning of the data, and we computed α for the data contained in that window. Then, we moved this window by 120 points (6 months) toward the right along the financial time series and again computed α. Iterating this procedure for the 1987 – 2000 series, we obtained a local measurement of randomness for the Dow Jones and Nikkey indices. The results are shown in figures 1 and 2. Figure 1 (b) reports the calculations for the Nikkey index where the α exponent value is mostly below 0.5 reflecting the estimated value (0.47) without the constructed "window observation". Clearly, there are some extreme events such as the October crash in 1987 and the period of the Gulf War in the 1990s. These events and the subsequent periods coincide with the smallest value exponent. Also, these events divide in two parts the pattern information propagation over time. After these periods, the exponent value is more stable ranged between 0.4 and 0.5. However, there are some potential outliers even in these periods. Notice a pattern of the exponent between June 26,1993 to June 26, 1995 and 1997 to June, 26, 2000. In these periods, in principle, investors could partially predict the future movements of the Nikkey, at least for a short horizon of 6 months.

Figure 2 (b) reports the calculations for the Dow Jones index where the α exponent value is again mostly below 0.5 reflecting the estimated value (0.45) without the constructed "window observation". Again the magnitude of the October crash in 1987 and the subsequent periods are well visible in the exponent value. However, we cannot detect a visible pattern in the exponent dynamics. Also, it should be noted that there exist some events such as the Mexico crisis occurred in the middle of 1994 that cause an abrupt decay on the exponent but did not in the Nikkey case. This observation reflects the idea that each market has a unique information propagation process.

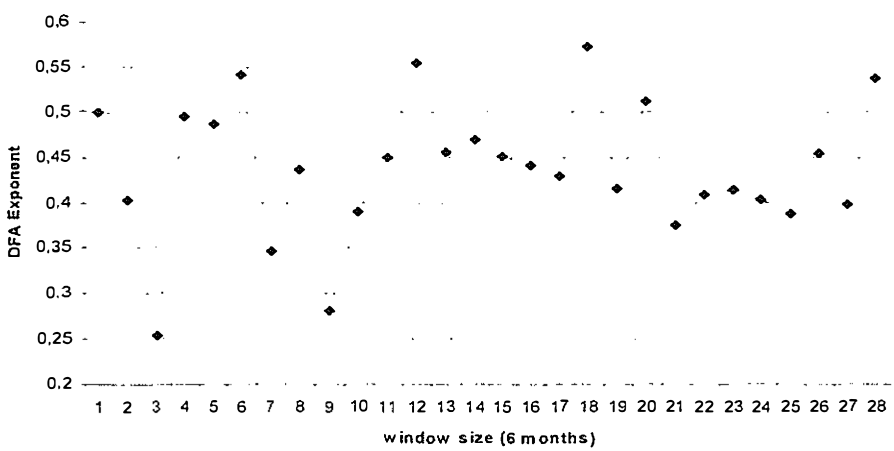
These results motivate several questions about the nature of the underlying dynamic systems. It seems that the Dow Jones index is more unstable than the Nikkey index, considering that the Dow Jones exponent dynamics does not demonstrate a clear pattern over the intervals of 6 months. On the other hand, the results from the Nikkey reveal visible trends in the exponent dynamics over the entire interval that could be used to predict the future movements of this market in a relatively short time interval of 6 months. Given the points raised above, we will test for the presence of chaos in these markets.

Figure 1 (a) - Nikkey Index



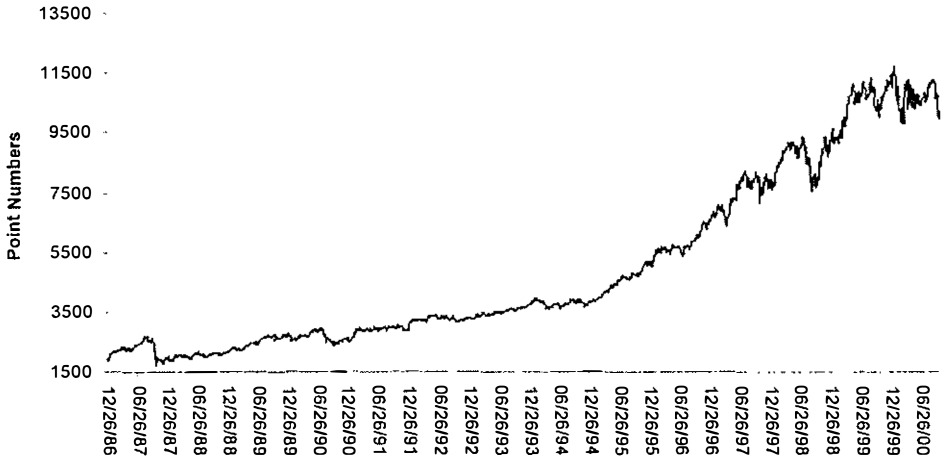
Notes:The evolution of the Nikkey index from 28 December,1986 to 20 October,2000.

