A NEOCLASSICAL FISCAL FRAMEWORK FOR A GROWING ECONOMY

Paulo Guedes

ABSTRACT

Neokeynesian models emphasize government policy to smooth out the short run business cycles; NEOCLASSICAL models point to the neutrality of anticipated counter cyclical policy, discarding feed-back rules from the short run state of the economy to the settings of monetary and fiscal policies.

My purpose in these notes is to formulate an expanded version (with public investment and public debt) of Friedman's (1957) NEOCLASSICAL FISCAL FRAMERWORK, whose policy proposals, as pointed out by Lucas (1981), may have increasing acceptance and influence at the light of new theoretical developments.

The neoclassical fiscal rules are derived from a dynamic optimization program concerned with "long run efficiency or prospects for growth of the economic system", as suggested by Friedman. Government expenditures, public capital formation, taxation and public debt emission are determined irrespectively of short run cyclical fluctuations.

A neoclassical fiscal framework whereby the government levies taxes to finance consumption, issues debt for financing public capital formation, charges a rental price for public capital services according to its marginal productivity and uses the proceeds to pay interest on public debt along the optimal growth path is shown to be an optimal public policy.

RESUMO

Modelos keynesianos atribuem ao governo o papel de suavizar as flutuações cíclicas de curto prazo. Modelos neoclássicos sustentam a neutralidade das políticas anti-cíclicas em razão das expectativas racionais dos agentes econômicos, descartando regras com feedback das variáveis macroeconômicas que descrevem a posição da economia a curto prazo para os instrumentos de política monetária e fiscal.

Meu propósito é formular uma versão ampliada (introduzindo investimentos públicos e emissão de dívida pública) do modelo fiscal neoclássico de Friedman (1957), cujas propostas de política econômica, como sugerido por Lucas (1981), podem ter aceitação e influência crescentes à luz da recente evolução da teoria macroeconômica.

As regras fiscais neoclássicas são deduzidas de um programa de otimização dinâmica objetivando "eficiência na alocação de recursos a partir das perspectivas de crescimento da economia", como indicado por Friedman. Os gastos correntes do governo, o investimento público, os impostos e a emissão de dívida são determinados independentemente das flutuações cíclicas de curto prazo.

O modelo fiscal neoclássico consiste em cobrar impostos para financiar o consumo corrente do setor público, financiar os investimentos através de emissões de dívida pública, cobrar pelo fornecimento dos serviços públicos para pagar os juros sobre a dívida pública ao longo da trajetória ótima de crescimento da economia.

A NEOCLASSICAL FISCAL FRAMEWORK FOR A GROWING ECONOMY*

Paulo Guedes**

"During the late nineteenth and early twentieth centuries the problem of the day were of a kind that led economists to concentrate on the allocation of resources and ecomomic growth, paying little attention to short-run fluctuations of a cyclical character. Since the great Depression of the 30's this emphasis has been reversed. Economists now tend to concentrate on cyclical movements as if any improvement, however slight, in control of the cycle justified any sacrifice, however large, in the long run efficiency or prospects for growth of the economic syster".

Milton Friedman, "A Monetary and Fiscal Framework for Economic Stability".

"... on the policy proposals advanced by Milton Friedman in "A Monetary and Fiscal Framework for Economic Stability (1957) ... some recent developments suggest that its acceptance and influence may be greater in the near future".

Robert Lucas, "Rules, Discretion, and the Role of the Economic Advisor".

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1. STABILIZATION ROLE VS ALLOCATIVE ROLE

Neokeynesian public policy models emphasize the role of government activity to neutralize short run business fluctuations through feedback rules (closed loop controls); neoclassical models point to the neutrality of anticipated counter cyclical policy, emphasing long run efficiency in the allocation of resources as the objective worth pursuing.

As posed by Sargent (1976). "The Central practical issue separating keynesian from non-keynesian economists is the nature of the optimal feedback rules for setting monetary and fiscal policy instruments. Keynesian economists have advocated ACTIVIST policies, which incorporate feedback from current and past observations on the state ot the economy to future settings of fiscal and monetary instruments (e.g., the deficit and the money supply). Usually, these feeback rules are though to imply that policy ought to lean against the wind, calling for increases in taxes in the boom, and lower taxes and higher growth in money when a recession is in the offing. On the other hand, non-keynesian economists such as Henry Simons and Milton Friedman have advocated that the government follow rules without feedback in setting fiscal and monetary policy. In essence, Simons and Friedman's advice to the government is three--fold: first, set government expenditures on the basis of cost-benefit analysis and don't manipulate government expenditures to try to combat the business cycle. Second, keep tax rates fixed at levels that, given the rate of government expanditures, make the growth rate of government debt average out over the business cycle to some desired level. Third, make the money supply grow at a constant percentage rate per year, regardless of the state of business conditions. The percentage rate should be set with a view to the average rate of desired inflation".

