# SUSTAINABLE CLUBS UNDER VARIABLE PARTICIPATION*

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## Abstract

In this paper we examine a two-period static club economy where individuals of distinct types decide whether or not to subscribe to clubs at each period. We show that neither facility sizes nor consumption group sizes are affected by variable participation rates because all individuals agree over the benefits of club quality. Variable participation rates and identical preferences for club quality, however, require the coexistence of part-time and full-time clubs. With an example, we show that the full-time clubs of the standard multiple-visit club model may not be sustainable.

## Resumo

Examinamos neste artigo um modelo estático de clubes com dois períodos onde indivíduos de tipos distintos decidem se afiliar ou não a um clube em cada período. Demonstramos que nem o tamanho do clube nem o tamanho do grupo de participantes do clube são afetados por taxas de participação variáveis uma vez que todos os indivíduos concordam sobre os benefícios da qualidade do clube. Participação variável e preferências idênticas por qualidade, no entanto, implicam na coexistência de clubes de tempo parcial e de tempo integral. Através de um exemplo demonstramos que os clubes de tempo integral do modelo de clubes de visitas múltiplas tradicional não é sustentável.

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## 1. Introduction

Congestible facilities are usually characterized by congestion levels that vary from one visitation period (e.g., a day) to the next. Theatres, museums, amusement parks, health and country clubs provide good examples. Equilibria with variable participation rates may result when individuals are heterogeneous. Equilibria with peak and off-peak consumption periods for congestible facilities are often associated with heterogeneous disutilities of congestion at any fixed visitation period. In their classic club theory paper, Scotchmer and Wooders (1987) consider a club economy with variable visitation rates where heterogeneous consumers may have identical preferences for club quality. Club quality is defined as a pair of facility size and aggregate congestion level, or total visitation rate. They show that mixed clubs are sustainable in equilibrium if all individuals agree on the benefits of club quality. Their equilibrium per-visit price, which is a function of aggregate congestion, equates the marginal social cost of crowding to the average cost of provision.

In this paper, we extend Scotchmer and Wooders' framework by explicitly accounting for idiosyncratic benefits over participation. Variable participation rates arise as a natural consequence of the fact that some, but not necessarily all, individuals derive sufficiently large private participation benefits. We show that, under perfect contestability conditions, variable participation rates constrain the set of feasible or sustainable visitation prices. In general, sustainable visitation prices must account for the marginal social cost of congestion *at each* visitation period. Our per-visit pricing concept is, therefore, "finer" than the one used by Scotchmer and Wooders. We provide an example to show that per-visit prices that account for aggregate congestion – coarse per-visit prices – are sustainable if and only if the club's cost of provision is linear in congestion.

We also show that variable participation rates leads naturally to the coexistence of part-time and full-time clubs. Indeed, their coexistence is necessary for the existence of equilibrium. Although we do not formalize an existence proof, the argument for its necessity is quite intuitive. If different numbers of consumers demand club goods at different visitation periods and the optimal consumption group size per visitation is the same across all periods, demand can only equal supply if there are more active clubs when demand is greater. Our model is closely related to a multiple visit club model described by Cornes and Sandler (1996). These authors provide optimal conditions concerning club quality levels, visitation rates and visitation prices for an economy where individuals have idiosyncratic preferences over club quality and visitation rates. Their model, however, assumes a predetermined number of clubs and that participants' preferences depend on aggregate congestion rather than the congestion level at each period. As a result, their equilibrium per-visit prices are coarse prices, and their model is unable to address the issue of coexistence of part-time and full-time clubs.