Figure 1 (b) - DFA Analysis



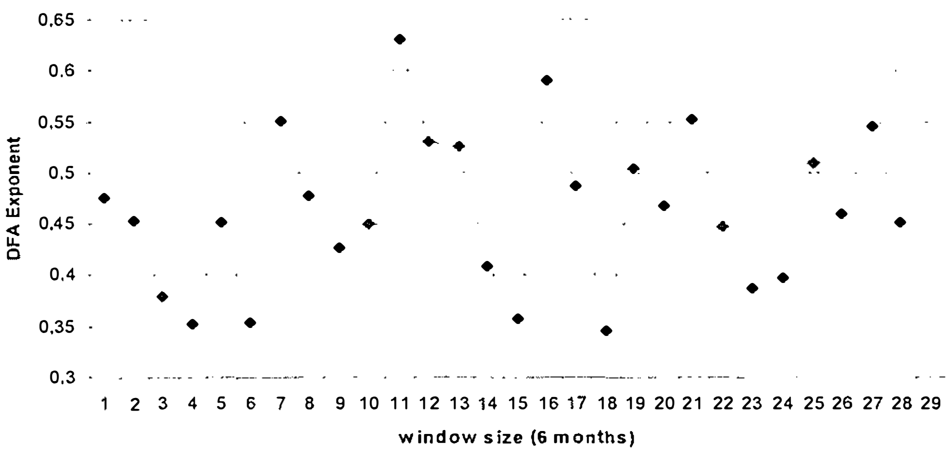
Notes: The evolution of the local value of α estimated with the DFA method for windows of 6 months.

Figure 2 (a) - Dow Jones Index



Notes: The evolution of the Dow Jones index from 28 December,1986 to 20 October,2000.

Figure 2 (b) - DFA Analysis



Notes: The evolution of the local value of α estimated with the DFA method for windows of 6 months.

4. Nonlinearity and Chaos.

To ensure that we truly capture any nonlinear structure present in the data, we should apply the correlation dimension method on stationary data. If the data are nearly nonstationary, in phase space the reconstructed attractor will be stretched along a ray, resulting in an underestimation of the true dimension. Grassberger and Procaccia (1983a) noted the same problem in continuous time processes sampled at very close intervals.

Since the Nikkey and Dow Jones stock prices are nonstationary, we perform the analysis on the rates of returns. To address the problem of correlated data, Brock (1986) suggested a residual diagnostic test. If a series is generated by deterministic chaos then the residuals from a linear, or smooth nonlinear transformation of the data, should yield the same correlation dimension as the original series. Thus, we filter the daily returns with linear AR models. Also, we should capture the dependence in the second moments because of the excess kurtosis. GARCH models (Bollerslev, 1986) are then estimated by maximum likelihood, and the data are filtered again. Since we are taking a smooth transformation of the data, the Brock residual theorem applies to this series as well[1].

To obtain additional evidence regarding the presence of nonlinearities and to further motivate our testing for chaotic structure, we perform the test suggested by Brock, Dechert, LeBaron and Scheinkman (1996) to the filtered series. As we have seen in the early sections, the asymptotic distribution is not appropriate when

[1] Based on the AIC (Akaike infonnation criterion), a Student-t GARCH (1,1) was adjusted for both the Nikkey and Dow Jones daily returns.

applied to the standardized residuals of ARCH models. We have used the critical values for the BDS test applied to GARCH standardized residuals reported by Brock et al. (1991).