Chile has already decided upon a money supply rule: it is endogenously determined by the growth rate of the demand for money as an implication of the fixed exchante rate system. The average rate of desired inflation is then the world inflation rate,[1] irrespectively of the domestic credit policy which ultimately will only determine changes in international reserves. A wise non discretionary credit policy implies a favorable balance of payments, providing for a stable growth of reserves.

The purpose of these notes is to formulate a neoclassical fiscal framework, in Friedman's sense, i.e., derived from a dynamic optimization program concerned with "long run efficiency or prospects for growth of the economic system", where expenditures, taxation and public debt emission are determined irrespectively of cyclical fluctuations in the short run.

A neoclassical fiscal framework where the government levies taxes to finance consumption, issues debt for financing public capital formation, changes a rental price for public capital services according to its marginal productivity and uses the proceeds to pay interest on public debt is shown to be an optimal public policy.

2. THE POLICY FRAMEWORK

From a keynesian viewpoint the government is not concerned with long run optimality;[2] its current fiscal program is the result of the accumulation of past and present stabilization policies.

The core of the fiscal framework fromulated in neokeynasian models is the expenditures program: the bulk of government current spending goes to public services programs of negligible marginal valuation, pointed out by Keynes (1936, Chapt. X, sect. VI) as most effective to fight unemployment. The "functional financing" of

(1) The more open the economy, that is, the larger the traded goods sector and the more it has of internationally tradable financial assets, the smaller will be the variability of its inflation and interest rates around the world levels and the faster will they converge to those levels once displaced by a shock.

(2) The "Burden of the debt" (Modigliani, 1961) and "money and growth" (Tobin, 1965) literatures had in their background a keynesian view of the role of government in modern mixed economies. Dynamic optimization models later denied their implications (Sidrausky, 1967, Barro, 1974).

such consumption expenditures through debt emission led to later concern with a burden of the public debt implied by less capital accumulation in a closed economy (Meade, 1958; Modigliani, 1961; Vickrey, 1961; Mishan, 1963; Diamond, 1965) and by more external indebtedness in an open economy (Guedes, 1978, Chapt. IV). Moreover, since public borrowing is not backed by real capital accumulation, interest payments may require future taxation as emphasized by Ricardo (1817), Bowen et.al (1960), Modigliani (1961).

Such fiscal framework described by a government that borrows to finance consumption and levies taxes to pay interest on public debt, set up by a short run stabilization effort, can hardly be rationalized in a long run optimization model.

This section formulates a neoclassical fiscal framework in Friedman's sense: (i) it is derived from a dynamic optimization program concerned with "long run efficiency or prospects for growth of the economic system"; (ii) expenditures, taxation and public debt emission do not change in response to cyclical fluctuations around the natural growth path. Under full employment, this framework fully satisfies the principles laid down by Samuelson (1960) in his apraisal of what a neoclassical framework would be.

The instruments of fiscal policy available to the government are linked by its budget constraint.

(1) $r(t)\ B^S(t) + G(t) + V(t) = T(t) + q(t)\ H^S(t) + B^S(t)$

where
- $r(t)$ = interest rate on public deb
- $B^S(t)$ = outstanding public debt
- $G(t)$ = public goods expenditures program
- $V(t)$ = public capital formation expenditures program
- $T(t)$ = taxation
- $q(t)$ = rental price for public capital services
- $H^S(t)$ = public capital stock

The privated sector budget is described by:

(2) $F[K(t),\ H^D(t),\ L(t)] + r(t)\ B^D(t) - q(t)\ H^D(t = T(t) + + C(t) + \dot{B}^D(t) + \dot{K}(t)$

where: $F(\ )$ = aggregate production function
$C(t)$ = private consumption spending
$K(t)$ = private capital stock
$L(t)$ = labor force
$B^D(t)$ = demand for public debt
$H^D(t)$ = demand for public capital services

Rewriting (1) and (2) in per capita terms we have:

(3) (a) $r(t)\, b^S(t) + g(t) + v(t) = \tau(t) + q(t)\, h^S(t) + \psi(t)$

(b) $f[k(t),\ h^D(t)] + r(t) b^D(t) - q(t) h^D(t) = \tau(t) + c(t) + \phi(t) + i(t)$

where: $\psi(t) = \frac{\dot{B}^S(t)}{L(t)}$: per capita public debt emission

$\phi(t) = \frac{\dot{B}^D(t)}{L(t)}$: per capita desired rate of public debt adquisition

$i(t) = \frac{\dot{K}(t)}{L(t)}$: per capita private investment rate

The private sector takes as given $h^S(t)$, $r(t)$, $b^S(t)$, $q(t)$, $\psi(t)$, $g(t)$ and $\tau(t)$, which are government policy instruments, and solves the dynamic optimization problem of:

$$\underset{c(t),\ \phi(t),\ i(t),\ h^D(t)}{\text{Max}} \int_0^\infty e^{-\delta t}\ U[C(t),\ g(t)]dt$$

Subject to (3) - (b) and the laws of motion for public debt acquisition and private capital formation:

(4) - (a) $\dot{b}(t) = \phi(t) - n\, b(t)$

(b) $\dot{k}(t) = v(t) - n\, k(t)$

where $n = \frac{\dot{L}(t)}{L(t)}$: constant growth rate of the labor force

δ: private rate of time preference

$U[c(t), g(t)]$: private utility function

Using the balance sheet of the private sector:

(5) $w(t) = k(t) + b^D(t) \Longrightarrow \dot{w}(t) = i(t) + \phi(t) - n(k(t) + b^D(t))$

and the private sector budget restriction given by (3) - (6), one can transform the two state variables problem in a simpler formulation separating the decisions relative to the optimal savings rate and the optimal portfolio allocation, as meant by Keynes (1936) and emphasized by Tobin (1961).

$$\underset{c(t),\, b^D(t),\, k(t),\, h^D(t)}{\text{Max}} \int_0^\infty e^{-\delta t}\, U[c(t), g(t)]dt$$

Subject to (6) (a) $\dot{w}(t) = f[k(t), h^D(t)] + r(t)\, b^D(t) - g(t)h^D(t) - \tau(t) - c(t) - n[k(t) + b^D(t)]$

(b) $w(t) = k(t) + b^D(t)$

(c) $b^D(t) \geq 0$

(d) $h^D(t) \geq 0$

The hamiltonian of the problem is:

(7) $H[c(t), b^D(t), k(t), h^D(t), \lambda(t), w(t), \mu(t), \Gamma_h(t), \Gamma_h(t)]$

$U[c(t), g(t)] + \lambda(t)\, \{f[k(t), h^D(t)] + r(t)\, b^D(t) - \tau(t) -$

$- c(t) - n(k(t) + b^D(t)\} + \mu(t)\, (w(t) - k(t) - b^D(t)) +$

$\Gamma_b(t)\, b^D(t) + \Gamma_h(t)\, h^D(t)$

and the necessary conditions implied by the Pontryagin's Maximum Principle are:

(8) - (i) $U_c[c(t), g(t)] - \lambda(t) = 0$

(ii) $\lambda(t)\, r(t) - \lambda(t)n - \mu(t) + \Gamma_b(t) = 0$

(iii) $\lambda(t)\ f_k[k(t),\ h^D(t)] - \dot{\lambda}(t) - \mu(t) = 0$

(iv) $\lambda(t)\ F_h[k(t),\ h^D(t)] - g(t) + \Gamma_h(t) = 0$

(v) $\dot{w}(t) = H_\lambda[c(t),\ b^D(t),\ \lambda(t),\ h^D(t), \mu(t),\ \Gamma_g(t),\ \Gamma_h(t)]$

(vi) $\dot{\lambda}(t) = \delta\lambda(t) - H_w\ [c(t),\ b^D(t),\ k(t),\ h^D(t),\ \lambda(t),$
$w(t),\ \mu(t),\ \Gamma_b(t), \Gamma_h(t)]$

(vii) $w(t) - b^D(t) - k(t) = 0$

(viii) $\Gamma_b(t) \geq 0,\ b^D(t) \geq 0,\ \Gamma_b(t)\ b^D(t) = 0$

(ix) $\Gamma_h(t) \geq 0,\ h^D(t) \geq 0,\ \Gamma_h(t)\ h^D(t) = 0$

and the transversality condition is:

(x) $\lim_{t\to\infty} e^{-\delta t}\ \lambda(t)\ w(t) = 0$

An implication for the interest rate paid on public debt follows from (8)-(ii), (iii) and (viii). If the government wants people holding public debt existing supply $b^S(t) > 0\ \forall\ t \in [0, \infty)$, i.e., $b^D(t) = b^S(t) > 0$, then $\Gamma_b(t) = 0\ \forall\ t \in [0, \alpha)$ by (8)-(viii). This implies by (8)-(ii) and (iii) that the interest rate paid by the government to public debt holders must be equal to the return on equities, i.e., the marginal productivity of capital along the optimal path.

The pricing policy for public capital services can guarantee equilibrium in such market making $h^D(t) = h^S(t) > 0\ \forall\ t \in [0, \infty)$, implying $\Gamma_h(t) = 0$ by (8) - (ix). By (8) - (iv) the private sector then chooses public capital services to as to equate their marginal productivity to rental costs.

Conditions (8) - (i) to (ix), considering the interest rate and pricing policies described above (that is, such as to make $b^D(t) = b^S(t) > 0$ and $h^D(t) = h^S(t) > 0\ \forall\ t \in [0, \infty)$, maintaining equilibrium in the bonds and public services markets), can be rewritten as:

$$(9) \quad \text{(i)} \quad U_c[c(t), g(t)] - \lambda(t) = 0$$

$$\text{(ii)} \quad r(t) - f_k[k(t), h^D(t)] = 0$$

$$\text{(iii)} \quad f_h[k(t), h^D(t)] - q(t) = 0$$

$$\text{(iv)} \quad w(t) - b^D(t) - k(t) = 0$$

$$\text{(v)} \quad \dot{w}(t) = f[k(t), h^D(t)] + r(t)\, b^D(t) - g(t)\, h^D(t) - \tau(t) - c(t) - n(k(t) + b^D(t))$$

$$\text{(vi)} \quad \dot{\lambda}(t) = \delta\, \lambda(t) - \lambda(t)\, r(t) + \lambda(t) n$$

At a given point of time the private sector state and costate variables are frozen, and so are the government policy instruments $g(t)$, $r(t)$ and $q(t)$.

Equations (9)-(i) to (iv) are then solved for the optimal private controls after differentiating totally such equations to discard arguments with zero partial derivatives in the control functions:

$$(10) - \quad \text{(i)} \quad U_{cc}(c,g)\, dc + U_{cg}(c,g)\, dg - d\lambda = 0$$

$$\text{(ii)} \quad dr - f_{kk}(k,h^D)\, dk - f_{kh}(k,h^D) dh = 0$$

$$\text{(iii)} \quad f_{hk}(k,h^D)\, dk + f_{hh}(k,h^D)\, dh^D - dg = 0$$

$$\text{(iv)} \quad dw - db^D - dk = 0$$

which is solved for:

$$(11) - \quad \text{(i)} \quad dc = \frac{1}{U_{cc}}\, d\lambda - \frac{U_{cg}}{U_{cc}}\, dg$$

$$\text{(ii)} \quad dk = \frac{f_{hh}}{f_{kk}\, f_{hh} - f_{hk}^2}\, dr - \frac{f_{kh}}{f_{kk}\, f_{hh} - f_{kh}^2}\, dg$$

$$\text{(iii)}\ dh^D = \frac{f_{kk}}{f_{kk}\, f_{hh} - f_{hk}^2}\, dg - \frac{f_{hk}}{f_{kk}\, f_{hh} - f_{kh}^2}\, dr$$

$$\text{(iv)}\ db^D = dw + \frac{f_{kh}}{f_{kk}\, f_{hh} - f_{hk}^2}\, dg - \frac{f_{hh}}{f_{kk}\, f_{hh} - f_{hk}^2}\, dr$$

implying that:

(12) (i) $c(t) = c[\lambda(t), g(t)]; \quad \frac{\partial c}{\partial \lambda} < 0, \frac{\partial k}{\partial g} > 0$

(ii) $k(t) = k[g(t), r(t)], \quad \frac{\partial b}{\partial g} < 0, \frac{\partial k}{\partial r} < 0$

(iii) $h^D(t) = h[g(t), r(t)]; \quad \frac{\partial h}{\partial g} < 0, \frac{\partial h}{\partial r} < 0$

(iv) $b^D(t) = b[w(t), g(t), r(t)]; \quad \frac{\partial b}{\partial w} > 0 \quad \frac{\partial b}{\partial g} > 0 \quad \frac{\partial b}{\partial r} > 0$

provided that the standard restrictions $U_{cc} < 0$, $U_{cg} > 0$, $f_{kk} < 0$, $f_{hk} > 0$, $f_{kk}\, f_{hh} - f_{hk}^2 > 0$ hold.