## 2. The Club Economy

We examine the provision of congestible public goods with variable participation. There are two goods, a private composite good (the numéraire), and a congestible public good (a swimming pool), and two visitation periods. Exclusion is assumed to be costless and perfect at each club and at each provided facility. Club owners make offers that induce individuals to subscribe to their clubs. The offers establish the responsibilities of the clubs and are used to coordinate the pattern of consumption of club goods at each period as well as across periods. The typical offer of an active full-time club (i.e., one that is open in both periods) consists of six parameters. $\{S_1, S_2, n_1, n_2, p_1, p_2\}$. The offer of a part-time club consists of only three parameters; that is, the relevant information for when the club is active. For each period $t$, $t = 1, 2$, $S_t$ is the facility size, $n_t$ is the consumption group size and $p_t$ is the visitation price. The facility and consumption group sizes at some period $t$, $(S_t, n_t)$, gives the quality of a club at that period. The visitation price at period $t$, $p_t$, accounts for the marginal social cost of crowding at that period.

Prior to the beginning of service provision and for given offers, agents decide whether or not to subscribe to clubs. Those who do not subscribe, do not participate in consumption of the club goods. Those who subscribe, choose when and how many times to use the club goods. If an agent decides to participate in both periods, her subscription cost equals the sum of the visitation prices. If she decides to participate only once, her subscription cost is equal to the visitation price at the chosen participation period.

Our framework focuses on a single representative provider. We assume that the representative provider faces actual or potential competition from other providers of the good; competition with free entry will be characterized by the assumption that the club owner must make an offer that maximizes the utility of a representative subscriber.

Our analysis considers cases where crowding effects are anonymous; that is, an agent who uses a club good at some period is affected by the number of other agents who, simultaneously, use the club good, but is indifferent to the types of these other agents. In our model, there are two types of agents, $a$ and $b$. Both types of agents are initially endowed with the same numéraire, $x_0$, and have identical preferences for the swimming pool's quality at each period. These preferences are described by $u(S_t, n_t), t = 1, 2$, where $u$ is concave, weakly increasing in $S_t$, weakly decreasing in $n_t$ and $u(0,0) = 0$.

Agents of different types have different (idiosyncratic) benefits over participation at each period. For example, a fraction of the population may enjoy going to the swimming club on weekends only whereas another fraction may enjoy going to the swimming club on weekends and weekdays. We denote agents' idiosyncratic benefits over participation at each period by $e_t(i), t = 1, 2, i = a, b$. Agents' decisions of whether or not to subscribe to a club depend on these private benefits over participation. We assume that each type of agent knows her own idiosyncratic benefits for both periods and the club owners know the values of the benefits for each type and period but cannot distinguish between individuals. Club owners also know the total populations of both types of agents, $N^a$ and $N^b$.

## 3. The Main Result

In this section we show that by introducing variable participation in the standard multiple-visit club model, an additional equilibrium condition arises: a participation constraint. It determines the type(s) of agent(s) who participate(s) at each period. Consider an arbitrary agent of type $i, i = a, b$. If she subscribes to a representative club for usage at both periods, she receives utility

$$x_0 - p_1 - p_2 + u(S_1, n_1) + u(S_2, n_2) + e_1(i) + e_2(i)$$

If she subscribes for usage at one but only one period $t$, her utility is

$$x_0 - p_t + u(S_t, n_t) + e_t(i)$$

If she does not subscribe, her utility is solely $x_0$.

Consider now the decision of an agent of type $i$ of whether or not to subscribe to a representative club. We start by examining the circumstances under which the agent does not subscribe to the club. The agent does not subscribe to the club if participation is not individually rational at any period. Participation is not individually rational at some period $t$ if the agent's utility from participation at that period is less than $x_0$:

$$x_0 - p_t + u(S_t, n_t) + e_t(i) < x_0$$

or simply

$$(1) \qquad u(S_t, n_t) + e_t(i) < p_t,$$

that is, whenever the total benefit from participation is less than the cost from participation. It follows from condition (1) that an agent of type $i$ participates at some period $t$, $t = 1, 2$, if and only if

$$(2) \qquad u(S_t, n_t) + e_t(i) \geq p_t$$

Therefore, an agent of type $i$ subscribes to the club if condition (2) is satisfied for at least one period.

