Monetary policy and exchange rate: effects on disaggregated prices in a FAVAR model for Brazil*

Elcyon Caiado Rocha Lima**
Thiago Sevilhano Martinez***
Vinícius dos Santos Cerqueira****

Abstract

This article investigates what type of nominal price rigidity there is in Brazil by testing if there are different price responses to sector-specific, monetary policy, and exchange rate shocks. We estimate the effects of these shocks on disaggregated prices of the Brazilian consumer price index (IPCA), from 1999 to 2011, using a factor-augmented vector autoregressive model (FAVAR). There is innovation in the Bayesian technique used to estimate the model, which allows for the consistent imposition of priors directly on the parameters of the structural form of the model and the use of sign restrictions in the impulse responses to identify the model. We combine the Gibbs sampling procedure developed by Bernanke, Boivin, and Eliasz (2004) to estimate a FAVAR with the Gibbs sampling procedure developed by Waggoner and Zha (2002 and 2007) to estimate a Bayesian structural vector autoregressive model (BSVAR). The main results are: (i) series-specific shocks are the main determinants of variances in highly disaggregated price series, but macroeconomic shocks have more influence over aggregated series; (ii) macroeconomic shocks are more persistent than series-specific shocks; (iii) almost all the signs of the price responses to monetary and exchange rate shocks are the ones predicted by the theory;

* Submitted on 19/01/2015. Revised on 12/01/2018.

** Professor at Rio de Janeiro State University (UERJ). E-mail: elcyon.lima@uerj.br.

*** Planning and Research Technician of the Directorate of Macroeconomic Policies and Studies (Dimac) at Ipea and doctoral student at FGV/EPGE Brazilian School of Economics and Finance. E-mail: thiago.martinez@ipea.gov.br.

**** Planning and Research Technician of the Directorate of Macroeconomic Policies and Studies (Dimac) at Ipea. E-mail: vinicius.cerqueira@ipea.gov.br.

and (iv) the magnitude of the responses to monetary and exchange rate shocks are heterogeneous across sectors.
Keywords: disaggregated prices, monetary and exchange rate shocks, FAVAR model.
JEL codes: E31, E52, E37.

1. Introduction

This article investigates what type of nominal price rigidity there is in Brazil by testing if there are different price responses to sector-specific, monetary policy, and exchange rate shocks. In addition, it is verified whether there is heterogeneity in the magnitude of these shocks by considering different activity sectors. To estimate the impacts of these shocks on prices, at different levels of disaggregation of the Brazilian consumer price index (IPCA), from 1999 to 2011, the FAVAR (factor-augmented vector autoregression) model was adopted. The FAVAR model adopted is estimated with a methodological innovation: the Gibbs sampling procedure developed by Bernanke, Boivin, and Eliasz (2004) to estimate a FAVAR model is combined with the Gibbs sampling procedure developed by Waggoner and Zha (2002 and 2007) to estimate a Bayesian structural vector autoregression (BSVAR) model. This methodological innovation enables the imposition of priors directly on the parameters of the structural form of the FAVAR, making the Bayesian procedure more consistent and enabling the model to be identified using sign restriction in the impulse-response functions.

New Keynesian macroeconomic models adopt the price rigidity hypothesis to explain the existence of short run effects of monetary policy on real variables. In empirical applications, in order to reproduce the degree of persistence observed in the data, a price duration period of around one year is typically considered (as is indicated, for example, in the review by Goodfriend and King, 1997). However, the empirical basis for this hypothesis has been questioned by research on the behavior of individual prices in microdata on comprehensive price indices. With data that cover 70% of the basket of the United States consumer price index (CPI), the pioneering study from Bils and Klenow (2004) calculates that the frequencies of individual price readjustments are much greater than usually assumed, which implies greater volatility and less price persistence. Various studies that cover other countries and price indices observe similar results, according to the literature review in Klenow and Malin (2010).

Boivin, Giannoni, and Mihov (2009) address the problem empirically by considering that sectoral price variation responses to macroeconomic shocks can be different from responses to idiosyncratic shocks. With data from the United States, they use a factor-augmented vector autoregressive (FAVAR) econometric model to identify the effects of sector-specific and macroeconomic shocks on disaggregated prices, the macroeconomic shocks being separated into monetary shocks and common shocks due to other causes. Among the most relevant results, the authors find that fluctuations in the specific components of the series are the main reason for the volatility of the disaggregated prices and that there are differences in the speed with which the shocks propagate, with quick responses to sector-specific shocks and slow ones to monetary and macroeconomic shocks in general. Thus, the high volatility found in the prices indices microdata would mainly be explained by specific shocks, which in the aggregation would tend to cancel each other out, leading to low volatility and high persistence of the inflation rate, which is typical of macroeconomic models.

Our paper adopts a FAVAR model to estimate the impacts of monetary and exchange rate shocks on prices at different levels of disaggregation of the Brazilian consumer price index (IPCA). In addition to these shocks, the effects of common factors and of the specific component are identified for 363 price variation series for disaggregations of the IPCA, which cover the period from August of 1999 to December of 2011 in monthly periodicity. In addition to the IPCA inflation series itself, there are four first hierarchical level disaggregation series, 15 second level series, and 343 series at the subitem level, the most disaggregated possible. The factor extraction procedure is applied to a set of 436 series, the 364 price variation ones plus another 72 series that capture general aspects of the economy. Identification by sign restriction is used in the responses of the macroeconomic variables that compose the FAVAR to monetary and exchange rate shocks. The response signs, the magnitude of the responses to the shocks in the different price categories, and the relative importance of the macroeconomic and specific shocks for the variance decomposition are analyzed.