Tables 3 and 4 report the BDS test statistics for the Nikkey and Dow Jones daily returns and their standardized GARCH residuals. In computing the BDS statistics, we have two important issues to deal with: the choice of m and ε. For a given m, ε cannot be too small because $C_m(\varepsilon)$ will capture too few points; also ε cannot be too large because $C_m(\varepsilon)$ will capture too many points. According to Hsieh (1989), we apply the BDS test to these sets of series for embedding dimensions of $m = 3, 4, 5$ and 10. For each m, ε is set to 0.5, 1.0, 1.5 and 2.0 standard deviations (σ) of the data.

We strongly reject the hypothesis that the Nikkey and Dow Jones daily returns are *i.i.d.* Also, it is clear that the BDS statistics all lie in the extreme positive tail of the standard normal distribution for the return series. When the BDS test is applied to Dow Jones GARCH residuals, none of the BDS statistics are significant when compared with the simulated values in Brock et. al (1991). On the other hand, we have mixed results with respect to Nikkey GARCH residuals, since some BDS statistics are significant at 5%. Thus, in principle, the behavior of the Dow Jones index appears to be well explained by a simple Student t – GARCH (1,1) model. With respect to the Nikkey index, the results suggest the presence of an unspecified omitted structure. These results motivate the investigation of a possible chaotic structure in the data.

Table 3 - BDS Test Results

		Nikkey daily returns	GARCH Residuals
Dimension	ε/σ		
m=3	0.5	18.61***	1.45
	1.0	18.27***	1.73**
	1.5	17.46***	2.01&
	2.0	16.15***	1.94&
m=4	0.5	23.12***	1.66
	1.0	21.05***	1.85**
	1.5	19.46***	1.95&
	2.0	17.69***	1.70&
m=5	0.5	29.61***	1.93
	1.0	24.34***	2.13**
	1.5	21.30***	2.22&
	2.0	19.05***	1.92&
m=10	0.5	134.72***	4.38
	1.0	51.18***	2.98**
	1.5	30.33***	2.01&
	2.0	23.68***	1.19&

Notes: The BDS tests for i.i.d., where m is the embedding dimension and ε is distance, set in terms of the standard deviation of the data (σ) to 0.5, 1.0, 1.5 and 2.0 standard deviations. The critical values for the BDS test applied to GARCH standardized residuals in the case of ε/σ=0.5 they are approximated by the 2.5% and 97.5% quantiles reported by Brock et al. (1991) on GARCH (1,1) standardized residuals for 1000 observations; in the case $\varepsilon\sigma$=1.0 they are approximated by the 2.5% and 97.5% quantiles reported by Brock et al. (1991) on GARCH (1,1) standardized residuals for 2500 observations. & indicates that the corresponding critical values for the BDS test statistic are not available and no hypothesis testing has been performed. *** indicates significance at the 0.01 level. ** indicates significance at the 0.05 level.

Table 4 - BDS Test Results

		Dow Jones daily returns	GARCH Residuals
Dimension	ε/σ		
m=3	0.5	8.00***	-0.91
	1.0	9.80***	-0.21$^{\&}$
	1.5	11.90***	1.07$^{\&}$
	2.0	13.72***	2.98$^{\&}$
m=4	0.5	9.76***	-0.91
	1.0	11.51***	-0.26
	1.5	13.15***	0.91$^{\&}$
	2.0	14.53***	2.79$^{\&}$
m=5	0.5	12.02***	-0.43
	1.0	13.35***	-0.03
	1.5	14.36***	0.88$^{\&}$
	2.0	15.19***	2.50$^{\&}$
m=10	0.5	33.25***	-0.39
	1.0	24.17***	0.95
	1.5	20.31***	1.35$^{\&}$
	2.0	18.47***	2.48$^{\&}$

Notes: The BDS tests for i.i.d., where m is the embedding dimension and ε is distance, set in terms of the standard deviation of the data (σ) to 0.5, 1.0, 1.5 and 2.0 standard deviations. The critical values for the BDS test applied to GARCH standardized residuals in the case of ε/σ=0.5 they are approximated by the 2.5% and 97.5% quantiles reported by Brock et al. (1991) on GARCH (1,1) standardized residuals for 1000 observations; in the case $\varepsilon\sigma$=1.0 they are approximated by the 2.5% and 97.5% quantiles reported by Brock et al. (1991) on GARCH (1,1) standardized residuals for 2500 observations. & indicates that the corresponding critical values for the BDS test statistic are not available and no hypothesis testing has been performed. *** indicates significance at the 0.01 level. ** indicates significance at the 0.05 level.