The total cost of providing a swimming pool at both periods is $C(S_1, n_1) + C(S_2, n_2)$, where $C$ is assumed to be increasing and convex in both arguments, and $C(0, 0) = 0$. The club owner provides the swimming pool at both periods if and only if

$$(3) \qquad n_1 p_1 + n_2 p_2 - C(S_1, n_1) - C(S_2, n_2) \geq 0,$$

that is, if the total revenue is at least as great as the total cost. With free entry, the club owner makes an offer $\{S_1, S_2, n_1, n_2, p_1, p_2\}$ which is feasible – that is, an offer which satisfies (2) and (3) – and which maximizes the surplus accruing to an average subscriber. (If the owner did not, another provider would offer a better arrangement and compete away her subscribers.) Since both types of agents have identical demands for the club good, the principal's problem is as follows[1]:

$$(P1) \qquad Max_{\{S_1,S_2,n_1,n_2,p_1,p_2\}} u(S_1,n_1) + u(S_2,n_2) - p_1 - p_2$$

subject to

$$u(S_t,n_t) + e_t(i) - p_t \geq 0, i = a,b; t = 1,2$$

$$n_1 p_1 + n_2 p_2 - C(S_1,n_1) - C(S_2,n_2) \geq 0$$

$$N > N^i \geq n_t \geq 0, i = a,b; t = 1,2$$

$$S_t \geq 0, p_t \geq 0, t = 1,2.$$

where $N = N^a + N^b$ is the total population size.

The assumption of free entry (or contestability) in the full and part-time clubs markets implies that not only condition (3) must hold with equality but also profits in each period must be equal to zero. Therefore, we can replace $p_1$ and $p_2$ with the respective average costs in problem (P1). Moreover, as the problem is symmetric (both agents have identical demands for the club good), we will have that in equilibrium $n_1 = n_2 = n$ and $S_1 = S_2 = S$. As the individual participation constraint (2) is slack, problem (P1) is equivalent to

[1] Given potential entry and identical demands for club quality, it follows that the principal behaves as a utilitarian social planner in that his objective function – the surplus that accrues to the average subscriber – is isomorphic to the sum of utilities of subscribers. The competitive equilibrium, therefore, is Pareto efficient.

choosing $n$ and $S$ to maximize $u(S,n) - c(S,n)/n$. The solution is characterized by the first-order conditions obtained by differentiating this expression with respect to $n$ and $S$ and setting them equal to zero. This discussion is summarized by the following proposition, which characterizes the solution to (P1) when the club provides the good in both periods.

**Proposition 1** *The conditions which characterize the solution with positive provision in both periods for (P1) are as follows:*

$$n_1 = n_2 = n \quad \textit{and} \quad S_1 = S_2 = S \tag{4}$$

$$n\frac{\partial u(S,n)}{\partial S} = \frac{\partial C(S,n)}{\partial S} \tag{5}$$

$$\frac{C(S,n)}{n} = \frac{\partial C(S,n)}{\partial n} - n\frac{\partial u(S,n)}{\partial n} \tag{6}$$

$$p(S,n) = \frac{C(S,n)}{n} \tag{7}$$

$$u(S,n) + e_t(i) - \frac{C(S,n)}{n} > 0, \tag{8}$$

*for at least one* $i, i = a, b$; *and* $t = 1,2$.

As both types of agents have identical demands for the club good, condition (4) states that optimal qualities must be identical across periods. Condition (5) is the Samuelson condition for optimal provision of facility size; it states that the sum of agents' marginal rates of substitution between public and private goods must equate the marginal social cost of provision. The left-hand size of (5) gives the marginal social benefit. The right-hand side of (5) is the marginal social cost of provision. Condition (6) gives the marginal social benefit from expanding the consumption group size at the club. The marginal user's cost sharing contribution is a benefit to other users of the good. The right-hand side of (6) gives the marginal social cost of crowding from expanding the consumption group size. The optimal consumption group size is obtained by equating marginal social benefit from cost sharing to marginal social cost of crowding.