The results obtained indicate that also for Brazil fluctuations in the most disaggregated price series are mainly explained by specific shocks, with approximately 70% of the variance in the prediction error, while for the IPCA inflation series the opposite occurs, with almost 70% of the variance being due to macroeconomic shocks. Of the total variability of the IPCA inflation series attributable to macroeconomic shocks, approximately 7% is due to monetary shocks and 13% is due to exchange rate shocks. The variance decomposition also shows that there is a greater lag in the responses to macroeconomic shocks compared to the responses to idiosyncratic shocks, which are faster.

Regarding sectoral heterogeneity, we observe that the pattern of responses is different depending on the sector considered. At the first level of disaggregation, prices are separated into four groups: food and drink (except food services), industrial goods, free services, and monitored or managed prices. The prices of the food and drink group are the ones that respond most to monetary shocks, with a greater intensity in tradable foods, and they respond little to exchange rate shocks. The price responses of industrial products, both to monetary and exchange rate shocks, are close to the IPCA mean response. In the services group, the responses to monetary and exchange rate shocks are typically non-significant, with the exception of the “food outside the home” categories, which present heightened responses to the two types of shocks, and “leisure, communication, and tourism”, whose responses to exchange rate shocks are significant. In the monitored prices, the responses to monetary shocks are non-significant in all the categories and the responses to exchange rate shocks are high, except for “energy (combustibles and electrical)”, whose responses to exchange rate shocks are non-significant.

1.1 Related literature

The article contributes to the debate on inflation and evidence from microdata on consumer price indices with results for Brazil. Studies on price behavior in the style of Bils and Klenow (2004) already exist for the country. Gouvea (2007) and Barros and Matos (2009) analyze microdata of the Getúlio Vargas Foundation Price Index (IPC/FGV), while Lopes (2008) investigates microdata on the Municipality of São Paulo Consumer Price Index calculated by the Economic Research Institute Foundation (IPC-FIPE). In general, similar results to those of the international studies are observed: high volatility of disaggregated prices, frequent price changes (the average duration in Brazil is even shorter than that observed for the United States and Europe), high magnitude price changes, as well as sectoral heterogeneity. The results obtained in our article indicate a preponderance of sector-specific shocks to explain the high volatility of the disaggregated price series and that there is heterogeneity in the responses of the sectors to different macroeconomic shocks.

Balke and Wynne (2007) analyze the effect of macroeconomic shocks on relative prices dispersion at a high level of disaggregation. The authors estimate the responses to monetary shocks of more than 600 components of the producer price index (PPI) in the United States, attaching the disaggregated price determination equations to a parsimonious VAR one by one. Responses with the opposite sign to the expected are observed – the so-called price puzzle, that is, an increase (decrease) in prices in response to a monetary contraction (expansion) – for approximately half of the prices. A similar methodology is applied to Brazil by Guillén and Garcia (2011), who investigate the effects of monetary and exchange rate shocks over 512 subitems of the Brazilian consumer price index (IPCA) between 1999 and 2006. The authors also observe a high occurrence of price responses with the opposite sign to expected, which occurs in 296 subitems for the monetary shock and 204 subitems for the exchange rate shock.

According to Sims (1992), in VARs with few variables, monetary shocks tend to generate the price puzzle due to the omission of variables that are important for monetary authority decisions. The FAVAR corrects this problem because the common factors condense the relevant macroeconomic information from many different series. Thus, in Boivin, Giannoni, and Mihov (2009) the price puzzle occurs in a very small quantity of sectors, contrasting with what is presented by Balke and Wynne (2007), and it does not occur at the aggregate level. In this article, we also observed a low occurrence of signs opposite to the expected ones. In the 343 subitems they appear in 16 (2.5% of the IPCA weighting) for the exchange rate shock and in only one (0.1% of the IPCA weighting) for the monetary shock.

The main reference for this article is Boivin, Giannoni, and Mihov (2009), whose most relevant results for the purposes of this study have already been presented. It is worth noting the novelties of this article in relation to Boivin, Giannoni, and Mihov (2009), as well as the fact that it is applied to Brazil. Regarding the results, in this article the responses to the exchange rate shock as well as to the monetary shock are analyzed, and the differences between sectors in the patterns of the responses to shocks are examined in detail, which are not the object of Boivin, Giannoni, and

Mihov (2009). The main innovation, however, is in the methodology. Although we also adopt the FAVAR model is in this article, the estimation procedure is different. Besides identification via sign restriction, the Gibbs sampling procedures from Bernanke, Boivin, and Eliasz (2004) and Waggoner and Zha (2002 and 2007) are combined in an unprecedented way in this literature for the Bayesian estimation of the FAVAR, enabling the imposition of priors directly on the structural form of the model.