4.1 Correlation Dimension Estimates.

In this section, we report the correlation dimension over the range of embedding dimensions $m = 1, 2, \ldots, 15$. Scheinkman and LeBaron (1989) suggest another diagnostic tool - shuffling the data. A shuffling of the original series results in a series without temporal dependence. For an *i.i.d.* process, randomizing will not affect the dimension, since the shuffled series will also be *i.i.d.* For a chaotic process of low dimensionality, shuffling will result in no saturation of the correlation dimension estimates. In this way, if we observe that the actual dimension estimate is less than that of every shuffled series, we find evidence in support of a nonlinear deterministic process underlying the data. Therefore, dimension calculations based on the shuffled data are a useful benchmark against which to compare actual dimension estimates. Few studies have passed the shuffle diagnostic after ARMA and ARCH filtering. Frank and Stengos (1989), and Scheinkman and LeBaron (1989) reported dimension estimates, in the range of 6 to 7 after filtering, that pass the shuffle diagnostic. Mayfield and Mizrach (1992) reported dimension estimates, in the range of 2 to 3 after GARCH filtering.

We use a pseudo-random number generator to create our shuffled series, which are constructed by randomly drawing with replacement from the associated original series. Tables 5 and 6 report correlation dimension estimates for the raw data and filtered series: AR (2) residuals and Student-t GARCH standardized residuals[2]. Clearly, the Nikkey and Dow Jones series appear to pass Brock's residual test when we use AR residuals. However, for both series the correla-

[2] The AR (2) model was selected based on the AIC, and fitted to both the Nikkey and Dow Jones daily returns.

tion dimension estimates for the GARCH residuals are significantly different from ones obtained for the daily returns. It should be noted that a possible reason for the noticeable increase in the correlation dimension estimates for the GARCH residuals is the tremendous filtering the Nikkey and Dow Jones are subjected to, given our relatively small sample.

Table 5 - Correlation dimension Estimates (Dow Jones)

	Dow Jones		AR residuals		GARCH residuals	
m	(a)	(b)	(a)	(b)	(a)	(b)
2	0.45	0.57	0.50	0.51	1.29	1.29
3	0.66	0.86	0.72	0.77	1.95	1.94
4	0.85	1.15	0.94	1.02	2.61	2.59
5	1.03	1.44	1.14	1.28	3.25	3.24
6	1.20	1.74	1.32	1.54	3.90	3.89
7	1.36	2.04	1.50	1.79	4.54	4.54
8	1.52	2.34	1.67	2.04	5.18	5.19
9	1.66	2.64	1.82	2.29	5.81	5.84
10	1.79	2.93	1.97	2.54	6.43	6.48
11	1.92	3.22	2.11	2.79	7.05	7.12
12	2.04	3.52	2.25	3.04	7.66	7.77
13	2.16	3.80	2.38	3.29	8.26	8.38
14	2.28	4.08	2.51	3.55	8.83	8.92
15	2.39	4.34	2.63	3.80	9.45	9.41

Notes: Colums (a) report estimates for original series and colums (b) report estimates for the shuffled series.

Table 6 - Correlation dimension Estimates (Nikkey)

	Nikkey		AR residuals		GARCH residuals	
m	(a)	(b)	(a)	(b)	(a)	(b)
2	1.01	1.17	1.01	1.16	1.04	1.43
3	1.43	1.75	1.43	1.73	1.56	2.15
4	1.82	2.33	1.81	2.30	2.07	2.86
5	2.17	2.92	2.16	2.88	2.57	3.56
6	2.49	3.51	2.47	3.46	3.08	4.26
7	2.78	4.10	2.76	4.04	3.58	4.97
8	3.03	4.69	3.02	4.62	4.06	5.68
9	3.28	5.29	3.26	5.19	4.54	6.38
10	3.50	5.88	3.49	5.75	5.02	7.08
11	3.71	6.46	3.69	6.32	5.48	7.76
12	3.90	7.05	3.89	6.88	5.94	8.48
13	4.08	7.64	4.07	7.44	6.38	9.26
14	4.25	8.28	4.24	8.02	6.80	9.90
15	4.42	8.85	4.41	8.61	7.22	10.58

Notes: Colums (a) report estimates for original series and colums (b) report estimates for the shuffled series.

