INPUT-OUTPUT ANALYSIS AND ECONOMIC PLANNING: A SURVEY

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SUMMARY

In this paper the author surveys the literature on Input-Output Analysis and its importance to Economic Planning. The contents involves: models, policies and plans, input-output frameworks, the construction and uses of social accounting matrix, endogenous investment and dynamic forms, construction and uses of population accounting matrix, sources material for econometric models, and many others related topics.

RESUMO

O autor sumariza o estado da arte no campo da análise de insumo-produto e sua importância para o planejamento econômico. É discutido: modelos, políticas e planos, a estrutura de insumo-produto, a construção e uso da matriz de contabilidade social, investimento endógeno e formas dinâmicas, construção e uso da matriz populacional, dados para modelos econométricos, e muitos outros tópicos relacionados.

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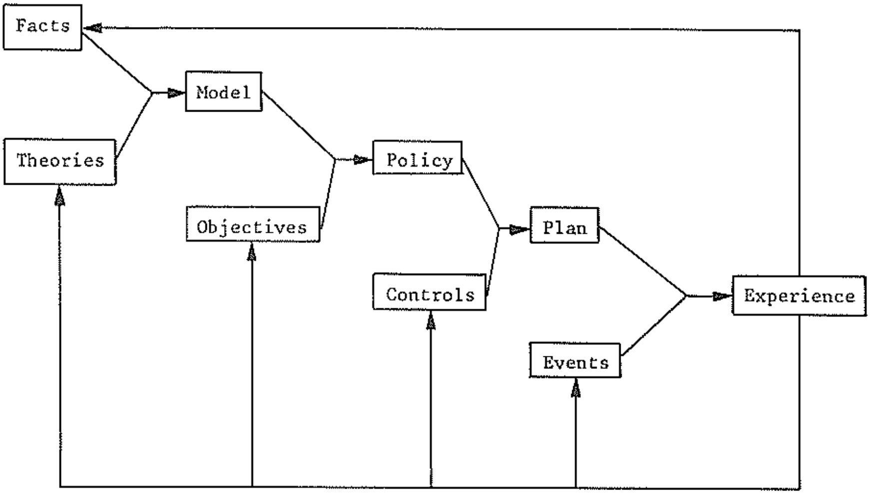
1. MODELS, POLICIES AND PLANS

Input-output analysis is a simple form of economic modelling, widely used to represent industrial interdependence but applicable in many other areas of economic and social life. Economic planning is a means of organising an economy or some smaller economic unit with a view to ensuring that certain aims can be realised.

Let me begin by showing how I think these terms are related. Diagram 1, which I have used for many years, brings out as simply as possible what seem to me to be the essential connections.

Diagram 1

MODELS, POLICIES AND PLANS



Let us start at the left-hand top corner with facts (or observations or data). Even if these were all we had, we might still be able to make a contribution to planning. For example, suppose we knew how much of each commodity we wanted to consume, how much of each input was needed to produce a unit of each output, the capacities of the different industries and the availabilities of the different primary inputs. Then we could work out whether our capacities and availabilities were sufficient to enable us to produce what we wanted. If they were not, we could attempt to discover, by trial and error, acceptable ways of reducing our demands or increasing our supplies.

In this situation a little theory would be a great help. Thus, if we knew Leontief's theory of input-output relationships, we could calculate the outputs needed to meet a given set of final demands in a single operation.

Thus, from a practical point of view, facts without theories enable us to make a start whereas theories without facts do not. It is best if we have both so that by combining them we can construct a model. Of course the facts may be inaccurate or out of date and the theories may be over-simplified; but hopefully we can construct a descriptive model of how the economic world works which is sufficiently realistic to be useful.

Such a model can be used to make projections given a set of initial conditions and assumptions about the future course of the exogenous variables. It will spell out the future changes to be expected in the endogenous variables given its form and the assumptions made. As a rule the structure and parameters of the model will be fixed; but, in medium- and long-term projections, some of the parameters, such as input-output coefficients, may be allowed to change in accordance with the model builder's expectations. The outcome may suggest that in the projection period a number of objectives, such as steady growth, a stable price level, high employment and balanced trade with the rest of the world, will be met or, alternatively, that only some of them or even none of them will be met.

If the objectives to be taken into account can be specified then they can be combined with the model in order to work out policies which would improve the chances of meeting them. This would bring the model builders into contact with those responsible for formulating objectives and policies, namely politicians. They might succeed in building the objectives into the model, that is to say in setting up a policy model in which the solutions respected the constraints represented by the objectives. This would go some way to formalising the decision-making process; but only some way, because the solutions might be found to embody undesirable features which had not been considered in formulating the objectives.

The formulation of policy is only one stage in mapping out a course of action; it is also necessary to see how that policy can be implemented. This is achieved in my diagram by what I have called controls. A simple example is provided by the targets and instruments approach described in Tinbergen (1952, 1956). If the objectives can be expressed in terms of certain variables taking specified values then, with an equal number of instruments, we can work out the values that these instruments must take if we are to hit the targets in the projection year, on the assumption that the system is in a steady state in that year. We can also gain considerable insight into the problems of control by calculating trade-off curves connecting a target variable with an instrument for fixed values of the other instruments and by calculating the values of a set of instruments which correspond to a given value of one of the target variables.

Again, we can try to build the control system into the model thereby obtaining a planning model. This may be extremely enlightening but in practice the complexity and uncertainties of the world are such that those responsible for action could not be expected to put themselves in the hands of even the most sophisticated model that can be built at the present time. A more pressing danger is that the difficulties of communication between politicians, administrators and model builders will be such that the contribution of models to decision-making will be less than it might be.

When the plan is put into operation it comes up against events and so we gain experience of the practical working of the methods we have adopted. This may lead us to revise not only our theories, objectives and controls but also our facts, that is to say the empirical correlates of the true facts with which we work.

It would be a simple matter to elaborate this description of the relatioship between knowledge and action and to introduce more connections and feed-back loops between the various boxes. But, as it is, the diagram is sufficient to show the comparatively modest nature of my subject: a single component at the model-building sta ge in the whole process of planning. Complications arise, quite apart from the extent to which we try to build the decision-making process into the model for a number of reasons. In the first place, there are many aspects of economic and social life to be modelled, for instance, the process of production, consumption and accumulation, the distribution of income and wealth, the monetary and financial system and a variety of demographic and social processes in which input-output plays an important role, though usually under a different name. In the second place, there are many levels at which models may be wanted, for instance, at the national, regional, industry or company level. In the third place, the planning period may be different, for instance, short, medium or long. And, finally, models may be of different types: linear or non-linear, open or closed, static or dynamic, deterministic or stochastic and so on.

Whatever the subject or form of the model, it is convenient to have a framework into which the main data on stocks, flows, prices and costs can be fitted. By now these are fairly standard and can be described as follows.

2. INPUT-OUTPUT FRAMEWORKS

National economic planning models are usually based on a system of national accounts, that is the economic accounts of the society being modelled. These accounts are brought together in a matrix in which each row and column pair represents an account with the incomings shown in the row and the outgoings shown in the column. Such a matrix is usually termed a social accounting matrix or SAM for short. The accounts it contains may be aggregated, so as to provide only variables needed for a macro-model, or they may be highly detailed.

The matrices which gave rise to the abbreviation SAM were constructed by the Cambridge Growth Project and published in CDAE (1962b). In the study describing the United Nations' system of national accounts, or SNA for short, UNSO (1968), increasingly disaggregated SAMs are set out in tables 1.5, 1.6 and 2.1.

For present purposes it is convenient to have a standard SAM but the only distinction I shall make is between endogenous and exogenous accounts, as in table 1 below.

Table 1

SYMBOLIC SAM

	Endogenous accounts	Exogenous accounts	Totals
Endogenous accounts	T_{11}	T_{12}	$t_{1.}$
Exogenous accounts	T_{21}	T_{22}	$t_{2.}$
Totals	$t'_{.1}$	$t'_{.2}$	

Table 1 relates to the transactions in an economic system closed by including a set of accounts relating to dealings with the rest of the world. The line between endogenous and exogenous accounts depends on the analysis to be made; if, for instance, we are constructing a simple input-output model of a productive system, the endogenous accounts will be the production accounts of the various branches. Thus, in this case, the elements of the matrix T_{12} relate to final output, those of T_{11} to intermediate output and the elements of the vector $t_{1.} \equiv T_{11}i + T_{12}i$ (where i denotes the unit vector) relate to the total incomings into the production accounts. Similarly, in the columns, the elements of T_{21} relate to primary inputs and those of $t_{.1} \equiv T'_{11}i + T'_{21}i$ (the prime suffix indicates the transposition of a vector or a matrix) relate to the total outgoings from the production accounts. The remaining submatrix, T_{22}, relates to transactions among the non-production accounts and, since all the accounts balance, $t_{1.} \equiv t_{.1}$ and $t_{2.} \equiv t_{.2}$.

The open input-ouput model follows easily from diagram 1. Let us write $T_{11} \equiv Y$, $T_{12}i \equiv x$ and $t_{1.} \equiv y$. An input-output coefficient matrix, A, is obtained by dividing the elements in the columns of Y by the corresponding element of y. If we denote a diagonal matrix by the symbol for a vector surmounted by a circumflex accent, then

$$A = Y\,\hat{y}^{-1} \tag{1}$$

From the first row of diagram 1 we can write

$$\begin{aligned} y &= Yi + x \\ &= Ay + x \\ &= (I - A)^{-1}x \end{aligned} \tag{2}$$

on substitution for Y from (1). The matrix $(I - A)^{-1}$ is the Leontief inverse and is also termed a matrix multiplier on an analogy drawn in Goodwin (1949) with the scalar multiplier in Kahn (1931) which plays an important role in *The General Theory*, Keynes (1936). As can be seen from (2) this matrix multiplier transforms the vector of total output, x, into the vector of total output, y.