The paper is divided into six more sections after this introduction. In the second section, the data used and the classification adopted for the intermediate levels of aggregation are presented. The third section discusses the methodology of the FAVAR model with identification using sign restriction and Gibbs sampling estimation. The results are analyzed in the following sections: the signs of the impulse-response functions in section four, the magnitude of the responses in section five, and the variance decomposition in section six. The conclusion summarizes the discussion of the main results, for the whole IPCA and by sector.

2. Database

The series of subitems used in this study are the percentage rates of price variation of the IPCA subitems, in monthly frequency, in the period from August of 1999 to December of 2011. During the period covered in this study, in July of 2006 there was a review of the IPCA weighting structure in order to make it compatible with the 2002-2003 POF, substituting the structure in place since 1999 based on the 1995-1996 POF.[1]

Due to changes in consumption patterns, it is normal for the list of goods that compose the price index to be altered in weighting structure reviews. This primarily occurs at the subitem level, which is the lowest level of aggregation of the IPCA. However, in the 2006 review, the quantity of series altered at the subitem level was

[1] The last reviews of the IPCA weighting structure occurred in August of 1999, July of 2006, and January of 2012. In January of 2012, of the 384 existing subitems, 50 were removed and 31 were added, resulting in 365 subitems. When this study was started, there were few observations available after the January 2012 review, so the decision was made to ignore the subsequent data in order to avoid any loss in the number of series. We ignored the data from the structure prior to 1999 as this was the year the current macroeconomic policy regime began, with the adoption of inflation targeting, floating exchange rate, and primary surplus goals.

quite high. This occurred because, in order to facilitate monthly monitoring, the IBGE altered the criteria for including subitems in the IPCA basket. The minimum weight from which a subitem is necessarily included was raised, and the minimum percentage of subitem cover for each item, the aggregation level above the subitem, was reduced. Considering removed, added, and merged categories, the balance was a reduction in the total number of subitems from 512 to 383 categories.

In this study, only the series of subitems kept in the 2006 review were considered, as well as those for which there is immediate correspondence between the two periods, verified by the translator of the structures available on the IBGE website (2006).[2] 342 series resulted from this procedure, with the remaining ones being aggregated into a single category called "9999999. Others". However, since most of the series excluded or added in the review have little weight, the 342 series cover more than 97% of the weighting of the index over the entire period.

In addition to the 342 subitems and to the "others" series, price variations series in sectoral aggregations were added. This was done because the results for the subitems will be presented by sector, so with the inclusion of these series it will be possible to compare the results for the subitems from each sector with those obtained for the corresponding aggregated series.

In the IPCA disaggregation levels used by the IBGE, the categories are formed according to finality of use by the consumer, so that types of products with discrepant characteristics with regards to price formation are often combined. In this study, the IPCA classification by nature of the products proposed in Martinez (2014) was adopted, which is comparable with the different disaggregations of the IPCA disclosed by the Brazilian Central Bank (BCB). Of the three levels of this classification, two were used, in four and 15 categories. The classification underwent some adaptations in order to take into consideration the aggregation and removal of some

[2] What is understood by immediate correspondence are series in which the identification code was kept and only the name changed, as well as those in which there was an aggregation or disaggregation in 2006 and there is direct equivalence between series from the two periods. In these cases, a single aggregated series was constructed for the two periods. For example, the series existing until 2006 known as "1102038. Pizza Dough", "1102040. Pastry Dough", and "1102043. Lasagna Dough", which together correspond exactly to "1102029. Semi-Prepared Dough" in the subsequent period, were aggregated into a single series also in the period up to 2006. In the aggregation, each series was weighted using the ratio between its weight in the IPCA in the month and the sum of the weights of all the aggregated series in that category.

of the series due to the compatibilization of the periods before and after 2006.[3] As well as the 19 series at these two levels of aggregation, inflation itself measured by the IPCA is added to the 343 at the subitem level.

For the extraction of the common factors, besides these 363 price variation series, a set of 72 series (totaling 436 series) was included. These series are related to credit granting, the consumption of electrical energy and combustibles, aggregated and disaggregated industrial production indicators, energy generation, the labor market, and the financial and capital market.[4] The aim was ensure greater efficiency in the process of identifying common factors, as was done by Boivin, Giannoni, and Mihov (2009).

The 436 series mentioned above were deseasonalized using the X12-ARIMA method. The ADF and Phillips-Perron unit root tests were used. The results of the ADF test, using the Schwarz lag selection criterion, show that only the "2104015. Bar of soap" subitem presents a unit root. As for the Phillips-Perron test, in no case was the presence of a unit root hypothesis accepted. In this context, the hypothesis that all the price variation series are stationary was adopted.

In the VAR equation, some macroeconomic variables are included based on the dynamic factors. For these, we construct the impulse-response functions by imposing sign restrictions, as presented in the next section. The variables for which sign restrictions were constructed were: Selic interest rate, nominal exchange rate, IPCA, 180 day (pre DI) swap, M1, and industrial production index. All these series were used in natural logarithm. The FMI commodities price index (combustibles and non-combustibles), in dollars and deseasonalized using the X12-ARIMA, were included in the VAR among the macroeconomic variables, but without sign restriction.

---

[3] One relevant alteration was the reclassification of the subitems "5104002. Ethanol" and "2201003. Charcoal", which are classified among the monitored energy ones by Martinez (2014), but in this study were reclassified as non-durable industrial products in order to standardize with the BCB classification. The classification is in the Appendix.