Figures A1 and A2 (Appendix) show that the correlation dimension estimates increase very slowly with embedding dimensions for the Dow Jones and Nikkey daily returns, and are well below the theoretical values for the shuffled series. If the time series is a realization of a stochastic process, the correlation dimension estimates should increase monotonically with the dimensionality of the space within which the points are contained. In finite data sets, however, stochastic data may yield correlation dimension estimates which are substantially lower than the embedding dimension m and which rise slowly with m while chaotic data may not deliver complete saturation.

Figures A3 and A4 (Appendix) report the correlation dimension estimates for GARCH residuals. The shuffle diagnostic applied to the Dow Jones GARCH residuals reveals that the GARCH model picks up the relevant structure of the data, considering that the shuffled series has the same dimension as the original series. This diagnostic confirms the results obtained by the BDS test: the BDS test did not reject the null hypothesis of an *i.i.d.* process. On the other hand, for the Nikkey index the nonlinear structure is not accounted by the GARCH model, since the correlation dimension estimates are well below the theoretical values for the shuffled series.

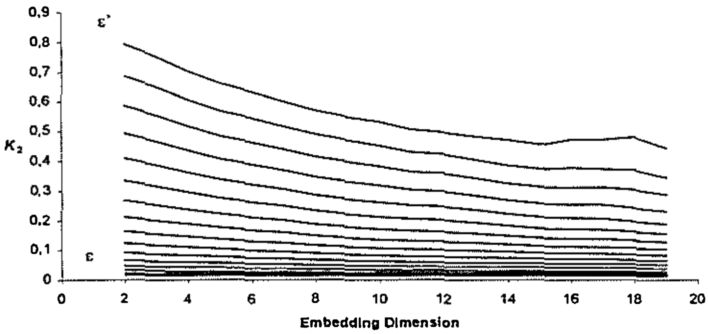
The correlation dimension estimates jointly with the shuffle diagnostic suggest the following: First, the daily series do not pass Brock's residual test since the dimension estimates for GARCH residuals are significantly different from those of the original series. However, it should be stressed that AR residuals show the same dimension from those of the original series. Second, the shuffle diagnostic applied to the Nikkey index suggests that some deterministic nonlinear structure may exist in the series, and is lost by shuffling. However, the same method suggests that the GARCH model picks up the relevant nonlinear structure of the Dow Jones index. Next, we compute the Kolmogorov entropy which measures the mean rate of creation of information of the system.

4.2 Kolmogorov Entropy Estimates.

In this section, we report the Grassberger and Procaccia (1983b) approximation to the Kolmogorov entropy, denoted as K_2. As previously indicated, if a time series is completely stochastic, then $K_2 = \infty$. Recall that the entropy measures the rate at which indistinguishable paths become distinguishable when the system is observed with only some finite level of accuracy. The lower the value of K_2, the more predictable the system. Figures 3 and 4 show the plot of $[\ln C_m(\varepsilon) - \ln C_{m+1}(\varepsilon)]$ against m over the scaling region for $\varepsilon \left(\varepsilon = 0.9^{20} \text{to} \varepsilon' = 0.9^{35}\right)$. These quantities do not converge to a constant value at relatively low embedding dimensions. The curves are decreasing, and as m increases and ε decreases these curves do not tend to constant values. Note that these estimates are the lower bound on the metric entropy and are consistent with the stochastic interpretation of the returns series since the entropy seems to diverge.