Equation (2), which forms the basis of the input-output representation of a productive system, relates to a single period and the entries in the transactions table are expressed in currency units. In principle, some of them could be expressed in physical units but there are many cases in which this would be difficult and others in which it would be impossible because, for example, there are no physical units in which to measure taxes and other transfers.

Before I go on to the difficulties that arise in applying input-output analysis and the ways in which it can be elaborated, I shall set out a standard demographic matrix which, on the analogy of SAM, we might call PAM standing for population accounting matrix. This is done in table 2.

Table 2

SYMBOLIC PAM

	Our country	Outside world	Totals
Our country	N_{11}	N_{12}	$\Lambda n_{1.}$
Outside world	N_{21}	N_{22}	$\Lambda n_{2.}$
Totals	$n'_{.1}$	$n'_{.2}$	

Table 2 relates to the population numbers in a demographic system closed by including a set of accounts relating to the outside world in addition to those for our country (or our region or our hospital system or whatever it may be). The outside world relates not only to other countries in this world but also to the other world from which births come and to which deaths go. Like SAM, PAM relates to a single period but, unlike SAM, the accounts in PAM do not balance and the formulation is essentially dynamic. The symbol Λ denotes the lag operator, $\Lambda^{\theta} n(\tau) \equiv n(\tau + \theta)$, which shifts in time the variable to which it is applied.

The accounts for our country relate to the population classified in any possible way: for instance they might relate to males classified by year of birth. In this case N_{11} would contain survivors from each age to the next in the leading subdiagonal and zeros everywhere else. The accounts for the outside world relate to births, deaths and migrations.

In general, the scheme can be described as follows. The elements of the row vector $n'_{.1}$ relate to the numbers in the population at the beginning of the period (the opening stock) in each category of the classification. In the course of the period these individuals will either pass into the outside world, by death or emigration, that is they will appear in N_{21}, or they will survive in our country until the end of the period, that is they will appear in N_{11}, The elements in a column of this submatrix show the number of survivors from a given initial state who are present in various states at the end of the period. Similarly the elements in a row of the same submatrix show the number of survivors from various initial states who are present in a given state at the end of the period. The elements of N_{12} show the entrants into our country during the period classified by their state at the end of it. Thus if we add together the elements in a row of N_{11} and N_{12} we obtain an element of $\Lambda n_{1.}$ that is the number of individuals in a given state in the closing stock.

In this case let us write $N_{11} \equiv S$, $N_{12}i \equiv b$ and $\Lambda n_{1.} \equiv \Lambda n$. Then we can form a coefficient matrix, C say, as

$$C = S\,\hat{n}^{-1} \tag{3}$$

and so we can write from the rows for our country in diagram 2

$$\begin{aligned} \Lambda n &= Si + b \\ &= Cn + b \end{aligned} \tag{4}$$

which states that the closing stock vector Λn is equal to the opening vector n, transformed by the coefficient matrix C, plus the vector of entrants b. If the population is in a state of stationary equilibrium, then $\Lambda n = n$ and (4) can be written as

$$\begin{aligned} n &= Cn + b \\ &= (I - C)^{-1} b \end{aligned} \tag{5}$$

which is of the same form as (2). In (5), $(I - C)^{-1}$ is a matrix multiplier which transforms the entrants of a period into the total population. This type of demographic model is developed in Stone (1971, 1973b), UNSO (1975) and elsewhere.

3. CONSTRUCTING AND USING SAMs

A social accounting matrix is, in the first place, a means of presenting data in an orderly way so that the implicit arithmetic and accounting identities connecting the data are evident; and, in the second place, a basis for model building. The taxonomic issues involved have been discussed in connection with the SNA, briefly, in UNSO (1968, ch. II) and, in more detail in UNSO (1973). Let us now look at some of these.

(a) Commodities and industries. In input-output analysis it is usual to define industries in terms of a group of establishments mainly engaged in producing a range of similar products which are characteristic of them. However, whatever classifications we use, it will always be found that some establishments in a group make products characteristic of other groups. As a consequence, industrial cost structures are rarely clean since they contain elements due to secondary production. This being so, it is desirable to distinguish commodities and industries and to start from two matrices: (i) a 'make' matrix, with industries in the rows and commodities in the columns, a row of which shows a given industry's output of its characteristic and secondary products; and (ii) an 'absorption' matrix, with commodities in the rows and industries in the columns, a column of which shows a given industry's use of its own and other characteristic products as intermediate inputs into current production.

With this information it is possible to transform the commodity x industry absorption matrix into either a commodity x commodity or an industry x industry matrix. In order to do this however it is necessary to make assumptions. The limiting cases are usually termed the assumption of a commodity technology which implies that a given commodity has the same input structure in whichever industry it is produced, and the assumption of an industry technology which implies that a given industry has the same input structure whatever the mix of its outputs. These limiting cases were explained in CDAE (1963). It is also possible to combine these extremes to produce hybrid technology assumptions as described in Gigantes (1970), Armstrong (1975) and UNSO (1968, 1973).

(b) Rectangular tables. Once it is agreed to distinguish between commodities and industries, it is natural to question a definition which makes them equal in number. On any ordinary view of the matter there are far more commodities than industries and this is recognised in Gigantes (1970). Rectangular tables have advantages over the usual square tables in that they make it possible to use more fully data that are often available, enable the relationships of production to be modelled more accurately and help in the application of input-output methods, for instance to cost-benefit analysis.

c) Domestic production and imports. The four main methods of treating imports in input-output tables are described and illustrated in UNSO (1973).

The first method consists of classifying imports by commodities, showing them as purchases from the rest of the world by the commodity accounts and combining them with similar domestic output in all sales by the commodity accounts. This treatment requires relatively little information but has the disadvantage that domestic and foreign sources of supply remain fixed in their base-period proportions.

The second method consists of classifying imports only by purchaser. Thus all commodities are domestically produced and each purchaser obtains foreign goods and services as a single item from the rest of the world. Although this method provides rather more information than the first variant, it has the disadvantage that imports are treated as a single item for each purchaser and are altogether excluded from the input-output table.

The third method consists of dividing commodities into two types: competitive commodities, which can be produced in the domestic economy; and complementary commodities, which can only be produced abroad. Domestic and imported competitive commodities are combined in the row for these commodities and complementary commodities are shown in a separate set of rows. This means that competitive commodities are treated as all commodities are treated under the first method while complementary commodities must come from the rest of the world and the demand for them cannot stimulate domestic production. This treatment is superior to the first method and may not involve much extra work: in Britain at any rate complementary imports are few in number but, in the aggregate, high in value.

The fourth method consists of a complete separation of domestically produced and imported commodities; and requires two sets of rows and columns, one for each type. Thus under this arrangement, there will not only be two vectors for each type of final buyer but also two input-output tables for inputs of domestic products and of imports respectively. Though demanding of data, this variant is extremely helpful in modelling the impact of demand on domestic and foreign supplies. Examples can be found in the British tables for 1963 and 1968 published in UKCSO (1970, 1973).

In model building it is likely to make a considerable difference which of the above variants is adopted. Clearly, the first should if possible be avoided, since it implies a fixed share of the domestic market for each import; and, equally, the fourth is the most flexible since it distinguishes domestic from foreign supplies in all cases.

It is of course necessary to ensure that domestic producers are not required to produce imports or the input into imports and so on; but this can be achieved by treating imports as a negative component of final demand.

(d) The valuation of product flows. Since product flows are measured in money terms, a branch of production is stimulated equally by a given expenditure on its products wherever this occurs. This raises the question of valuation since, quite apart from problems of product mix, different buyers may pay very different prices for the same product. Thus, households usually buy through retail shops whereas businesses buy wholesale and so avoid the final link in the distributive chain. Some producers, such as farm families which consume part of the food they have produced, avoid both wholesale and transport charges. Thus it may make a great deal of difference whether goods and services are valued at purchasers' or at producers' values.

Indirect taxes and subsidies operate in a similar way; the tax rate charged when the buyer is a household may be reduced when the buyer is a business and remitted altogether on exports. Thus we may expect to get a still more homogeneous treatment if we value products at factor costs rather than at producers' values just as homogeneity is increased by choosing producers' rather than purchasers' values.

In the SNA, a concept intermediate between factor cost and producers' values is introduced. It is termed basic values, and its true and approximate forms are explained in UNSO (1968, annex to ch. IV). I shall not discuss this concept here since its purpose is solely to get over certain practical difficulties of measurerement.

(e) Constant-price comparisons. Although outputs and intermediate product flows may be thought of as quantities measured in physical units, they are, as we have seen, usually measured in money units. This means that comparisons over time involve adjustment to a common price structure, say the price structure of one year chosen as a base. In principle, on the assumption that commodities are homogeneous so that the same price is appropriate to each element in a row of the transaction table, the adjustment is relatively simple: the constant-price coefficient matrix is obtained by a similarity transformation of the current price matrix, the transforming factor being a diagonal matrix of the price ratios in the two periods. In practice, however, the position is much more difficult since uniform price measures cannot be used all along the rows of the transactions table. For instance, if chemicals n.e.s. is a commodity, its input into agriculture consists mainly of fertilisers, into rubber mainly of synthetic rubber, into manufacturing n.e.s. mainly of plastics, into motor vehicles and construction mainly of paint, and so on. As a result, many if not most of the cells of the transactions table have to be treated individually.