[4] The series were obtained from the Brazilian Central Bank (credit granting and capital market), Eletrobrás (electrical energy consumption), the National Petroleum Agency (consumption of combustibles) IBGE (industrial production and labor market indicators), the National Association of Automotive Vehicle Manufacturers (vehicle production), the National Electricity System Operator (energy production), and the Institute of Applied Economic Research (real minimum wage).

3. Methodology: the FAVAR model and the identification using sign restriction

The econometric methodology used in this study enables the estimation of a structural model with a large amount of information. The FAVAR model, adopted in this article, was also used by Bernanke, Boivin, and Eliasz (2005) to investigate the impact of monetary shocks on a broad set of US economy variables. The model was applied for data on the Brazilian economy by Almeida, Alves, and Lima (2012). Using Bayesian estimation techniques, the authors analyzed the effects of monetary and exchange rate shocks on Brazilian economic variables.

In this article, an innovation is adopted in relation to the Bayesian procedure used in the literature. The Gibbs sampling developed by Bernanke et al. (2005) and the Gibbs sampling developed by Waggoner and Zha (2002 and 2007) are combined to estimate the Bayesian Structural FAVAR model (BSFAVAR) adopted. A detailed description of the methodology is found in Appendix C.

The FAVAR model can be represented by the following equations:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \tag{1}$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + DJ + \nu_t \tag{2}$$

Where: Y_{t} is a vector with a small number M of macroeconomic variables, including the monetary policy and exchange rate variables; F_{t} is a vector with a small number K of unobserved factors, extracted from the 436 series, which summarizes the relevant macroeconomic information not captured by Y_{t}; X_{t} is a vector of time series with the 363 price variation subitems and other aggregations of the IPCA; $\Phi(L)$ is a polynominal of the lags operator L of a finite order p, which can contain prior restriction over its values; Λ^f and Λ^y are factor loading matrices; D is a matrix *(M+K) x h* of exogenous variable parameters; J is a vector *h x 1* of exogenous variables; e_t is a vector of white noise errors, uncorrelated contemporaneously; v_t is a vector of white noise errors.

Equation (1) shows the impact of the unobserved factors. monetary and exchange rate shocks on disaggregated prices. Equation (2) is a VAR in (Y_{t}, F_{t}).

The estimation of the model is carried via a Bayesian method that uses Gibbs sampling, which is one of the approaches presented by Bernanke et al. (2005).

The procedure developed by Ahn and Horenstein (2013) was adopted to choose the number of common factors, it indicates one factor as the optimal number.

The model in state-space form is given by:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^y \\ 0 & I \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + DJ + \nu_t \tag{4}$$

In which (3) is the measurement or observation equation and (4) is the transition equation.

The set of parameters of the model $\theta = (\Phi\ L\ ,\ \mathrm{D},\ \Lambda^f\ ,\ \Lambda^y\ ,\ R,\ Q)$ has each element treated as a random variable. The estimation of the parameters and of the unobserved factors F_t is carried out via a multi-move Gibbs sampling (Carter and Kohn, 1994). Considering $Z_t=(X_t,\ Y_t)$, $\varepsilon_\mathrm{t} = (\mathrm{e_t},\ 0)$, and $G_t=(F_t,\ Y_t)$, equations (3) and (4) can be represented as:

$$Z_t = \Lambda G_t + \varepsilon_t \tag{5}$$

$$G_t = \Phi(L) G_{t-1} + DJ + \nu_t \tag{6}$$

The structural version of equation (6) is:

$$HG^t = H\Phi(L)G_{t-1} + HDJ + H_{\nu t}$$

The matrix H is full rank and is chosen so that: $\mathrm{u_t} = \mathrm{H}\nu_\mathrm{t}$, $\mathrm{cov(u_t) = I}$. Let $\mathrm{H}\Phi\ L)$ $= \mathrm{C_1L} + \ldots + \mathrm{C_pL^p}$, $\Psi = [C_1 ... C_p D]$, and $\mathrm{x_t} = [\mathrm{G_{t-1}\ G_{t-2}\ ....\ G_{t-p}\ \ J}]'$, considering a VAR with p lags. Thus, the structural form of equation (6) becomes:

$$G_t' H' = x_t' \Psi' + u_t'$$

In equation (5) the unknown parameters of Λ are the factor loading matrices, composed of Λ^f and Λ^y, and $P = cov\ (\varepsilon_t,\ \varepsilon_t')$ is the covariance matrix of $\varepsilon_t = (e_t,\ 0)$.

Since the model will be estimated using Bayesian methods, the parameters are treated as random variables. Thus, if $\tilde{X}_t = (X_1, X_2, ..., X_t)$ is the history of X between periods 1 and T, than $\tilde{F}_T = (F_1, F_2, ..., F_T)$. If $p(\tilde{F}_T, \theta)$ is the combined posterior density of $\tilde{F}_T$ and of the set of parameters θ, the posterior marginal densities of $\tilde{F}_T$ and θ will be given by:

$$p(\tilde{F}_T) = \int p(\tilde{F}_T, \theta) d\theta \tag{7}$$

$$p(\theta) = \int p(\tilde{F}_T, \theta) d\tilde{F}_T \tag{8}$$

The estimates of $\tilde{F}_T$ and θ are their values at the mode of the distributions given by the densities in (7) and (8). The approximations for these densities were obtained by applying the Gibbs sampling multi-move procedure to the model represented by equations (5) and (6). There are three stages in the procedure, described in detail in Appendix C:

I. Choose $\theta^{(0)}$ the set of initial values for the θ parameters of the model;

II. Conditional on $\theta^{(s)}$ and on the data in $\tilde{Z}_T$, obtain a draw $\tilde{F}_T^{(s)}$ from the conditional distribution $p(\tilde{F}_T^{(s)}/\tilde{Z}_T, \theta^{(s)})$;

III. Conditional on the data in $\tilde{Z}_T$ and the $\tilde{Z}_T^{(s)}$ drawn in the previous stage, a new draw of the set of parameters $\theta^{(s+1)}$ is obtained from the conditional distribution $p(\theta/\tilde{Z}_T, \tilde{F}_T^{(s)})$.

The Gibbs sampling adopted in the draw of Ψ and of H, from the distribution $p((\Psi, H)/\tilde{Z}_T, \tilde{F}_T^{(s)})$, was developed by Waggoner and Zha (2002 and 2007) and is described in detail in Appendix C. The same Gibbs sampling enables draws of the impulse-response functions that satisfy the sign restrictions. Considering $\tilde{F}_T^{(s)}$ and $\tilde{Z}_T$ as the data, equation (6)', and a Bayesian procedure with priors similar to that of Litterman (1986) the algorithm allows for a draw "s" of Ψ and H, $\Psi^{(s)}$, and $H^{(s)}$.

Waggoner and Zha use 5 hyperparameters $\lambda_0, \lambda_1, \lambda_2, \lambda_3$, and λ_4 for the priors and in our article their values were set at 0.5, 0, 0.25, 1, and 100. Since we haven't used dummy observations, the parameter λ_2 was set to zero.

3.1 Obtaining a draw of the impulse-response function that satisfies the sign restrictions:

(a) For each simulation "s" described previously, draw a matrix $\tilde{W}$ from an independent normal standard distribution of dimension r x r (r = number of endogenous variables of the BVAR) and let $\tilde{W} = \tilde{Q}\tilde{R}$ be a decomposition QR of $\tilde{W}$ with the diagonal of $\tilde{R}$ normalized to be positive.

(b) Let $H^{(s)} = H^{(s)}\tilde{Q}$. Compute the impulse-response function $\mathrm{IRF}^{(\mathrm{s})}$ given $\mathrm{H}^{(\mathrm{s})}$ and $\Psi^{(s)}$.

(c) If the $\mathrm{IRF}^{(\mathrm{s})}$ satisfies the sign restrictions it is kept, otherwise it is rejected.

After a large number of simulations all the simulations of the impulse-response functions that satisfy the sign restriction are kept.

3.2 The identification of the shocks

The shocks are identified using sign restrictions. As in Almeida, Alves, and Lima (2012), the sign restrictions come from the Mundell Fleming Dynamic Stochastic model presented in Lima, Maka, and Alves (2009). The sign restrictions used in the identification of the two types of shocks are presented in Table 1.

Table 1
Sign restrictions imposed in the identification of the FAVAR model (responses from 1 to 5 steps ahead were restricted).

Type of shock	Response of the variables				
	Selic Rate	IPCA	Production	M1	Exchange rate
Monetary policy	≥ 0	≤ 0	≤ 0	≤ 0	≤ 0
Exchange rate	≥ 0	≥ 0	≥ 0	≥ 0	≥ 0

Source: Elaborated by the authors.

3.3 Obtaining a draw of the impulse-response function for the specific and common shocks.

Consider equation (1) described previously and presented again below:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \tag{9}$$

Let $C_{i,t}^{(s)} = \Lambda_i^{(s),f} F_i^{(s)} + \Lambda_i^{(s),y} Y_t$ be the value for the first portion of equation i, belonging to the set of equations (1), in simulation "s" of the model in period t. Let $e_{i,t}^{(s)}$ be the value of the residual of equation i in simulation "s" in period t.

For each simulation "s" and for each equation i an autoregression is estimated using the values of $C_{i,t}^{(s)}$, $t = 1, ..., T$, and the impulse-response function is obtained from this autoregression using a shock equal to one standard deviation of the residual. This is the impulse-response function of the common shock in simulation "s".

With the impulse-response function computed and the common (which includes the factors) and specific shocks identified, it is possible to calculate the variance decomposition of the prediction errors.

4. Signs of the responses to monetary and exchange rate shocks

This section presents the signs of the impulse-response functions of the 343 series of subitems (including the "others" category), in the first 12 months, to one standard deviation shocks in the monetary policy and exchange rate variables, in contractionary direction for the monetary shock and depreciation direction for the exchange rate shock.[5]

The following procedure was adopted when computing the significance of the impulse-response functions:

i. For each subitem k_1 = 1,..., 343, j = 1,..., 1000 responses to the shocks in question, in 12 months, are calculated;

[5] The "others" category was merely added to complete the 100% weighting of the IPCA. In this and in the next sections, its results will not be commented on because they concern a residual of discontinuous series, without economic significance.

ii. For each subitem, considering the distribution formed by the 1000 responses, the confidence intervals (quantiles: 0.16; 0.5, and 0.84) were obtained for each one of the 12 months;

iii. For each subitem, it is verified whether in any of the 12 months there were significant responses: with the expected sign, in the opposite direction, in both directions (in different months), or whether in all the months the responses are non-significant;

iv. The subitems in each one of the four situations are counted and the results are reported at different levels of aggregation, considering the weight of the subitem in the IPCA or not.