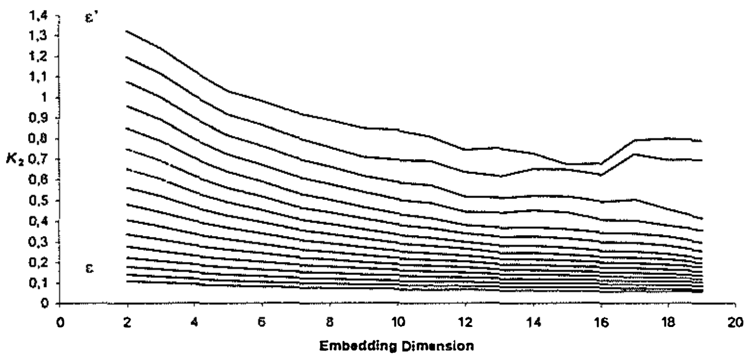
Taken the evidence from all diagnostic tools for chaos together, nonlinear determinism cannot be supported as a representation of the data generating process for the Dow Jones and the Nikkey indices. The results indicate that GARCH processes explain most of the nonlinear dependence in the Dow Jones index. With respect to the Nikkey, the results suggest some unknown nonlinear stochastic structure in the data.

Figure 3 - Approximation to Kolmogorv Entropy (Dow Jones)



Notes: ε=0.9^{20} and ε'=0.9^{35}. See the text for the definitions.

Figure 4 - Approximation to Kolmogorv Entropy (Nikkey)



Notes: ε=0.9^{20} and ε'=0.9^{35}. See the text for the definitions.

5. Remarks.

In this paper we examined certain properties of the Dow Jones and the Nikkey indices, exploring possible deterministic nonlinear structures. Using the detrended fluctuation analysis (DFA) statistical method, we investigated the information propagation over time in these markets. We constructed a local measurement of randomness which permitted to identify some extreme events and their impacts on the underlying randomness of the systems. Also, it seems that each market react in a different way with respect to some common events, resulting in a distinct dynamics of the information propagation measured by the DFA exponent.

With respect to chaos analysis, our results do not suggest evidence of deterministic nonlinear structures. In fact, the shuffle diagnostic for the Dow Jones index showed that GARCH processes can account its relevant structure. This diagnostic confirmed the results obtained by the BDS statistics. With the methods used in this paper, we cannot determine the nature of the apparent randomness exhibited by the Nikkey index. However, the estimated Kolmogorov entropy suggested that this market creates information such that the entropy of its system diverges, contrary to the chaos hypothesis.

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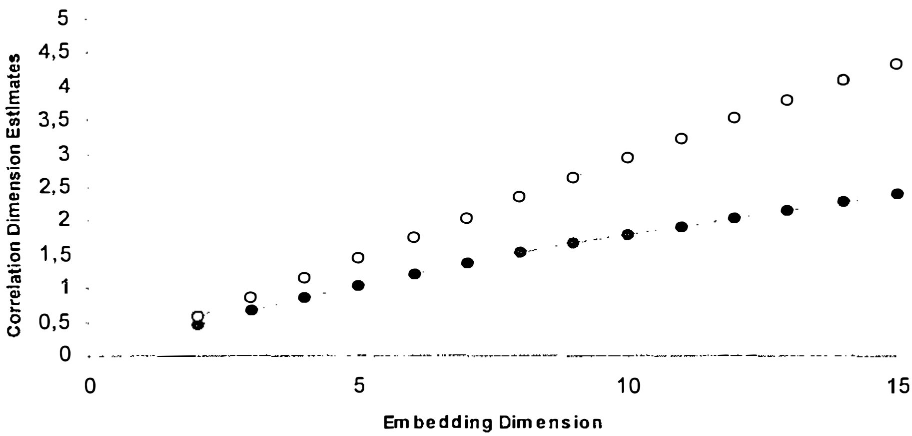
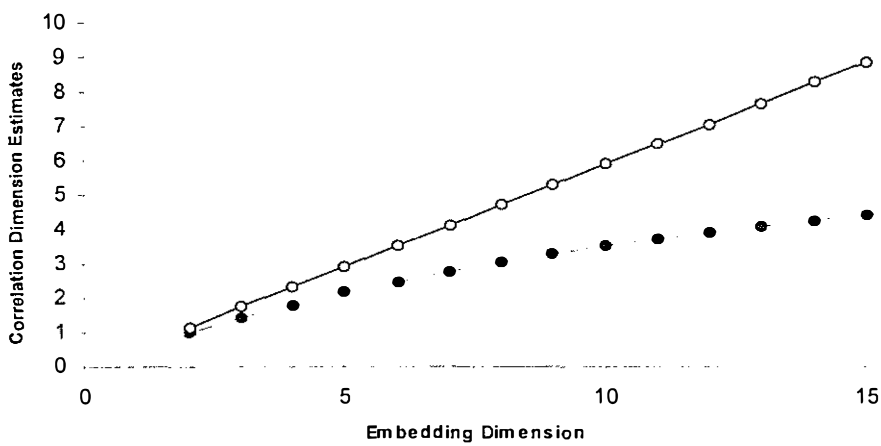
Appendix.