(f) The stability of coefficients, updating and projection. The methods we have considered so far are designed to make the best use of the data available and to arrange it in a coherent and flexible form for analytical purposes. In building planning models, attention to these details is necessary, but it forms only part of what is required of the input-output economist. Almost all input-output tables relate to the past whereas planning relates to the future. Nowhere is Burke's dictum that 'you can never plan the future by the past' more true than in input-output analysis. The reason is that input-output coefficients tend to change over time as a consequence of changes in the techniques of production and in relative prices. Consequently, some effort has to be devoted to up dating and projection if input-output methods are to be used in planning.

In discussing the techniques available for these purposes, it is conveniente to arrange them according to their information content, in which they very considerably. In practice it is not very likely that the better methods can be applied to all coefficients and so a number of methods of varying reliability will have to be used.

(i) A revision of the model. Since our immediate difficulty arises from the fact that coefficients are assumed constant in the input-output model which are not constant in real life, any fundamental improvement requires that we complicate the model. This can be done in a number of ways.

In CDAE (1968), Wigley carried out a detailed study of the demand for energy products. He was particularly concerned with the replacement of coal by oil and the position was made more difficult by the recent discovery of North Sea gas. His estimates of changing input-output coefficients for coal and oil were obtained by combining stock-adjustment models of the relative use of these two products in various industries with equations expressing outputs in terms of fuel inputs. The exogenous variable in the stock-adjustment models was the ratio of coal price to oil price so that if this ratio were stabilised the coal coefficients would eventually reach a minimum and the oil coefficients a maximum for moderate levels of gas consumption. For higher levels of gas consumption, oil might reach a maximum and, subsequently, itself begin to be replaced by gas.

An entirely different method has been used in Peterson (1978 a and b). The second of these papers, which is also concerned with fuels, makes use of a generalised cost function which gives Leontief's fixed coefficients as a special case. Econometric analysis shows for both industries and final consumers that: any attempt to explain changes in the pattern of fuel consumption in Britain over the period 1955-75 without using time trends as proxies for technical progress or changes in consumer taste is rejected at a very high level of significance; and that the same can be said for the influence of relative prices.

An alternative model, which has often been applied to final oonsumers' demand but not, as far as I know, to intermediate demand, is provided by the linear expenditure system: see, for instance, Stone (1954), Deaton (1975). In this model the Engel curves, both for expenditures and quantities, are linear through the origin (and therefore like Leontief's constant coefficients) wherever committed expenditure is zero: that is where $c_j = 0$ in the usual terminology. The estimates given in Deaton (1975, table 5.2) suggest that typically this condition is not fulfilled and, further, that time trends in the supernumerary budget shares, the b_j are often significant.

These studies confirm what is only to be expected, namely that input-output coefficients are not very likely to be constant. However, the methods described require a great deal of data and in many cases less demanding methods will have to be used.

(ii) The Delphi method. Consulting the oracle or, in the present case, engineers, chemists and other experts concerned with production processes, can be extremely useful in compiling and projecting input-output tables. The method has been applied on several occasions by the Battelle Memorial Institute and is described in Fisher and Chilton (1972). It has the advantage that it goes directly and knowledgeably to what is required, namely estimates of coefficients, and avoids the difficulties of deriving these from records, usually incomplete, of sales and purchases. Its disadvantage is that it requires good contacts with the technical side of industry and so can be applied systematically only by a large organisation. On a more modest scale it may be extremely useful in quantifying general tendencies of change.

(iii) Time trends. If these methods fail, it is necessary to fall back on a purely empirical approach. If a coefficient has been changing in the past it is likely that in some degree the change will continue and projections may be made by means of time trends. Clearly for this purpose a sequence of comparable tables at constant prices is of great value.

The main difficulty in this case is to decide on the rate of change that is probable in the future. Many changes in coefficients are associated with a new technique of production that penetrates the establishments in that branch at a certain rate until they have all adopted it. At this point the change comes to an end. Accordingly, it seems desirable to adopt a sigmoid form of trend which will eventually reach an upper or a lower bound. This has been done on a large scale in the Maryland model and is described in Almon and others (1974, pp. 157-64).

(iv) Mechanical adjustments. By means of the methods described so far it should be possible to project the main changes in input-output coefficients. However, if the associated intermediate product flows are calculated and introduced into a projected SAM, it is not very likely that all the production accounts will balance though the discrepancies may not be very large. Having done all that can be done with more informative methods, a final balance can be obtained by the application of a mechanical adjustment tecnique. The one most generally used, the RAS technique, is described, for instance, in Stone (1962), CDAE (1963) and Bacharach (1970). Other methods are available, for instance in Matuszewski, Pitts and Sawyer (1964) and Nijkamp and Paelinck (1974).

(v) Projections of prices and quantities. The projection of input-output tables is usually treated as a matter of projecting flows expressed in physical or constant-price units without an explicit consideration of the accompanying prices. This is not altogether satisfactory since projected prices (or at least relative prices) are also needed in a complete model and independent projections of these may not be consistent with the projected quantities. A means of reconciliation is proposed in Stone (1968) and in a slightly earlier paper by Fontela and others (1970). This method is not a mechanical one but an application of the adjustment of conditioned observations based on a variance matrix arrived at subjectively. The severe computing problems involved have been resolved successfully in Byron (1978).

The fact that input-output coefficients change, the reasons for this and the problems to which it gives rise have been studied since the beginning of input-output analysis. The first edition of The Structure of American Economy, Leontief (1941) contains two input-output tables, for 1919 and 1929, and the second edition adds a third, for 1939. The matter is further discussed in Leontief and others (1953). Among many other studies, I will mention Arrow and

Hoffenberg (1959), Tilanus (1966) and Carter (1970). In Sevaldson (1963, 1970, 1976) we find a series of studies of the stability of coefficients based on Norwegian experience. And in Barker (1975 a and b) we find some results of updating and projecting intermediate demands based on British experience.

4. EXTENSIONS AND GENERALISATIONS

The usefulness of input-output techniques is not confined to the analysis of industrial structure and change. I shall now give examples of their extension to the study of pollution and of their generalisation to cover the interactions of two or more divisions, such as production and consumption, in an economic system.

(a) The extension to pollution. A method of analysing the economic effects of pollution by means of input-output techniques is set out in Leontief (1970b). The usual n x n table of intermediate product flows is extended by the addition of m additional rows and columns relating to pollutants and treatment services which, in this exposition, are assumed to be in one-to-one correspondence. Each of the m rows relates to the quantity of treatment required measured by the emissions from the various industries of the corresponding pollutant: and each of the m columns contains the cost strucutre of one of the treatment services.

Taken literally, this model implies that all pollutants are to be treated; but this is not essential and might lead to an undesirable reduction in the supply of regular goods and services. Clearly it might be in the social interest to reduce pollution without necessarily eliminating it. The question of how much to eliminate and how to decide this question is considered in Stone (1972b). In Meade (1972) it is pointed out that the citizen is likely to be more interested in the cleaner air, land or water resulting from treatment than in the amount of the treatment itself.

Leontief's original idea was applied to the problem of air pollution in Leontief and Ford (1972). Since then many other writers have contributed to the subject, such as Ayres (1974), Cumberland and Stram (1976), Hartog and Howeling (1976, Thoss (1976) and Thoss and Wiik (1978).

(b) Generalisations beyond the productive system. There is no reason why input-output analysis should be confined to the study of inter-industry relationships: it can equally well be applied to a system containing households and government agencies too, as was recognised many years ago in Goodwin (1949). An interesting example of this genralisation, in which the matrix multiplier is partitioned into multiplicative components, is provided in Pyatt, Roe and associates (1977).

These authors are primarily concerned with problems of equitable distribution in the consumption sector rather than with those of efficient allocation in the production sector. Accordingly, they disaggregate the income and outlay accounts of the system and, in particular, those for the household sector, as well as the production accounts. The current accounts for producers and consumers form the endogenous part of the system while the exogenous part consists of the capital accounts and accounts for the rest of the world. In comparison with equation (2) above the multiplier analysis now runs as follows.

The coefficient matrix, A, is now partitioned and can be written

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} + \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$$

$$= B + C \qquad (6)$$

say. Equation (2) becomes

$$\begin{aligned} y &= A\,y + x \\ &= (I - A)^{-1} x \\ &= [I + (I-B)^{-1}C]\,[I - (I-B)^{-1}C(I-B)^{-1}C]^{-1}\,(I-B)^{-1}\,x \\ &= M_3\,M_2\,M_1\,x \\ &= M\,x \qquad (7) \end{aligned}$$

say. In (7), first

$$M_1 = \begin{bmatrix} (I-A_{11})^{-1} & 0 \\ 0 & (I-A_{22})^{-1} \end{bmatrix} \tag{8}$$

and so M_1 includes the multiplier effects which arise from the repercussions of the initial injection within the group of accounts (or subsystem) which it originally entered, and so may be said to measure the intra-group effects. For instance, $(I-A_{11})^{-1}$ is the usual Leontief inverse.

Second,

$$M_2 = \begin{bmatrix} D & O \\ O & E \end{bmatrix} \tag{9}$$

where

$$D = [I - (I - A_{11})^{-1}A_{12}\ (I - A_{22})^{-1}A_{21}]^{-1} \tag{10}$$

$$E = [I - (I - A_{22})^{-1}A_{21}\ (I - A_{11})^{-1}A_{12}]^{-1} \tag{11}$$

and so M_2 includes the multiplier effects which arise from the repercussions of the initial injection when it has moved to the other group and returned to the one which it had originally entered, and so may be said to measure the inter-group effects.

Third,

$$M_3 = \begin{bmatrix} I & (I-A_{11})^{-1}A_{12} \\ (I-A_{22})^{-1}A_{21} & I \end{bmatrix} \tag{12}$$

and so M_3 includes the multiplier effects which arise from the repercussions of the initial injection when it has moved outside its original group without returning to it, and so may be said to measure the extra-group effects.