Observe also that:

(13) (i) $\frac{\partial b}{\partial g} = - \frac{\partial k}{\partial g}$

(ii) $\frac{\partial b}{\partial r} = - \frac{\partial k}{\partial r}$

(iii) $\frac{\partial b}{\partial w} = 1$

Plugging functions (12)-(i) to (iv) on (9)-(v) and (vi) one determines the motion of $w^*\ (t_o;t)$ and $\lambda^*\ (t_o;\ t)$ for preannounced government policy functions $g(t_o;\ t)$, $\tau(t_o;\ t)$, $r(t_o;\ t)$, $g(t_o,t)$, given $w(0) = w_0$ and $\lim_{t\to\infty} w(t) = \bar{w}$ (such as to satisfy the transversality condition, what is sufficient to stablish the optimality of a Pontryagin path).

Since the government does follow its preannounced polices perfectly anticipated by the private sector, the private optimal control solutions below turn out to be a rational expectations equilibrium

(14) (i) $c^*(t_o;t) = c[\lambda^*(t_o;t), w^*(t_o;t), g(t_o;t), r(t_o;t), q(t_o;t)]$

(ii) $k^*(t_o;t) = k[\lambda^*(t_o;t), w^*(t_o;t), g(t_o;t), r(t_o;t), q(t_o;t)]$

(iii) $h^*(t_o;t) = h[\lambda^*(t_o;t), w^*(t_o;t), g(t_o;t), r(t_o;t), q(t_o;t)]$

(iv) $b^*(t_o;t) = b[\lambda^*(t_o;t), w^*(t_o;t), g(t_o;t), r(t_o;t), q(t_o;t)]$

The public sector also determines its policy instruments path solving a dynamic optimization problem:

$$\max_{g(t),\, r(t),\, q(t),\, \tau(t),\, v(t),\, \Psi(t)} \int_0^\infty e^{-\delta t}\, U[c(t), g(t)]dt$$

Subject to:

(15) - (i) $r(t)\, b^S(t) + g(t) + v(t) = \tau(t) + q(t)\, h^S(t) + \psi(t)$

(ii) $\dot{b}^S(t) = \psi(t) - nb^S(t)$

(3) The dynamic game theoretic framework is a continuous version of Kydland's (1975).
Players are assumed to have rational expectations in the sense that the expectaction of the other's action turn out to be the actual outcome.
The government is assumed to be the dominant player; in making his decision, it takes into account the reaction functions of the non dominant player. With rational expectations he correctly foresees such functions. The private sector decides what his optimal decision is taking the government instruments as given. Under rational expectations, the government controls path turn out to be exactly what the private sector was expecting when solving its optimization problem.

(iii) $\dot{h}^S(t) = v(t) - n\, h^S(t)$

(iv) $h^S(t) = b^S(t) = 0$: the public sector balance sheet[4];

(v) $\dot{w}(t) = f[k(t), h^D(t)] + r(t)\, b^D(t) - q(t)\, h^D(t) - \tau(t) - c(t) - nk(t) - n\, h^D(t)$: the private sector budget restriction;

(vi) $c(t) = c[\lambda(t), g(t)]$, $k(t) = k[q(t), r(t)]$, $h^D(t) = h[q(t), r(t)]$, $b^D(t) = b[w(t), q(t), r(t)]$: the private policy functions.