Table 2 presents the main results concerning the signs in the first 12 months of the responses to monetary and exchange rate shocks. For each shock, the first line shows the count of subitems in the four situations, while the second line exhibits the weighting in the IPCA corresponding to these subitems for December of 2011. The negative column shows the products in which for any of the 12 months significant responses occur only in the direction of a reduction in the rate of price variation, which is the response expected by the theory in the case of the contractionary monetary shock and opposite to expected in the case of the exchange rate depreciation shock; the "non-signif." column shows the series in which the response is not significantly different to zero over the 12 months; the subitems for which all the significant responses are an increase in the rate of price variation are in the "positive" column, which is the response expected by the theory in the case of the exchange rate shock and counter-intuitive for the monetary shock; and finally, the "pos./neg." column indicates that over the period significant responses are presented in both directions.

For both shocks, the quantity of subitems in which there are significant responses with the opposite sign to the one expected by the theory is small, the so-called price puzzle[6]. Among the responses to the monetary shock, this occurs in only one subitem, with a weight of 0.1% in the IPCA. For the exchange rate shock, there

[6] The same occurs with specifications that use two or three factors. See results in tables B and C in the appendix.

Table 2
Analysis of the sign of the responses in the first 12 months to monetary and exchange rate shocks for the set of IPCA subitems.

		Total	Signs observed in the responses			
			Negative	Non-signif.	Positive	Pos./Neg.
Monetary shock	n. of subitems	343	177	165	1	0
	weight in the IPCA	100%	50,0%	49,9%	0,1%	0,0%
Exchange rate shock	n. of subitems	343	11	203	124	5
	weight in the IPCA	100%	0,8%	57,1%	40,4%	1,8%

Source: Elaborated by the authors.

are 16 subitems, which add up to 2.5% of the weight of the IPCA, but which is nonetheless quite a low number.

Regarding the significance, half of the weight of the IPCA has responses to monetary shocks in the direction expected by the theory, while in the other half the responses are not significant in the 12 months. In the responses to the exchange rate shock, the weight of the subitems that follow in the expected direction is a little lower and equal to 40%, and the weight of those with non-significant responses adds up to 57%.

In Table 3, the composition of the responses to the monetary and exchange rate shocks among the four situations is presented for the two hierarchical levels, already introduced in the second section of the text. The "IPCA weight" column shows the weight of each category in the IPCA in December of 2011. For each shock, the values in the columns, which add up to 100% on the line, indicate the proportion of subitems of the category whose response signs are in the direction indicated, aggregated using the ratio of the weight of each subitem in relation to the weight of the category.

For the monetary shock, the first line of Table 3, already presented in Table 2, indicates that in the IPCA as a whole there is an equal division of the weighting between the subitems that have significant responses in the expected direction and those without significant responses. In the first level disaggregation, the greatest deviation from this proportion occurs in food and drink, a group in which more than

60% of the weighting presents negative responses, while in the other three groups a little less than half of the weighting has negative responses.

At the second level there is greater discrepancy, including within each group. For food and drink, the tradables respond more and the non-tradables less. In the industrial products, the semi-durables are within the mean of the IPCA, the non-durables have more negative responses, and the durables have more non-significant responses. In the services the heterogeneity is even greater: there are three categories with a high proportion of negative responses (various, housing, and food outside the home, the latter with 100%) and three with a high percentage of non-significant responses in all the months (education, with 100%, personal services, and the leisure, tourism, and communication category). For the monitored items, almost 100% of the energy subitems do not present significant responses, public transport has a high proportion of negative responses, and the other monitored items are closer to the mean for the IPCA.

For the exchange rate shock, on the first line it is observed that 40% of the IPCA weighting only has significant price variation responses in the positive direction, which is expected by the theory in the case of an exchange rate depreciation shock. In the first level disaggregation, the closest situation to the IPCA set is that of the industrial products. Both in the foods and drink and in the services, only 22% of the weighting of the group has significant responses only in the direction expected. Food and drink also stand out due to a reasonable proportion of responses in the opposite direction to expected, which occur in 14% of its weighting. In the monitored prices the opposite takes place, and there are a high proportion of subitems with responses only in the direction expected, corresponding to 66% of the weighting of the group.