As can be seen M_1 and M_2 are block-diagonal matrices while M_3 is not.

For many purposes it is convenient to have an additive decomposition of M. This can be done by considering what is added to the initial injection by introducing the different effects in the appropriate order. Thus

$$M = I + (M_1 - I) + (M_2 - I)M_1 + (M_3 - I)M_2M_1 \tag{13}$$

Apart from the original statement and illustration of this generalisation in Pyatt, Roe and associates (1977), complete numerical examples of the various matrices are given in Stone (1978 a and b).

5. ENDOGENOUS INVESTMENT AND DYNAMIC FORMS

Up to this point, investment has been treated as exogenous. Part of it no doubt is; but another part is endogenous, being needed to provide for higher output levels in the future. In the simplest case, this part can be introduced into an input-output model by replacing equation (2) above by

$$y = Ay + K\Delta y + x \tag{14}$$

where K denotes a matrix of capital coefficients and $\Delta \equiv \Lambda - 1$ denotes the first-difference operator. The elements k_{rs} of K measure the direct demand on industry r required to produce the capital goods needed to enable industry s to produce an additional unit of output. This formulation implicitly assumes that all additional equipment, that for simplicity replacement investment can be ignored and that there is a one-year lag in all forms of investment activity.

If, in these circumstances, the elements of x are assumed to grow exponentially in the future at rates given by the elements of a vector r, then it is not difficult to show, as in Stone and Brown (1962), that (14) leads to

$$y = (I-A+K)^{-1} \left\{ \sum_{\theta=0}^{\infty} [K(I-A+K)^{-1}]^{\theta} (I+\hat{r})^{\theta} \right\} x \tag{15}$$

in which, with a suitable change of dating, the multiplier is equivalent to an infinite version of the first row of Leontief's dynamic inverse with fixed values of A and K, as can be seen by comparing it with equation (5) in Leontief (1970a).

In the paper by Stone and Brown just cited, it is shown that (15) can also be written in the form

$$y = (I-A)^{-1} \{ \sum_{\theta=0}^{\infty} [K(I-A)^{-1}]^{\theta} \hat{r}^{\theta} \} x \qquad (16)$$

and that, on either formulation, we could allow for a changing technology if we could project A and K. In CDAE (1962a) the same authors showed how to take account of different production periods for capital goods. In principle it would seem desirable to take account of this complications but in fact I have never encountered a case in which the necessary data were available.

The solutions given by (15) and (16) involve infinite sums though, in practice, it seems likely that only the first few terms in them need be considered: this would certainly be the case if the growth rates of exogenous demands were assumed to be linear rather than exponential. However, it is shown in Mathur (1964) that the problem can be reformulated so that the solution requires summation over exogenous demands rather than over time. In Mukerji (1964) a general solution is given in place of our particular solution. This general solution is expressed in a way that involves K^{-1}. The matrix K is necessarily singular, since not all commodities enter into capital formation, and so it might be thought that there was an insuperable difficulty here; but it is shown in Livesey (1976) that this difficulty can be got round. This is also true of the question of replacement demands, mentioned earlier, which was discussed in Gossling (1974).

Although capital coefficients are widely used to measure the investment requirements of given increases in output, it should not be assumed that they are particularly well suited for this purpose. Just as intermediate demands are unlikely to depend only on output levels, so it is unlikely that investment demands depend only on the change in output levels. Accordingly, we are likely to obtain a better model if we can set up investment demand functions on some acceptable theory of producers' behaviour and then convert the demands for investment goods into commodities and the components of value added by means of a classification converter. This treatment of investment demands is similar to the treatment of private consumers' demands, except of course that the form of the demand functions is different. For some years now we have followed this approach, in preference to using capital coefficients, in the Cambridge Growth Project. A discussion of the problems involved and the form of the functions adopted is given in Pet'

6. CONSTRUCTING AND USING PAMs

Some of my audience may wonder why I think it relevant to discuss population accounting matrices in this paper despite the fact that, as we have seen in section 2 above, they are amenable to analysis by input-output methods. My answers is that many of their uses are relevant to economic planning: they can be applied for instance to the study of industrial and occupational change, to the duration of unemployment, to the operation of non-market services such as health and education and to many other matters which are surely relevant to economic planning. Even the study of internal migration and intrinsic population growth rates in different regions, which sounds highly demographic, may become of great economic importance as soon as economic planning is given a regional dimension. So I am not in favour of cutting off economics from other disciplines, a tendency which in my opinion has already gone too far.

Like SAM, PAM is also a means of presenting data in an orderly way and a basis for model building. The taxonomic issues involved have been discussed in: Stone (1971), largely in connection with educational systems; in UN (1975), in a much more general way leading to a system of social and demographic statistics (SSDS); and in Rees and Wilson (1977) in connection with regional demography.

Let us now look at some of the problems to which PAMs give rise.

(a) *Stocks and flows*. In social demography, information on stocks, that is on the number of human beings in various states at particular dates, is fairly plentiful; but information on flows, that is changes of state over the corresponding interval, is scarce. It is only possible to determine net flows from data on stocks, and to estimate the distribution of these flows use can be made of the additive constraints imposed by PAM and beliefs about which flows are zero or negligible. In order to obtain gross flows assumptions must be made of varying plausibility. For instance, it may be unlikely that a child who starts going to nursery school will be withdrawn before going on to some form of primary education; but it is quite likely that one or both partners to a marriage which is annulled will have married again before the end of the year. However, it may be possible to do something with this kind of information, as is illustrated by the tables in Stone (1971).

In order to make reliable estimates of gross flows it is necessary to collect flow data, and this can most easily be done by means of retrospective questionnaires. For instance Ministries of Education often conduct annual censuses of schools which provide a great deal of information about pupils at the date of the census. It should not be very difficult to collect at the same time information about pupils a year ago, thus providing information on transitions over the yearly interval. An example of this method is given in NCBS (1969). Other methods of collecting flow data are described briefly in UNSO (1975, Ch. IV).

(b) The definition of age. Since age is usually an important criterion of classification in PAMs, it is essential that a uniform definition be adopted. The usual one is year of birth or the equivalent, age at 1 January. This means that all information expressed in terms of the age at which something happens, for instance at which an individual leaves school or enters employment or dies, must be adjusted to the standard definition.

(c) Residence. Another question which has to be settled is whether the matrix should relate to the actual or the normal residents of a given country. For most purposes it is convenient to work with the concept of actual residents. A number of complications arise especially if the calendar year is abandoned as the standard interval in favour of a shorter period. In discussing movements over national or regional boundaries it is useful to distinguish betweeen migrants, that is people who intend to make a permanent change of address, from visitors, for whom the change is intended only to be temporary.

(d) Multiple activities and the use of time. In assigning individuals to states it is frequently the case that the principal state that is relevant can readily be identified. Thus, if we are primarily interested in movements into, within and out of the system of full-time formal education, individuals will be assigned to some branch of this system and their other activities will be ignored. However, much education is undertaken on a part-time basis and training on the job can be regarded as a largely informal kind of technical education. Further, time budgets, as in Szalai and others (1972), show that principal activities, whatever they may be, take up only a limited part of the day. Complications arise if subsidiary activities are to be introduced into the definition of states and, moreover, the number of states rapidly becomes extremely large. Perhaps the main taxonomic question in this field is to introduce the distinctions that are important in some area while keeping the number of states down to a manageable number.

(e) Prices and costs. Equations (4) and (5) are quantity equations, the variables they contain being expressed in numbers of human beings. In economic input-output there is a dual equation which enables us to express product prices in terms of primary input costs per unit of output. An analogous state of affairs holds in the present case as can readily be demonstrated by an example relating to education.

Let m denote a vector whose elements measure the educational costs that must be incurred this year to educate an individual now in a given state of the system. On the assumption that m remains fixed in the future, the total cost to be incurred from now on to educate, or complete the education of, an individual now in a given state is an element of a vector, k say, where

$$\begin{aligned} k &= m + C'm + C'^2m + \dots \\ &= m + C'k \\ &= (I - C')^{-1}\, m \end{aligned} \tag{17}$$

where C' is the transpose of the transition matrix C in (4) and (5) above. If we can project m and C we can allow for changing unit costs and changing transitions and, in any case, there is no difficulty in any case, there is no difficulty in allowing for the discounting of future costs. These developments are set out in UNSO (1975, pp. 45-6).

(f) *Stability, updating and projection*. There is little to add to what was said in section 3 (f) above beyond the fact that changes in coefficients are quite as important in social demography as they are in economics. In many parts of the world there has been a strong tendency in the past generation for an increasing proportion of children to remain at school after the minimum leaving age leading to sigmoid trends in the relevant transition proportions. At the same time changes of regulation imposed by the school system may also lead to minor changes in some of the coefficients and occasionally to major ones, as when the school leaving age is raised.

(g) *Forecasting*. The simple model based on equation (4) of section 2 above implies assumptions about the working of the system. Thus in (4) the numbers in different states in the future will depend on the numbers in different states in the past. If we are talking about an educational system, this implies that the number of places required in the future will be made available, that is the educational authorities are adaptive; and, if we are talking about a business firm, this implies that there are sufficient vacancies to accommodate those who qualify for promotion. On such assumptions, (4) can be used for making projections of n contingent on a knowledge of the future values of the exogenous vector b. Thus if we operate on (4) with Λ we obtain

$$\Lambda^2 n = C^2 n + Cb + \Lambda b \tag{18}$$

and, in general,

$$\Lambda^\tau n = C^\tau n + \sum_{\theta=0}^{\tau-1} C^\theta \Lambda^{\tau-\theta-1} b \tag{19}$$

which shows the population vector in the future year τ as made up of the survivors from the initial stock and the survivors from subsequent entrants. If we wish to allow for changes in the C-matrix, there is no difficulty in principle but a more complicated expression takes the place of (19).