Figure A1 - Dimension Estimates of Daily Returns (Dow Jones)



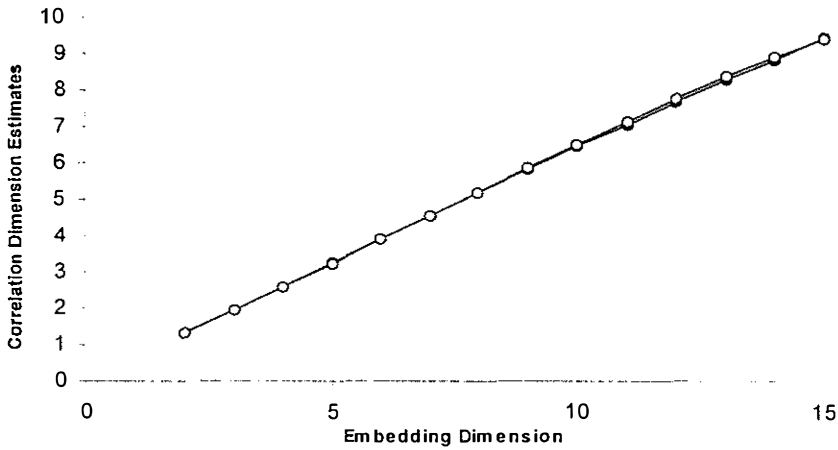
Notes: The black points (•) represent estimates for the ordered original data. The dimension estimates for the shuffled data are represented by the light points (o).

Figure A2 - Dimension Estimates of Daily Returns (Nikkey)



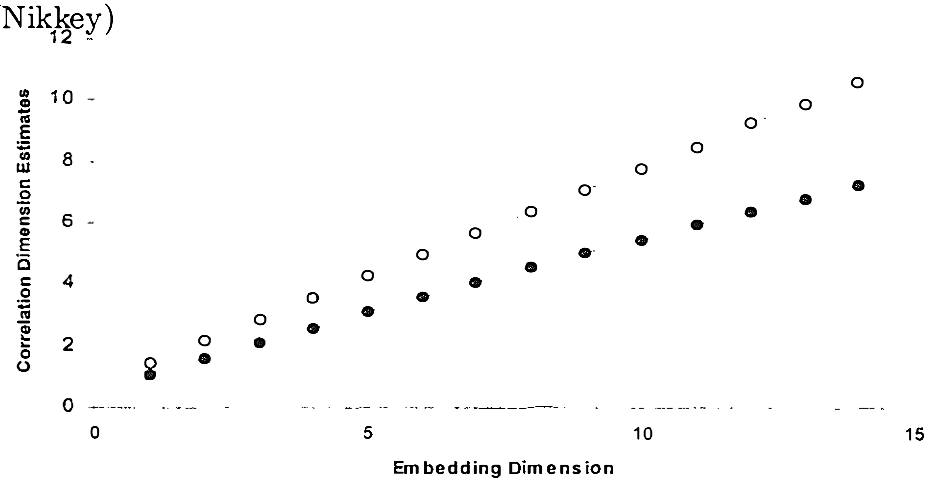
Notes: The black points (•) represent estimates for the ordered original data. The dimension estimates for the shuffled data are represented by the light points (o).

Figure A3 - Dimension Estimates of Standardized GARCH (Dow Jones)



Notes: The black points (•) represent estimates for the ordered original data. The dimension estimates for the shuffled data are represented by the light points (○).

Figure A4 - Dimension Estimates of Standardized GARCH Residuals (Nikkey)



Notes: The black points (•) represent estimates for the ordered original data. The dimension estimates for the shuffled data are represented by the light points (○).

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