For the second level, as in the case of the monetary shock there are discrepancies within the groups. In food and drink, the proportion of expected responses is below the mean for the IPCA in the two categories, principally the non-tradables, with only 5%, and both present a percentage of 12% of subitems with significant responses in the two directions. In the industrial products, the difference with the IPCA mean in the proportion of positive responses is around 10% less for the durables and 10% more for the semi-durables and non-durables. For the services, in half of the categories (education, personal services, and various) all or almost all the responses

Table 3
Analysis of the sign of the responses in the first 12 months to a contractionary monetary shock and to an exchange rate devaluation shock
(% of the weight of the category)

Grouping	IPCA Weight	Monetary shock				Exchange rate shock			
		Neg.	N.S.	Pos.	P./N.	Pos.	N.S.	Neg.	P./N.
Total subitems (IPCA)	100,0	50,0	49,9	0,1	0,0	40,4	57,1	0,8	1,8
First level disaggregation									
1. Foodanddrink	15,0	62,4	37,6	0,0	0,0	22,3	63,9	2,1	11,7
2. Industrialproducts	21,9	47,8	52,0	0,3	0,0	43,3	55,4	1,3	0,0
3. Services	33,6	49,0	51,0	0,0	0,0	21,9	77,7	0,4	0,0
4. Monitored	27,2	41,8	58,2	0,0	0,0	66,1	33,9	0,0	0,0
Others	2,3	100,0	0,0	0,0	0,0	100,0	0,0	0,0	0,0
Second order disaggregation									
Foodanddrink									
11. Tradables	12,5	66,0	34,0	0,0	0,0	25,6	60,2	2,4	11,7
12. Non-Tradables	2,4	43,6	56,4	0,0	0,0	5,2	82,4	0,7	11,7
Industrial products									
21. Durables	7,1	31,3	68,7	0,0	0,0	27,9	69,9	2,2	0,0
22. Semi-durables	9,2	49,9	50,1	0,0	0,0	52,2	47,8	0,0	0,0
23. Non-durables	5,5	64,2	34,8	1,0	0,0	48,2	49,5	2,3	0,0
Services									
31. Foodoutsidethehome	8,4	100,0	0,0	0,0	0,0	35,3	64,7	0,0	0,0
32. Education	6,0	0,0	100,0	0,0	0,0	0,0	100,0	0,0	0,0
33. Housing (rent, condominium)	5,0	59,3	40,7	0,0	0,0	40,7	59,3	0,0	0,0
34. Personal services	5,9	18,0	82,0	0,0	0,0	0,0	100,0	0,0	0,0
35. Leisure, tourism, andcommunication	4,2	21,8	78,2	0,0	0,0	51,8	48,2	0,0	0,0
36. Various (maint., transp., health)	4,1	76,3	23,7	0,0	0,0	4,4	92,1	3,5	0,0
Monitored									
41. Energy(combustibles and electricity)	8,8	0,9	99,1	0,0	0,0	2,0	98,0	0,0	0,0
42. Health plans and medication	6,2	43,5	56,5	0,0	0,0	100,0	0,0	0,0	0,0
43. Publictransport	6,1	88,4	11,6	0,0	0,0	92,8	7,2	0,0	0,0
44. Various (telephone, fees, and games)	6,1	49,5	50,5	0,0	0,0	92,9	7,1	0,0	0,0

Source: Elaborated by the authors. Note: “Neg.” = negative, “N.S.” = non-significant, “Pos.” = positive, and “P./N.” = positive and negative.

are non-significant, while in the rest the proportion of positive responses is closer to the mean for the IPCA. In the group of monitored items, the energy category has almost 100% non-significant responses and clearly contrasts with the others, in which all or almost all the subitems have a positive response in some of the months.

5. Magnitude of the responses to shocks

The responses accumulated in 12 months to monetary and exchange rate shocks of one standard deviation, for the IPCA and at the two levels of disaggregation, are presented in this section. For the comparison of the impacts on the different categories of prices, the mean of the effects on the subitems that compose each category was calculated, as well as the impacts on the aggregated series themselves, with confidence intervals.

The procedures used when computing the weighted mean per category of the impact of each shock on the subitems was:

i. For each subitem $k_1 = 1,..., 343$, $j = 1,..., 1000$ responses accumulated in 12 months are calculated;

ii. Using the weight of each subitem k_1, corresponding to the weighting in the IPCA in Dec/2011, each one of the 1000 responses was aggregated by hierarchical level;

iii. For each level of aggregation, considering the distribution formed by the 1000 responses, the confidence intervals were obtained (quantiles: 0.16; 0.5, and 0.84).

With the aggregated series of the categories, or specific rates of inflation, the procedure was:

i. For each category $k_2= 344,..., 363$ (20 series: 4 first level, 15 second level, and IPCA), $j = 1,..., 1000$ responses accumulated in 12 months are calculated;

ii. Considering the distribution formed by the 1000 responses, in each category the confidence intervals were obtained (quantiles: 0.16, 0.5, and 0.84).

The results for both shocks are exhibited in Table 4. Both for the subitem means and for the aggregated series of the category, the extremes and the median of the confidence interval are shown.

Regarding the monetary shock, it is immediately noted that in each category the impacts are usually greater in the aggregated series than in the subitem means, which indicates the existence of correlations that enhance the effects of the shock. In addition, as would be expected, the relative effectiveness of the monetary policy between the categories is similar whether we take the means or the aggregated series.