Differentiating (15) - (iv), using (15) - (i), (ii), (iii) and (iv) in the result, the problem is transformed into:

$$\underset{g(t),\, \tau(t),\, r(t),\, q(t),\, b^S(t)}{\text{Max}} \int_0^\infty e^{-\delta t}\, U[c(t), g(t)]\, dt$$

Subject to:

(16) - (i) $\tau(t) + [q(t) - r(t)]b^S(t) - g(t) = 0$

(ii) $\dot{w}(t) = f[k(t), h^D(t)] + r(t)\, b^D(t) - q(t)\, h^D(t) - \tau(t) - c(t) - n\, w(t)$

(iii) $c(t) = c[g(t), \lambda(t)]$, $k(t) = k[q(t), r(t)]$
$h^D(t) = h\,[q(t), r(t)]$, $b^D(t) = b[w(t), q(t), r(t)]$

The hamiltonian for the public sector optimization problem is:

$$M(g(t), \tau(t), r(t), q(t), b^S(t), \theta(t), w(t)\, \sigma(t) =$$
$$= U\,[c(g(t), \lambda(t), g(t)] + \theta(t)\,[\tau(t) + (q(t) - r(t))b^S(t) - g(t)]$$
$$+ \sigma(t)\,\{f[k(q(t), r(t)), h(q(t), r(t)] - r(t)\, b(w(t), q(t), r(t))$$
$$- g(t)\, h(q(t), r(t)) - \tau(t) - c(g(t), \lambda(t)) - n\, w(t)\}$$

(4) Samuelson (1958) attributed to money the intergenerational intermediation role; public debt is the logical successor in a real (non monetary) model. The young generation buys public debt, the government invests the proceeds in public capital and pays interest: to the same generation in the future (then older) out of the forthcoming benefits of public capital formation.

and the necessary conditions by the Pontryagin's Maximum Principle are:

(17) (i) $U_c(c,g)\,\frac{\partial c}{\partial g} + U_g(c,g) - \sigma\frac{\partial c}{\partial g} = 0$

(ii) $\theta - \sigma = 0$

(iii) $-\theta\, b^S + \sigma\, f_k\,\frac{\partial k}{\partial r} + \sigma\, f_h\,\frac{\partial h}{\partial r} - \sigma\, b^D -$

$- \sigma r\,\frac{\partial b}{\partial r} - \sigma\, g\,\frac{\partial h}{\partial r} = 0$

(iv) $\theta b^S + \sigma\, f_k\,\frac{\partial k}{\partial q} + \sigma f_h\,\frac{\partial h}{\partial q} - \sigma\, r\,\frac{\partial b}{\partial q}$

$- \sigma h^D - \sigma\, g\,\frac{\partial h}{\partial q} = 0$

(v) $\theta(q - r) = 0$

(vi) $\tau + (q - r)b^S - g = 0$

(vii) $\dot{w} = f(k,\, h) + r\, b^D - qh^D - \tau - c - n\, w$

(viii) $\dot{\sigma} = \delta\sigma - \sigma r\,\frac{\partial b}{\partial w} - \sigma n$

A first implication from (17) - (v) is that the government must charge for public capital services a rental rate equivalent to the interest rate it pays on public debt, that is, $q(t) = r(t)$.

The marginal productivity of public capital will then be equal to the marginal productivity of private capital, since the private optimization policies equate the public services rental rate to the first and the public debt interest rate to the latter, as given by (9) - (ii) and (iii).

Using this result ($f_k = r = q = f_h$), (17) - (ii), (15) - (iv) and (13) in (17) - (iii) and (iv) we get:

(17) - (iii) $b^S(t) = b^d(t)$

(iv) $h^S(t) = h^D(t)$

that is, the public services pricing policy and interest rate to public debt holders will be such as to generate equilibrium respectively in the public capital services and the bond markets.

Still using $r(t) = q(t)$ in (17) - (vi), the government must pay all current spending on public goods consumption out of current taxation:

(17) - (vi) $g(t) = \tau(t)$

By (17) - (ii) the shadow price of public resources $\theta(t)$ is equal to the shadow price $\sigma(t)$ attibuted by the government to private resources.

By (9)-(vi) and (17)-(viii) one observes that the shadow price $\lambda(t)$, attributed by the private sector to its resources, and the shadow price $\sigma(t)$, used by the government to evaluate the opportunity cost of such funds, follow the same law of motion. Their paths will coincide if the initial condition $\lambda(0) = \sigma(0)$ holds.

Assuming that the government does use the shadow price of private funds as determined by private optimization to evaluate the opportunity cost of such resources, i. e., $\lambda(t) = \sigma(t)$, and using (17)-(ii) in (17-(i) it follows that:

(18) $[U_c(c,g) - \lambda] \frac{\partial c}{\partial g} + U_g(c,g) - \lambda = 0$

whereby it is shown that along the optimal path the marginal utility of private consumption equals the marginal utility of public goods consumption (since $U_c - \lambda = 0$ by (9)-(1)).