A special case of (19) which is of some interest arises as follows. Suppose that a population is initially in a state of stationary equilibrium, with $b = b_0$ and $n = n_0$, and that, in a future year, b changes to b_1. The (19) takes the form

$$\begin{aligned}\Lambda^{\tau} n &= C^{\tau} n_0 + \sum_{\theta=0}^{\tau-1} C^{\theta} b_1 \\ &= C^{\tau} n_0 + (I - C^{\tau})(I - C)^{-1} b_1 \\ &= C^{\tau} n_0 + (I - C^{\tau})\, n_1 \qquad (20)\end{aligned}$$

The matrix C^{θ} is a null matrix for all values of θ which exceed the human life span and we can see from (20) that a single, sustained step in b will completely work itself out in one human life span, during which time the population vector will be a changing weighted sum of the elements of the initial and final population vectors, n_0 and n_1.

(h) Structural analysis. As in the economic case, PAMs lend themselves to all kinds of structural analysis. To take a simple example, suppose a PAM relates to males classified only by year of birth. Then the transition matrix C contains survival rates in the leading subdiagonal and zeros everywhere else. The matrix multiplier, $(I - C)^{-1}$, has a number of interesting properties. First, if the survival rates relate to exact years of age, the column sums of $(I - C)^{-1}$ are the expectations of life at the exact age shown at the head of the column; and if the survival rates relate to a group of individuals aged λ at a particular date, the column sums will equal the above expectations increased by half an interval, that is half a year. Second, the diagonal elements of the inverse provide estimates of the time on average spent in each of the states by someone who is just entering them. Third, the off-diagonal element in row r and column s of the inverse provides an estimate of the time spent on average in state s multiplied by the probability of reaching it from state r. And, finally, the sum of the elements in column s of the inverse gives the expectation of life of someone entering state s and so from a life table we can find out the average age at which individuals enter that state. For populations in stationary equilibrium these results hold quite generally even if age is not a criterion; but in other cases the elements of the

C-matrix need adjustment. An example of this is provided in Stone (1972a).

The significance of this association with the life table is that it applies however the population is classified. For instance, everyone passes through what I have called the active sequence starting as an infant at home, continuing through a series of educational states, and later through a series of earning states, and ending up, in retirement, at home again. Whatever categories are used, the elements in the columns of the inverse derived from the transition matrix represent expected mean staying times and add up to the average expectation of life on entering the state shown at the head of the column.

The population studied my relate to a country or to a more restricted group, such as individuals covered by a system of medical care, as in Baldwin (1971) and in Wright and Jones (1976), the staff of a business firm, as in Bartholomew (1967) and in Mahoney and Milkovich (1971), or the names on an unemployment register, as in Fowler (1968) and Cripps and Tarling (1974). The price equation can be used to compare the net profit or loss from alernative treatments which are spread out through time, as in Meredith (1973). These and other examples are discussed in Stone (1973b) and UNSO (1975).

7. SOURCE MATERIAL FOR ECONOMETRIC MODELS

There are so many econometric models recently constructed or under construction that it is quite impossible to keep track of them without a good deal of organised research. Fotunately this has been set in train first and foremost by Dr. Goetz Uebe of the Institute of Statistics and Research of the Technical University of Munich. He has earned the thanks of all builders and users of econometric models by compiling a computerised bibliography of macro-econometric models and deserves the co-operation of all model builders in making his bibliography as complete as possible. In addition there are other sources which, though on a smaller scale, are well worth the attention of the searcher.

(a) The Munich bibliography of macro-econometric models. This is a computerised bibliography, at present containing nearly two thousand entries, from which printouts and special surveys can be obtained. For instance a recent printout lists models by country and, within countries, by date, author and a number of characteristics, one of which is the number of input-output branches, if any. If we look down this list we find that the first input-output entry is for Argentina under the name of Adelman. From the bibliography we can trace this to Adelman and Thorbecke eds. (1966).

(b) ECE reports on current research. A decision was taken at the fourth meeting (1966) of the Senior Economic Advisers to ECE Governments to initiate a system of annual reports on the use of mathematical methods in economic analysis. This has now become a regular annual feature circulated as UNECE (1973-). These reports provide information about the progress of planning models but, as their title implies, their scope is much wider than this.

(c) ECE: SEA papers. Much useful material will be found in the volumes of papers presented to meetings of the Senior Economic Advisers to ECE Governments or to conferences sponsored by them. Examples are provided by UN ECE (1967, 1970, 1971, 1973, 1974, 1975).

(d) Proceedings of the International Conferences on Input-Output Techniques. At irregular intervals since 1950, six international conferences on input-output techniques have been held and a seventh is planned for 1979. The proceedings give a good impression of the development of input-output studies and contain much material relating to planning models. They are listed here under the editors: NEI (1953), Barna (1955, 1963), Carter and Brody (1970), Brody and Carter (1972) and Polenske and Skolka (1976).

(e) Input-output bibliographies. A comprehensive bibliography of inter-industry research was published in Riley and Allen (1955). This work was continued for the period 1955-60 by Charlotte E. Taskier and published in UN (1961). Later issues have appeared in Statistical Papers Series M: UNSO (1964, 1967, 1972). From time to time the International Statistical Institute invites a review of input-output studies: the latest two are published in Armstrong and Upton (1969) and Stone (1975).

8. MODELS AND APPLICATIONS

I have done my best in this paper to show how model building is related to the planning process and the ways in which input-

-output techniques can be used in model building. I have set out the main problems to which, the construction and use of input-output tables give rise both in a strictly economic context and in the context of social demography which, it seems to me, enters into many economic planning problems. Finally, I have indicated ways into the voluminous literature on econometric models in general and input-output methods in particular.

If we look through this literature we can find input-output used in econometric model building at every level of sophistication, in a wide range of countries and in a large number of areas of application. It has to be remembered that input-output is usually only a part and sometimes a relatively minor part of a complete model and so while, other things being equal, a model may be expected to perform better if the input-output work is well done, other things may not be equal and a model may perform badly in spite of careful attention to input-output.

Let us now consider various aspects of models designed to be useful in policy formation and planning.

(a) Degree of disaggregation. The fact that I have been talking about problems of disaggregation does not of course imply that all models used for planning purposes are disaggregated; on the contrary most models designed to help in answering short-term policy questions are not. Disaggregation is more usual in medium and long-term models where industrial structure is likely to be important. Once disaggregation is agreed on, a demand tends to develop for a large number of branches mainly to avoid the criticism that it is meaningless to treat such and such an industry as a single industry and to enable industry committees to identify themselves with something in the model. On the whole I think this tendency should be resisted, particularly if it is likely to carry the number of branches above about fifty. The reasons are: first, that if the model is at all sophisticated the amount of data processing will greatly increase; and, second, that as the model gets more detailed simple relationships become less and less appropriate. One way of getting round this difficulty is to break the model up into a linked system of submodels, as in CDAE (1968).

(b) Sophistication in modelling. A modest acquaintance with the literature shows that models can be found at every level of sophistication. For instance, in the 1920s, an input-output table was constructed in the Soviet Union and published in USSRCSB (1926). It was a response to the needs of a planned economy and provided, in money terms and at a fairly low level of disaggregation, the kind of information needed to produce by traditional bureaucratic planning methods balances of supplies and demand, as described in Montias (1959). But at the time, despite Walras (1874) and Dmitriev (1904), there were no input-output equations: these came later in Leontief (1936, 1941). At the other end of the spectrum, we find in a recent report of the Japanese Committee for Econometric Model Analysis, publisched in JEPA (1977), the first attempt by dhe Japanese Government to apply the turnpike model to actual long-term planning.

(c) Statics and dynamics. Until recently most disaggregated models of national economies have been static, being concerned with a target year and not with the path by which that year is approached. This has been the case with the model of the Cambridge Growth Project which was begun in 1960 and has passed through a succession of stages of increasing elaboration until about three years ago. A short account of its history is given in Stone (1977) and a detailed account of its final form is set out in Barker ed. (1976).

This model has now been adapted to form a multisectoral dynamic model, MDM for short, so that given a set of initial values in a base year and time series of future values of the exogenous variables, estimates of the future values of all endogenous variables can be made year by year. At present the base year is 1972 and the model is solved through 1985. It is a large model and when I last looked into the matter, about a year ago, it contained 2551 endogenous variables and 4721 predetermined variables. The endogenous variables are matched by an equal number of equations, of which 603 are stochastic and the remaining 1948 are identities, input--output relationships and the like. Of the predetermined variables, 3876 are lagged values and 845 are exogenous, made up of 794 policy variables and 51 others.

(d) Endogenous objectives and controls. A number of disaggregated models, though usually not the very large ones are optimising models and seek to maximise or minimise an objective function, usually by means of linear or non-linear programming: thus policy criteria are built into them. We have not gone quite as far as this in the Cambridge Growth Model but, instead, have adopted the targets and instruments approach described in Tinbergen (1952, 1956). So far this method has been applied to the static version of the model so that up to ten targets can be set for the planning year and a solution worked out in which they are all hit. This is ensured by the calculation of appropriate values of the instruments.

There are a number of advantages in building a control system into a model.

First, if run without constraint, a model may indicate in the target year an undesirable state of affairs which, if it were believed in, would provoke action to falsify it. In these circumstances it seems desirable that the model builder should at least be able to indicate changes in instruments which would ameliorate the position in the target year and even be able to calculate the magnitude of the changes needed to render the position 'acceptable'.