Condition (7) determines the optimal price of usage. It states that the optimal price must equate the average cost of provision at each period. By (6), we can see that the optimal price of usage must also equate the marginal social cost of crowding. As illustrated by conditions (4) through (7), free entry implies that there cannot be cross-subsidization across periods. Condition (8), the participation constraint, is novel. Positive provision at each period requires that at least one type of agent decides to participate.

The optimal mechanism determined by conditions (4) through (8) does not depend on which type of agent visits the club at any period; the conditions are the same if both types decide to visit the club simultaneously. However, the number of active clubs in equilibrium at any period depends on which type of agent visits the club as well as on whether one or both types of agents participate. At any period, many more clubs are needed when both types participate than when only one type of agent does so. Variable participation inevitably requires some clubs to provide the club good in both periods (full-time clubs) and some clubs to provide the good in only one of the periods (part-time clubs).

Since agents subscribe to clubs prior to service provision and providers know agents' idiosyncratic benefits over participation for both periods as well as the overall distribution of agents' types in the population, the optimal mix of full-time and part-time clubs is known at the offer-making stage. For example, if both types of agents decide to participate in the first period (that is, condition (8) is satisfied for both types) and only agents of type $b$ decide to participate in the second period, equilibrium requires $\frac{N}{n}$ clubs to provide the good in the first period and $\frac{N^b}{n}$ clubs to provide the good in the second period. Since $\frac{N^b}{n}$ clubs provide the good in both periods, this is the necessary number of full-time clubs that should exist for the optimal mix between full-time and part-time clubs to be reached. Similarly, there should be $\frac{N^a}{n}$ part-time clubs.[2]

It is worth noting that the equilibrium allocation characterized in Proposition 1 corresponds to the allocation that arises from the maximization of an utilitarian social welfare function:

$$W = [N^a(I_1^a + I_2^a) + N^b(I_1^b + I_2^b)][u(S,n) - \frac{C(S,n)}{n}]$$

subject to

$$I_t^i = 1 \quad \text{if} \quad u(S,n) + e_t(i) \geq \frac{C(S,n)}{n}, i = a, b; t = 1, 2$$

[2] It should be clear that we implicitly assume away the integer problem. A formal analysis of the integer problem would require us to impose more stringent conditions than those imposed for the conventional club model (see, e.g., Wooders (1978,1980)).

$$I_t^i = 0 \quad \text{if} \quad u \quad ,n \ + e_t \ \imath \ < \frac{\quad}{n}, \imath = a, \ ; t = 1, 2$$

$$S \geq 0, N > N^i \geq n \geq 0, i = a, b.$$

where the first bracketed term of the objective function gives the total number of participants in each visitation period and the second bracketed term gives the consumer surplus of the average subscriber at each visitation period.

## 4. An example and a comparison with the standard multiple-visit club model

In this section, we provide an example of our analysis and use this example to illustrate the effects of variable participation on the standard multiple-visit club model. Suppose that agents of one type make more visits to clubs than agents of the other type. Let any active full-time club make an offer $\{S_1, S_2, n_1, n_2, p_1, p_2\}$, where $S_1 = S_2 = S, n_1 = n_2 = n, p_1 = p_2 = p\}$, and $S$, $n$ and $p$ solve (5), (6), and (7). One of the main consequences of Proposition 1 is that the offer of any active club is not affected by the type of agent who visits the club at any period.