The neoclassical framework where the government levies taxes for financing consumption expenditures, issues debt to finance public capital formation and charges for public capital services according to its marginal productivity, using the proceeds to pay interest on debt, is shown to be an optimal public policy.

The public sector solution and the private sector (simultaneously) implied solution are then determined by:

(19) - (i) $U_g[c(g(t), \lambda(t)), g(t)] - \lambda(t) = 0$

(ii) $\tau(t) = g(t)$

(iii) $b^S(t) = b^D(w(t), q(t), r(t))$

(iv) $b^S(t) = h^D(g(t), r(t))$

v) $r(t) = q(t)$

(vi) $\dot{w}(t) = f[k(g(t), r(t)), h(g(t), r(t)) - \lambda(t) - $
$- c(g(t), \lambda(t)) - n\, w(t)$

(vii) $\dot{\lambda}(t) = \delta\lambda(t) - n\,\lambda(t) - r(t)\,\lambda(t)$

describing the paths for $g(t)$, $\tau(t)$, $r(t)$, $q(t)$, $b^S(t)$, $\lambda(t)$ and $w(t)$ given the initial and the transversality conditions.

Eliminating $b^S(t)$, $\tau(t)$ and $q(t)$ the system reduces to:

(20) - (i) $U_g\ [c(g(t), \lambda(t)), g(t)] - \lambda(t) = 0$

(ii) $b^D\ (w(t), r(t), r(t)) = h^D(r(t), r(t))$

(iii) $\dot{w}(t) = f[k(r(t), r(t)), h(r(t), r(t)) - g(t) - $
$- c(g(t), \lambda(t)) - n\, w(t)$

(iv) $\dot{\lambda}(t) = \delta\lambda(t) - n\,\lambda(t) - r(t)\,\lambda\ (t)$

At a given point in time $w(t)$ and $\lambda(t)$ are frozen with (20)-(i) and (ii) respectively determing public goods consumption and the interest rate (the latter basically equating the demand for bonds to the marginal productivity of public capital):

(20) - (i) $g(t) = g[\lambda(t)]$

(ii) $r(t) = r[w(t)]$

Plugging such fuctions n(20) - (iii) and (iv) the private sector costate and state paths are determined:

(20) - (iii) $\dot{w}(t) = f[k(r(w(t)), h(r(w(t))] - g\ [\lambda\ (t)] - $
$- c[g(\lambda(t)), \lambda(t)] - n\, w(t)$

(iv) $\dot{\lambda}(t) = (\delta - n)\ \lambda(t) - r[w(t)]\ \lambda(t)$

with $w(0) = w_0$ and $\lim_{t\to\infty} w(t) = \hat{w}$

Taking the solution $w^*(t_0;t)$, $\lambda^*(t_0;t)$ and also $g(t_0;t) = g[\lambda^*(t_0;t)]$, $r(t_0;t) = r[w^*(t_0;t)]$, $q(t_0;t) = r(t_0;t)$ as given by (20)-(i), (ii) and (19)-(v) into (14), the private sector path is entirely determined.

The paths for the public sector variables $g(t)$, $\tau(t)$, $r(t)$ $q(t)$, $b^S(t)$, $h^S(t)$ follow respectively from (20)-(i), (19) - (ii), (20)-(ii), (19)-(v), (19)-(iv), (15)-(iv) after plugging $w^*(t_0;t)$ and $\lambda^*(t_0;t)$.

Systems (9) and (19) are indeed simultaneously solved by the private and public sectors with each sector having perfect foresight of what the other is doing.

Aggregating the budgets of boths sectors as given in (3) -(a) and (b) we have:

(21) $F(k(t), h(t)) = g(t) + c(t) + v(t) + i(t)$

having used stock and flow equilibrium conditions in the bonds markets:

$(t)\ \Psi(t) = \phi(t)$ and $b^D(t) = b^S(t)$

Aggregating the balance sheets we have:

(22) $w(t) + h(t) - b^S(t) = k(t) + b^D(t) + h(t) - b^S(t) = k(t) + h(t)$

implying that:

(23) $\dot{w}(t) = \dot{k}(t) + \dot{h}(t) = v(t) + i(t) - n(k(t) + h(t))$

and the problem for the private and public sectors is to derive simultaneously the optimal path for the economy:

$$\max_{g(t),\, c(t),\, k(t),\, h(t)} \int_0^\infty e^{-\delta t}\, U(c(t), g(t))\, dt$$

subject to:

(16) (a) $w(t) - h(t) - k(t) = 0$

(b) $\dot{w}(t) = F(k(t), h(t)) - g(t) - c(t) - n\, w(t)$

where the latter follows from (21) and (23)

The hamiltonian for the problem is given by:

$$P(c(t),\ g(t),\ k(t),\ h(t),\ w(t),\ \lambda(t),\ \mu(t) =$$
$$= U(c(t),\ g(t)) + \lambda(t)\ \{F(k(t),\ h(t)) - g(t) - c(t) - n\ w(t)\}$$
$$+ \mu(t)\ (w(t) - k(t) - h(t))$$

and the Maximum Principle necessary conditions are:

(17) (i) $U_c\ (c(t),\ g(t)) - \lambda(t) = 0$

(ii) $U_g\ (c(t),\ g(t)) - \lambda(t) = 0$

(iii) $\lambda(t)\ F_k(k(t),\ h(t)) - \mu(t) = 0$

(iv) $\lambda(t)\ F_h(k(t),\ h(t)) - \mu(t) = 0$

(v) $\dot{w}(t) = F(k(t),\ h(t)) - c(t) - g(t) - n(k(t) + h(t))$

$w(0) = w_0$

(vi) $\lambda(t) = \delta\lambda(t) - n\ \lambda(t) - \mu(t)$

(v) $w(t) - k(t) - h(t) = 0$

and the transversality condition $\lim_{t\to\alpha} e^{\delta t}\ \lambda(t)\ w(t) = 0$

whose solution must be exactly the previously obtained with the government as a dominant player and the private sector reacting to that.

3. SOME REMARKS ON THE BRAZILIAN EXPERIENCE

Brazil had a hard experience during the last years and a lot of it had to do with not having adopted a neoclassical fiscal framework as described here. The rate of inflation went from 45% in June/79 (last 12 months) to 140% in June/83 and the country lost US$ 12 billions in such period.

A major reason for that was A DISCRETIONARY EXPENDITURES PROGRAM stabilishing new priorities (agriculture and exports) without specifying non inflationary methods of financing. The III National Development Plan determined an over expansionary credit policy by the Central Bank through a Movement Account charging 1% nominal interest (yearly rate). Due to the relative size of the banks (Banco do Brasil is the biggest rural bank in the world), a relatively moderate credit expansion by Banco do Brasil implies an extremely high growth rate for the monetary base and ultimately for the money supply. The institutional setting is such that an expansionary credit policy leads to lack of monetary control.

Hence, any increase in public spending (usually with subsidized credit) which is not followed by increased taxation leads to more inflation. After the inflation rate sky rocked in Brazil, the tax rate on financial transactions was raised from less than 5% to 25%, transforming a previously unimportant item into the 3rd major source of fiscal resources.

This has been the pattern: an expenditures program starts without a cost-benefit analysis or provisions for funding and then taxes are raised after inflation sky rocked.

Another interesting violation of the neoclassical fiscal framework here proposed is the "Complementary Law number 12", relative to public debt management. Public bonds with monetary correction helped to curb inflation after the 64 Revolution, when Brazil suffered from a "fiscal" inflation. In 69 the country was booming, inflation coming down and people changing their portfolios towards equities and out of public debt. The expenditures with interest payments and amortization began to exceed the new placements of Indexed bonds. The Treasury removed such item from its budget and transferred it to the Monetary Buget, under the Central Bank control. This undesirably avoided a healthy conflict, such as the one between the Fed and the US Treasury, ended with the Fed independence in 53.

The Central Bank trying to do a nice job while managing the debt for the Treasury, allways places enough debt to pay interest and amortization expenses, printing enough money to make sure it happens. It can be easily shown that the per capita public debt in real terms explodes to infinity by the Complementary Law mechanism, unless the government starts manipulating the monetary correction, what already happens.

Also, the Central Bank resorts to snowball public debt printing because it didn't spend wisely the proceeds of the placements.

A final remark also emphasizes the need for investing in capital formation the resources obtained from debt placements.

Two thirds of Brazilian external indebtedness (around US$ 60 billion today) is due to the public sector, mainly, state owned companies. The country will have to effect future transfers of real resources to service such debt. There will be a real burden, what is not certain in a closed economy. Unless such resources (actually obtained through the current account) are invested in productive purposes, fostering public capital formation, the country may regret in the future for not having had a neoclassical fiscal framework in the past.

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