Second, this extension of the model makes it possible to explore the probable consequences of proposed changes in the instruments. Thus, in making the kind of calculation I have described, it is possible to work out a large number of trade-offs, such as the trade-off between the balance of trade and the exchange rate for constant values of the standard rate of tax. Such trade-off curves give a good deal of insight into the relative merits of different instruments for different purposes. However, it must be borne in mind that with the kind of model I am describing in this section we are always talking about normal years and steady states. This means that it is implicitly assumed that the changes in the instruments have been introduced well in advance of the target year and that we agree to ignore their possibly unfavourable effects in the first years of their introduction.

Finally, the methods I have described make it possible to experiment with optimal instrument packages. This is desirable because changes in instruments have bad as well as good effects and so are best used in combination. Further, as circumstances change, we do not want to have to change the instruments more than necessary. The essence of a good control system is that, having got the economy on to a steady path, it changes as little as possible the conditions under which economic decisions are taken.

We are now trying to learn how to control the dynamic model. There seems to be no doubt that this is feasible but it is a large task because the number of target and instrument settings are greatly increased, and moreover, the settings are connected over time.

(e) The subject-mix of models. In building economic models we usually begin with the kind of variables and relationships that economists are accustomed to handling: production, consumption, accumulation, foreign trade, input-output relation-ships, production functions, consumption functions and so on. I have argued above that in all kinds of ways we may find it desirable to pay more attention to demographic and social variables and I have sketched out some methods, which have a distinctively input-output flavour. The environment is another area that should not be neglected. Many writers would like to introduce political considerations into their models. A conference, organised by a group of political scientists, to consider this problem in the context of world modelling is reported in Deutsch and other eds. (1977).

(f) Geographical and institutional coverage. We can find models whose area coverage ranges from the world to a single city or, within one of these areas, to a single institution such as an educational system or a large corporation.

(i) World models. There are already a number of world models and more are under construction. Among the more strictly economic models some, like MEGISTOS a world income and trade model described in Duprez and Kirschen eds. (1970), are single models while others, like Project LINK described in Ball ed. (1973) and Waelbroeck ed. (1976), are formed by connecting a number of country models. Another recent example, the UN study described in Leontief and others (1977), is based on a multiregional world input-output system and quantifies, among other things, some of the problems of pollutants and their abatement. More speculative and controversial models seeking to link living standards, natural resources, pollution, population and production have been developed in Forrester (1971), Meadows and others (1972) and Mesarovich and Pestel (1974).

(ii) National models. These are so numerous that I shall not attempt to list even the more important ones. They vary in all possible characteristics. An important feature, which I think is coming to be recognised, is that one cannot think of a large econometric model as being completed in a fixed span of years. It may be useable after a comparatively short time but it is always capable of improvement and elaboration. In our present state of knowledge the end of the road is definitely not in sight: indeed I should say that we can only see a limited way along it and have no idea how long it really is. We can only improve our chances of learning from experience if we accept this fact and keep good models in being as long as they continue to develop.

(iii) Regional models. These are sometimes an aspect of, or adjunct to, a national model and sometimes confined to one or more regions of a country. An example of the first kind is the French regional-national model REGINA which began in 1972 and became operational in 1975. It has been developed at the University of Paris X-Nanterre by Raymond Courbis and his associates and among the large number of publications in which it has been described I will mention Courbis and others (1973), Courbis and Valled (1976) and Courbis (1978). An example of the second kind is an input-output study of Brabant in relation to the other regions of Belgium (Flanders and Wallonia) and to the rest of the world. This work is described by Vanwynsberghe (1974, 1976). A project on a somewhat different scale is the immense multi-regional input-output model for the United States, in which seventy-eight industries and forty-four regions (states or groups of states) are distinguished. It was carried out at the Harvard Economic Research Project and is described in Polenske (1970 a and b, 1972) and in Polenske, Anderson and Shirley (1972).

(iv) City models. It is difficult to construct input-output models for small, open areas largely because of the problems of tracing the provenance of inputs and the destinations of outputs. However, such models would be useful in connection with development planning and probably a number have been constructed. For instance, a study of the structure of the economy of Stockholm in 1950 is given in Artle (1959). An input-output table for Antwerp in 1958 was published in BSEA (1964). The Dutch regional accounting study for 1960, published in NCBS (1968) contains not only provincial input-output tables but also tables for the communities of Amsterdam and The Hague.

(v) Models for non-market services. The planning of these services is an obvious field for the application of programming methods and for the combination of economic and socio-demographic input-output techniques. Econometric models have, in particular, been applied to education such as those formulated in Tinbergen and Bos (1965) as part of the OECD educational planning programme. An annotated bibliography of analytical techniques was published in OECD (1969). I have already refered to my contribution to this work in Stone (1971). This was followed by a survey of mathematical models for the educational sector published in OECD (1973, 1974). Interesting studies using input-output and programming methods are to be found in OECD (1967), Armitage, Smith and Alper (1969), Bowles(1969), Thonstad (1969), Bermant and others (1972) and Schiefelbein and Davis (1974).

(vi) Corporate planning models. Companies not only use input--output and programming models but also build such models of their own operations. The results obtained are not usually published but their existence is evident from the contributions of business economists to the proceedings of conferences such as the one reported in Gossling ed. (1970, sessions I and VII). Discussions on industrial applications have been organised at recent meetings of the International Conference on Input-Output Techniques, and Polenske and Skolka (1976) contains papers on this subject. In Stone (1973a) I proposed an input-output framework in which commodities, processes and ownership or control groups were distinguished. This made possible a discussion at the theoretical level of such matters as indirect profitability and some of the circumstances conducive to takeover bids.

9. CONCLUSION

At this point I shall stop. There is much more that could be said but I think I have succeeded in bringing out the role of accouting matrices in general and input-output tables in particular in organising and synthesising statistical data for the purpose of model building. The framework proposed, though capable of endless elaboration, is simple enough to enable a start to be made even with limited information. Data for a complete as opposed to a partial system provide great insight even if they are not very accurate and detailed, and they have perhaps done more than anything else to change attitudes to the problems and possibilities of economic policy.

The taxonomy of accounting matrices will certainly improve but I do not expect overwhelming changes in this direction. In the matter of modelling I think we still have a long way to go and that in the not very distant future the input-output model, as opposed to its organised data base, will be a barely discernible component in econometric models. But we should not forget that it is this simple model that has enabled us to take off. Without it, we should not even have reached the position we are in today.

A LIST OF WORKS CITED

ADELMAN, Irma, and Erik THORBECKE (1966). *The Theory and Design of Economic Development.* Johns Hopkins Press, Baltimore, 1966.

ALMON, Clopper, Jr., and others (1974). *1985: Interindustry Forecasts of the American Economy.* D.C. Heath & Co., Lexington, Mass., 1974.

ARMITAGE, Peter, Cyril SMITH and Paul ALPER (1969). *Decision Models for Educational Planning.* The Penguin Press, London, 1969.

ARMSTRONG, A.G. (1975). Technology assumptions in the construction of U.K. input-output tables. In *Estimating and Projecting Input-Output Tables* (eds. R.I.G. Allen and W.F. Gossling). Input-Output Publishing Co., London, 1975.

ARMSTRONG, A.G. and D.C. UPTON (1969). A review of input-output applications. *Bulletin of the International Statistical Institute*, vol. XLIII, bk. 1, 1969, pp. 113-30.

ARROW, Kenneth J., Marvin HOFFENBERG and others (1959). *A Time Series Analysis of Interindustry Demands.* North-Holland, Amsterdam, 1959.

ARTLE, Roland (1959). *Studies in the Structure of the Stockholm Economy.* Business Research Institute, Stockholm School of Economics, Stockholm, 1959.

AYRES, R.U. (1974). *Application of Input-Ouput Models for Environmental Analysis.* International Research and Technology Corporation, Arlington, Virginia, 1974.

BACHARACH, Michael (1970). *Biproportional Matrices and Input-Output Change.* Cambridge University Press, 1970.

BALDWIN, J.A. (1971). *The Mental Hospital in the Psychiatric Service: A Case-Register Study.* Oxford University Press, for the Nuffield Provincial Hospitals Trust, 1971.

BALL. R.J., ed. (1973). *The International Linkage of National Economic Models.* North-Holland, Amsterdam, 1973.

BARKER, T.S. (1975a). Some experiments in projecting intermediate demand. In *Estimating and Projecting Input-Output Coefficients* (eds. R.I.G. Allen and W.F. Gossling). Input-Output Publishing Co., London, 1975.

BARKER, T.S. (1975b). An analysis of the updated 1963 input-output transactions table. In *Estimating and Projecting Input-Output Coefficients* (eds. R.I.G. Allen and W.F. Gossling). Input-Output Publishing Co., London, 1975.

BARKER, T.S., ed. (1976). *Economic Structure and Policy*. No. 2 in *Cambridge Studies in Applied Econometrics*. Chapman and Hall, London, 1976.

BARNA, Tibor, ed. (1955). *The Structural Interdependence of the Economy*. Wiley, New York; Giuffre, Milano; 1955.

BARNA, Tibor, ed. (1963). *Structural Interdependence and Economic Development*. Macmillan, London, 1963.

BARTHOLOMEW, D.J. (1967). *Stochastic Models for Social Processes*. Wiley, New York 1967; 2nd edn., Wiley, New York, 1973.

BELGIUM, STUDIECENTRUM VOOR DE EXPANSIE VAN ANTWERPEN (1964). *Input--Output Tabellen voor het Arrondissement Antwerpen*. Studiecentrum voor de Expansie van Antwerpen, Mededelingen, no. 2, 1964.