Suppose agents' benefits from participation are described as follows:

$$u(S, n) + e_1(i) > p > 0, i = a, b(\text{period1})$$

$$u(S, n) + e_2(a) < p, \quad \text{and} \quad u(S, n) + e_2(b) > p > 0(\text{period2})$$

Then, agents of type $a$ participate in period 1 only and agents of type $b$ participate in both periods. The utilities from participation for both types of agents are

$$x_0 + u(S,n) - p(S,n) + e_1(a)(\text{type a})$$

$$x_0 + 2[u(S,n) - p(S,n)] + e_1(a) + e_2(b)(\text{type b})$$

The number of active clubs in period 1 is $\frac{N}{n}$ and in period 2 is $\frac{N^b}{n}$. Since the number of active clubs decreases in period 2, when only agents of type $b$ visit the club, the optimal mix of clubs is composed of $\frac{N^b}{n}$ full-time clubs and $\frac{N^a}{n}$ part-time clubs. Part-time clubs are open in the first-period and closed in the second. Although it seems that the coexistence of full-time and part-time serves to screen agents according to their participation rate – that is, agents of type $a$ subscribe to part-time clubs and agents of type $b$ subscribe to full-time clubs – it should be clear from our analysis above that there is no reason for this to be so. As both types of agents have identical preferences regarding quality levels, they can be mixed in any possible way at any type of club.

We now use this example to compare our equilibrium formulation with the one described by Scotchmer and Wooders (1987). They consider an economy composed of two types of agents, a private good and a club good, and where agents visit clubs which provide the club good at different rates. They define a competitive equilibrium as a triple of initial endowments, per-visit prices and visits made by each type of agent, $\{(w^i), P(S,c), [v^i(S,c)]\}$, such that if $v^i(S,c) > 0$, then:

$$\{S, v^i, c\} = \arg\max u^i[w^i - v^i(S,c)P(S,c), S, v^i(S,c)]$$

$$P(S, c) = \frac{C(S, c)}{c}.$$

where $c = \sum_{i=a,b} n^i v^i$ is the total congestion or the total number of visits made at a representative club. Using Scotchmer and Wooders' equilibrium formulation for the example above, we obtain the following equilibrium per-visit (per-period) price:

$$P(S, 2n) = \frac{C(S, 2n)}{2n}$$

as total congestion at a representative full-time club is $2n$. If we assume that the optimal facility size is unchanged under either type of equilibrium formulation, we obtain the following utilities from participation:

$$x_0 + u(S, n) - p(S, 2n) + e_1(a)(\text{type a})$$

$$x_0 + 2[u(S, n) - p(S, 2n)] + e_1(a) + e_2(b)(\text{type b})$$

Note that if there are constant returns to scale to $n$, our equilibrium formulation is identical to the one presented by Scotchmer and Wooders, since in this case we have:

$$\frac{C(S, 2n)}{2n} = \frac{2C(S, n)}{2n}$$

However, if the cost function is strictly convex in $n$, our equilibrium formulation leads to lower per-visit prices, since:

$$\frac{C(S, 2n)}{2n} > \frac{2C(S, n)}{2n}$$

In this case, Scotchmer and Wooders, by letting the per-visit price depend upon aggregate congestion rather than the congestion level at each visit, obtain a per-visit price which is not sustainable in equilibrium. Both types of agents would be better off with a finer per-visit price that accounts for the congestion level at each visit.

## 5. Conclusion

In this note we examined a two-period static club economy where individuals of distinct types decide whether or not to subscribe to clubs at each period. Individuals subscribe to the club at period $t$, $t = 1, 2$, if and only if it is individually rational to do so. Differences in participation rates arise naturally as a consequence of differences in private benefits over participation. Neither facility nor consumption group sizes are affected by variable participation rates because all individuals agree over the benefits of club quality. Variable participation rates and identical preferences for club quality, however, require the coexistence of part-time and full-time clubs.

With a simple example, we compare our results with those of Scotchmer and Wooders (1987). They consider provision of club goods by full-time clubs only, where prices reflect total congestion rather than per-visit congestion. When the cost function is strictly convex in congestion, we show that their clubs may not be sustainable in equilibrium in the sense that agents would prefer a per-visit price which only accounts for the congestion level at each visit.

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