BERMANT, M.A., and others (1972). *Mathematical Models and Educational Planning*. Izdatelstvo Nauka, Moscow, 1972.

BOWLES, Samuel (1969). *Planning Educational Systems for Economic Growth*. Harvard University Press, 1969.

BRODY, A., and A.P. CARTER, eds. (1972). *Input-Output Techniques*. North-Holland, Amsterdam, 1972.

BYRON, Ray P. (1978). The estimation of large social accounting matrices. *Journal of the Royal Statistical Society, series A*, 1978 (forthcoming).

CAMBRIDGE, DEPARTMENT OF APPLIED ECONOMICS (1962a). *A Computable Model of Economic Growth*. No 1 in *A Programme for Growth*. Chapman and Hall, London, 1962.

CAMBRIDGE, DEPARTMENT OF APPLIED ECONOMICS (1962b). *A Social Accounting Matrix for 1960*. No. 2 in *A Programme for Growth*. Chapman and Hall, London, 1962.

CAMBRIDGE, DEPARTMENT OF APPLIED ECONOMICS (1963). *Input-Output Relationships, 1954-1966*. No. 3 in *A Programme for Growth*. Chapman and Hall, London, 1963.

CAMBRIDGE, DEPARTMENT OF APPLIED ECONOMICS (1968). *The Demand for Fuel, 1948-1975*. No. 8 in *A Programme for Growth*. Chapman and Hall, London, 1968.

CARTER, Anne P. (1970). *Structural Change in the American Economy*. Harvard University Press, 1970.

CARTER, A.P. and A. BRODY, eds. (1970). *Input-Output Analysis*: vol. 1, *Contributions*: vol. 2, *Applications*. North-Holland, Amsterdam, 1970.

COURBIS, Raymond (1978). The REGINA model: presentation and first contribution to economic policy. In *Econometric Contributions to Public Policy* (eds. R. Stone and A.W.A. Peterson). Macmillan, London, 1978.

COURBIS, Raymond, and Dominique VALLET (1976). An interindustry interregional table of the French economy. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambrigde, Mass., 1976.

COURBIS, Raymond, and others (1973). Le modèlè RÉGINA: analyse économique du modèle. Report no. 43, G.A.M.A., University of Paris-X, Nanterre, 1973.

CRIPPS, T.F., and K.J. TARLING (1974). An analysis of the duration of male unemployment in Great Britain, 1932-72. *The Economic Journal*, vol. 84, no. 333, 1974, pp. 289-316.

CUMBERLAND, John H., and Bruce N. STRAM (1976). Empirical application of input-output models to environmental problems. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

DEATON, Angus (1975). *Models and Projections of Demand in Post-War Britain*. No 1 in *Cambridge Studies in Applied Econometrics*. Chapman and Hall, London, 1975.

DEUTSCH, Karl W., and others, eds. (1977). *Problems of World Modeling*. Ballinger, Cambridge, Mass., 1977.

DMITRIEF, V.K. (1904). *Economic Essays on Value, Competition and Utility*. Richter, Moscow, 1904. English transl. by D. Fry, Cambridge University Press, 1974.

DUPREZ, C., and E.S. KIRSHEN, eds. (1970). *MEGISTOS*. North-Holland, Amsterdam, 1970.

FISHER, W. Halder, and Cecil H. CHILTON (1972). Developing ex ante input-output flow and capital coefficients. In *Input-Output Techniques* (eds. A. Bródy and A.P. Carter). North-Holland, Amsterdam, 1972.

FONTELA, E., and others (1970). Forecasting technical coefficients and changes in relative prices. In *Applications of Input-Output Analysis* (eds. A.P. Carter and A. Bródy). North-Holland, Amsterdam, 1970.

FORRESTER, Jay W. (1971). *World Dynamics*. Wright-Allen, Cambridge, Masss., 1971.

FOWLER, R.F. (1968). *Duration of Unemployment on the Register of Wholly Unemployed*. Studies in Official Statistics, Research Series no. 1. H.M.S.O., London, 1968.

GIGANTES, T. (1970). The representation of technology in Input-Output systems. In *Contributions to Input-Output Analysis* (eds. A.P. Carter and A. Bródy). North-Holland, Amsterdam, 1970.

GOODWIN, R.M. (1949). The multiplier as matrix. *The Economic Journal*, vol. LIX, no. 236, 1949, pp. 537-55.

GOSSLING, W.F. (1974). Correct fixed-capital replacement in input-output growth models. *The Review of Economic Studies*, vol. XLI (4), no. 128, 1974, pp. 525-31.

GOSSLING, W.F., ed. (1970). *Input-Output in the United Kingdom*.Frank Cass, London, 1970.

HARTOG, H. den, and A. HOWELING (1976). Pollution, pollution abatement, and the economic structure of the Netherlands. In *Advances in Input-Output Analysis* (es. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

JAPAN, ECONOMIC PLANNING AGENCY (1977). *Econometric Models for the National Economic Plan for the Second Half of the 1970's*. Economic Planning Agency, Government of Japan, August 1977.

KAHN, R.F. (1931). The relation of home investment to unemployment. *The Economic Journal*, vol. XLI, no. 162, 1931, pp. 173-98.

KEYNES, John Maynard (1936). *The General Theory of Employment, Interest and Money*. Macmillan, London, 1936. Reissued in *The Collected Writings of John Maynard Keynes*, vol. VII; Macmillan, London, 1973.

LEONTIEF, Wassily W. (1936). Quantitative input and output relations in the economic system of the United States. *The Review of Economic Statistics*, vol. XVIII, no. 3, 1936, pp. 105-25.

LEONTIEF, Wassily W. (1941). *The Structure of American Economy*. 1st edn. [*1919-1929*], Harvard University Press, 1941; 2nd edn. [*1919-1939*], Oxford University Press, New York, 1951.

LEONTIEF, Wassily W. (1970a). The dynamic inverse. In *Contributions to Input-Output Analysis* (eds. A.P. Carter and A. Bródy). North-Holland, Amsterdam, 1970.

LEONTIEF, Wassily W. (1970b). Environmental repercussions and the economic structure: an input-output approach. In *A Challenge to Social Scientists* (ed. Shigeto Tsuru) Asahi, Tokyo, 1970. Reprinted in *The Review of Economics and Statistics*, vol. LII, no. 3, 1970, pp. 262-71.

LEONTIEF, Wassily W., and Daniel FORD (1972). Air pollutionand the economic structure: emprical results of input-output computations. In *Input-Output Techniques* (eds. A. Bródy and A.P. Carter).North-Holland, Amsterdam, 1972.

LEONTIEF, Wassily W., and others (1953). *Studies in the Structure of the American Economy*. Oxford University Press, New York, 1953.

LEONTIEF, Wassily W., and others (1977). *The Future of the World Economy*. Oxford University Press, New York, 1977.

LIVESEY, D.A. (1976). A minimal realization of the Leontief dynamic input-output model. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

MAHONEY, Thomas A. and George T. MILKOVICH (1971). The internal labor market as a stochastic process. In *Manpower and Management Science*, English Universities Press, 1971.

MATHUR, P.N. (1964). Output and investment for exponential growth in consumption - an alternative formulation: and derivation of their technological upper limits. *The Review of Economic Studies*, vol. XXXI, no. 85, 1964, pp. 73-6.

MATUSZEWSKI, T., R.R. PITTS and J.R. SAWYER (1964). Linear programming estimates of changes in input coefficients. *The Canadian Journal of Economics and Political Science*, vol. 30, no. 2, 1964, pp. 203-10.

MEADE, J.E. (1972). Citizens' demands for a clean environment. *L'industria*, no 3/4, 1972, pp. 145-52.

MEADOWS, Donella H. and Dennis L., and others (1972). *The Limits to Growth*. Universe Books, New York, 1972.

MEREDITH, Jack (1973). A Markovian analysis of a geriatric ward. *Management Science*, vol. 19, no. 6, 1973, pp. 604-12.

MESAROVIC, Michajlo, and Eduard PESTEL (1974). *Mankind at the Turning Point*. Dutton, New York, 1974.

MONTIAS, J.M. (1959). Planning with material balances in Soviet-type economies. *American Economic Review*, vol. XLIX, no. 5, 1959, pp. 963-85.

MUKERJI, V. (1964). Output and investment for exponential growth in consumption - the general solution and some comments. *The Review of Economic Studies*, vol. XXXI, no. 85, 1964, pp. 77-82.

NETHERLANDS, CENTRAL BUREAU OF STATISTICS (1968). *Regionale Rekeningen 1960*. 2 vols. N.C.B.S., The Hague, 1968.

NETHERLANDS, CENTRAL BUREAU OF STATISTICS (1969). *An Educational Matrix of the Netherlands for 1967*. N.C.B.S., The Hague, 1969.

NETHERLANDS ECONOMIC INSTITUTE, eds. (1953). *Input-Output Relations*. Stenfert Kroese, Leiden, 1953.

NIJKAMP, P., and J.H.P. PAELINCK (1974). Some methods for updating input-output tables. *Foundations of Empirical Economic Research*, no 1974/2, Netherlands Economic Institute, Rotterdam, 1974.

O.E.C.D. (1967). *Mathematical Models in Educational Planning*. O.E.C.D., Paris, 1967.

O.E.C.D. (1969). *Systems Analysis for Educational Planning*. O.E.C.D., Paris, 1969.

O.E.C.D. (1973). *Mathematical Models for the Education Sector: a Survey*. O.E.C.D., Paris, 1973.

O.E.C.D. (1974). *Mathematical Models for the Education Sector: Supporting Material to the Survey*. O.E.C.D., Paris, 1974.

PETERSON, A.W.A. (1976). Investment. In *Economic Structure and Policy*, no. 2 in *Cambridge Studies in Applied Econometrics*. Chapman and Hall, London, 1976.

PETERSON, A.W.A. (1978a). Factor demand functions. In *The Measurement of Capital: Theory and Practice* (eds. K.E. Schott and K.D. Patterson). Macmillan, London, 1978.

PETERSON, A.W.A. (1978b). Fuel use in the U.K.: a study of substitution responses. To be published.

POLENSKE, Karen R. (1970a). Empirical implementation of a multiregional input-output gravity trade model. In *Contributions to Input-Output Analysis* (eds. A.P. Carter and A. Bródy). North-Holland, Amsterdam, 1970.

POLENSKE, Karen R. (1970b). *A Multi-Regional Input-Output Model for the United States*. EDA Report no. 21, Harvard Economic Research Project, 1970.

POLENSKE, Karen R. (1972). The implementation of a multiregional input-output model for the United States. In *Input-Output Techniques* (eds. A. Bródy and A.P. Carter). North-Holland, Amsterdam, 1972.

POLENSKE, Karen R., Carolyn W. ANDERSON and Mary M. SHIRLEY (1972). *A Guide for Users of the U.S. Multiregional Input-Output Model*. U.S. Department of Transportation, Washington, 1972.

POLENSKE, Karen R., and Jiri V. SKOLKA, eds. (1976). *Advances in Input-Output Analysis*. Ballinger, Cambridge, Mass., 1976.

PYATT, Graham, Alan R. ROE and associates (1977). *Social Accounting for Development Planning with special reference to Sri Lanka*. Cambridge University Press, 1977.

REES, P.H., and A.G. WILSON (1977). *Spatial Population Analysis*. Arnold, London, 1977.

RILEY, Vera, and Robert Loring ALLEN (1955). *Interindustry Economic Studies*. The Johns Hopkins Press, Baltimore, 1955.

SCHIEFELBEIN, Ernesto, and Russell G. DAVIS (1974). *Development of Educational Planning Models and Application in the Chilean School Reform*. Heath, Lexington, Mass., 1974.

SEVALDSON, Per (1963). Changes in input-output coefficients, In *Structural Interdependence and Economic Development* (ed. Tibor Barna). Macmillan, London, 1963.

SEVALDSON, Per (1970). The stability of input-output coefficients. In *Applications of Input-Output Analysis* (eds. A.P. Carter and A. Bródy). North-Holland, Amsterdam, 1970.

SEVALDSON, Per (1976). Price changes as causes of variations in input-output coefficients. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

STONE, Richard (1954). Linear expenditure systems and demand analysis: an application to the pattern of British demand. *The Economic Journal*, vol. LXIV, no. 255, 1954, pp. 511-27.

STONE, Richard (1962). Multiple classifications in social accounting. *Bulletin of the International Statistical Institute*, vol. XXXIX, no. 3, 1962, pp. 215-33. Reprinted in *Mathematics in the Social Sciences and Other Essays*, Chapman and Hall, London, 1966.

STONE, Richard (1968). Input-output projections: consistent prices and quantity structures. *L'industria*, no. 2, 1968, pp. 212-24. Reprinted in *Mathematical Models of the Economy and Other Essays*, Chapman and Hall, London, 1970

STONE, Richard (1971). *Demographic Accounting and Model Building*. O.E.C.D., Paris, 1971.

STONE, Richard (1972a). The fundamental matrix of the active sequence. In *Input-Output Techniques* (eds. A. Brōdy and A.P. Carter). North-Holland, Amsterdam, 1972.

STONE, Richard (1972b). The evaluation of pollution: balancing gains and losses. *Minerva*, vol. X, no. 3, 1972, pp. 412-25.

STONE, Richard (1973a). Process, capacity and control in an input-output system. *L'industria*, no. 1/2, 1973, pp. 3-17.

STONE, Richard (1973b). Transition and admission models in social demography. *Social Science Research*, vol. 2, no. 2, 1973, pp. 185-230; also in *Social Indicator Models* (eds. K.C. Land and S. Spilerman), Russell Sage Foundation, New York, 1975.

STONE, Richard (1975). The expanding frontiers of input-output analysis. *Bulletin of the International Statistical Institute*, vol. XLVI, bk. 1, 1975, pp. 306-21.

STONE, Richard (1977). The evolution of the Cambridge Growth Project. In *Beiträge zur Strukturpolitik*, Materialien zum Siedlungs - und Wohnungswesen und zur Raumplanung, vol. 18 (eds. W. Ernst and R. Thoss), University of Münster, 1977.

STONE, Richard (1978a). The disaggregation of the household sector in the national accounts. Paper presented at the World Bank SAM Conference, Cambridge, England, 1978.

STONE, Richard (1978b). Multipliers for Quesnay's *Tableau*. Paper presented at the World Bank SAM Conference, Cambridge, England, 1978.

STONE, Richard, and J.A.C. BROWN (1962). Output and investment for exponential growth in consumption. *The Review of Economic Studies*, vol. XXIX, no 80, 1962, pp. 241-5.

SZALAI, Alexander, and others (1972), *The Use of Time*. Mouton, The Hague/Paris, 1972.

TASKIER, Charlotte E. (1961). *Input-Output Bibliography, 1955-1960*. U.N., New York, 1961.

THONSTAD, Tore (1969). *Education and Manpower*. Oliver and Boyd, Edinburgh and London, 1969.

THOSS, Rainer (1976). A generalized input-output model for residuals management. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

THOSS, Rainer, and Kjell WIIK (1978). Optimal allocation of economic activities under environmental constraints in the Frankfurt metropolitan area. In *Econometric Contributions to Public Policy* (eds. R. Stone and A.W.A. Peterson). Macmillan, London, 1978.

TILANUS, C.B. (1966). *Input-Output Experiments: The Netherlands 1948-1961*. Rotterdam University Press, 1966.

TINBERGEN, J. (1952). *On the Theory of Economic Policy*. North-Holland. Amsterdam, 1952.

TINBERGEN, J. (1956). *Economic Policy: Principles and Design*. North-Holland, Amsterdam, 1956.

TINBERGEN, Jan, and H.C. BOS (1965). *Econometric Models of Education*. O.E.C.D., Paris, 1965.

U.K., CENTRAL STATISTICAL OFFICE (1970). *Input-Output Tables for the United Kingdom, 1963*. Studies in Official Statistics, no, 16. H.M.S.O., London, 1970.

U.K., CENTRAL STATISTICAL OFFICE (1973). *Input-Output Tables for the United Kingdom, 1968*. Studies in Official Statistics, no. 22. H.M.S.O., London, 1973.

U.N., ECONOMIC COMMISSION FOR EUROPE (1967). *Macro-Economic Models for Planning and Policy-Making*. U.N., Geneva, 1967.

U.N., ECONOMIC COMMISSION FOR EUROPE (1970). *Multi-Level Planning and Decision-Making*. U.N., New York, 1970.

U.N., ECONOMIC COMMISSION FOR EUROPE (1971). *Long-Term Planning*. U.N., New York, 1971.

U.N., ECONOMIC COMMISSION FOR EUROPE (1973). *Long-Term Aspects of Plans and Programmes*. U.N., New York, 1973.

U.N., ECONOMIC COMMISSION FOR EUROPE (1974). *Factors and Conditions of Long-Term Growth*. U.N., New York, 1974.

U.N. ECONOMIC COMMISSION FOR EUROPE (1975). *Use of Systems of Models in Planning*. U.N., New York, 1975.

U.N., ECONOMIC COMMISSION FOR EUROPE (1973-). *Reports on Current Research During the Year ... on the Use of Mathematical Methods in Economic Analysis*. U.N., E.C.E., Geneva, annually since 1973.

U.N., STATISTICAL OFFICE (1964, 1967, 1972). *Input-Output Bibliography, 1960-1963, 1963-1966, 1966-1970*. Statistical Papers, Series M, nos. 39, 46 and 55 (3 vols.). U.N., New York, 1964, 1967 and 1972.

U.N., STATISTICAL OFFICE (1968). *A System of National Accounts*. Studies in Methods, series F, no. 2, rev. 3. U.N., New York, 1968.

U.N., STATISTICAL OFFICE (1973). *Input-Output Tables and Analysis*. Studies in Methods, series F, no. 14, rev. 1. U.N., New York, 1973.

U.N., STATISTICAL OFFICE (1975). *Towards a System of Social and Demographic Statistics*. Studies in Methods, series F, no. 18. U.N., New York, 1975.

U.S.S.R., CENTRAL STATISTICS BOARD (1926). Balance sheet of the national economy of the U.S.S.R. for 1923-24. *Transactions of the U.S.S.R. Central Statistics Board*, vol. 29, Moscow, 1926.

VANWYNSBERGHE, Dirk (1974). *De Brabantse Economie in een Interregionaal en Intersectorieel Perspectief. Eeen Input-Output Benadering*. Regional Economic Council for Brabant, Brussels, 1974.

VANWYNSBERGHE, Dirk (1976). An operational nonsurvey technique for estimating a coherent set of interregional input-output tables. In *Advances in Input-Output Analysis* (eds. K.R. Polenske and J.V. Skolka). Ballinger, Cambridge, Mass., 1976.

WAELBROECK, Jean L., ed. (1976). *The Models of Project LINK*. North-Holland, Amsterdam, 1976.

WALRAS, Léon (1874). *Éléments d'Économie Politique Pure*. 1st. edn., Corbaz, Lausanne, 1874; definitive edn., Pichon et Durand-Auzias, Paris, 1926. English transl. by William Jaffé, Allen and Unwin, London, 1954.

WRIGHT; Nic, and David JONES (1976). Patient referrals in the NHS. *Health and Social Service Journal*, 6 March 1976.