For the IPCA, in the subitem mean a monetary contraction of one standard deviation produces an average reduction effect of 0.15 percentage points in the rate

Table 4
Responses accumulated in 12 months to monetary and exchange rate shocks, by category – weighted mean of the subitems and aggregated series (percentage points)

Grouping	Monetary shock						Exchange rate shock					
	Mean subitems			Aggregated series			Mean subitems			Aggregated series		
	inf.	md.	sup.	inf.	md.	sup.	inf.	md.	sup.	inf.	md.	sup.
IPCA	-0,26	-0,15	-0,04	-0,90	-0,50	-0,10	0,10	0,25	0,41	0,10	0,58	1,19
First level disaggregation												
1. Foodanddrink	-0,37	-0,23	-0,08	-0,83	-0,48	-0,14	0,00	0,17	0,37	-0,14	0,27	0,73
2. Industrialproducts	-0,27	-0,16	-0,05	-0,71	-0,38	-0,06	0,10	0,23	0,38	0,07	0,41	0,86
3. Services	-0,24	-0,13	-0,02	-0,69	-0,33	0,00	0,07	0,21	0,38	0,18	0,59	1,10
4. Monitored	-0,21	-0,08	0,04	-0,50	-0,19	0,12	0,11	0,29	0,47	0,10	0,46	0,87
Others	-0,82	-0,43	-0,07	-0,82	-0,43	-0,07	0,27	0,72	1,24	0,27	0,72	1,24
Second order disaggregation												
Food and drink												
11. Tradables	-0,40	-0,25	-0,09	-0,79	-0,45	-0,12	0,00	0,19	0,41	-0,15	0,27	0,75
12. Non-Tradables	-0,22	-0,14	-0,06	-0,63	-0,36	-0,09	-0,01	0,07	0,17	-0,14	0,15	0,47
Industrial products												
21. Durables	-0,27	-0,15	-0,03	-0,53	-0,24	0,02	0,05	0,18	0,33	-0,02	0,28	0,60
22. Semi-durables	-0,26	-0,15	-0,05	-0,76	-0,37	-0,07	0,12	0,24	0,37	0,28	0,65	1,10
23. Non-durables	-0,36	-0,19	-0,03	-0,62	-0,32	-0,03	0,06	0,26	0,50	-0,03	0,31	0,72
Services												
31. Food outside the home	-0,61	-0,37	-0,12	-0,83	-0,44	-0,10	0,12	0,41	0,81	0,13	0,57	1,10
32. Education	-0,18	-0,04	0,08	-0,29	-0,06	0,17	-0,01	0,13	0,30	-0,06	0,17	0,46
33. Housing (rent, condominium)	-0,26	-0,06	0,14	-0,42	-0,14	0,13	-0,01	0,23	0,50	0,00	0,31	0,66
34. Personal services	-0,15	-0,02	0,11	-0,18	-0,01	0,19	-0,07	0,06	0,23	-0,11	0,11	0,35
35. Leisure, tourism, and communication	-0,14	-0,05	0,04	-0,48	-0,18	0,07	0,08	0,18	0,30	0,06	0,38	0,75
36. Various (maint., transp., health)	-0,22	-0,12	-0,03	-0,44	-0,17	0,07	-0,02	0,10	0,24	-0,03	0,24	0,53
Monitored												
41. Energy (combustibles and electricity)	-0,19	-0,05	0,10	-0,31	-0,06	0,16	-0,07	0,09	0,28	-0,15	0,10	0,38
42. Health plans and medication	-0,31	-0,13	0,03	-0,47	-0,20	0,06	0,26	0,44	0,67	0,23	0,55	0,92
43. Public transport	-0,35	-0,11	0,14	-0,44	-0,15	0,19	0,06	0,36	0,68	0,03	0,41	0,87
44. Various (telephone, fees, and games)	-0,22	-0,06	0,10	-0,41	-0,11	0,13	0,15	0,34	0,53	0,22	0,51	0,89

Source: Elaborated by the authors.

of price variation, while on the rate of inflation the effect is a half percentage point reduction. In both cases the effect is significantly different to zero, but for the subitem mean the spread is small, while in the aggregate the reduction in inflation after 12 months fluctuates between one decimal to almost one percentage point. In the first level disaggregation, the food and drink group is generally more affected by the monetary policy, industrial products and services are similar to the IPCA at the subitem mean, and the monitored group is the least affected, with non-significant responses both for the aggregated series and for the subitem mean.

In the second level disaggregation, there are two categories with mean response considerably higher than that observed for the IPCA set: food services outside the home and tradable foods, which are also the second level series that respond most to the monetary shock. The responses of the monitored and services categories, with the exception of foods outside the home, are non-significant for all the aggregated series and almost all the means. In the responses of the industrial products, the subitem means are close to the IPCA set, and in the aggregated series the response is non-significant for the durables category. The non-tradables foods have a lower response than the tradable ones, but it is significant and close to the IPCA at the subitem mean.

Regarding the effect of the exchange rate depreciation shock, also in this case the effects are typically greater in the aggregate of the series compared with the subitem mean and the comparative analysis is generally similar for the means or aggregated series, with some discrepancies. The response of the IPCA is significant both in the subitem mean and for the aggregated inflation, with medians of 0.25 and 0.58 percentage points, respectively. At the extremes of the confidence interval, an exchange rate shock of one standard deviation causes a rise in the rate of price variation of between 0.10 and 0.41 points for the subitem mean, while for the aggregate inflation the effect can be much greater, at between 0.10 and 1.19 points.

For the first level disaggregation, there is some difference in the comparisons between groups depending on whether the mean or the aggregate is taken. For the subitem mean, the groups of industrial products and services have values close to the IPCA, as in the case of the monetary shock, but there is an inversion in the other two groups: the monitored items have a greater median response and the food and drinks have a smaller one, which even becomes non-significant. For the aggregated series, the greatest median response occurs in the services group, followed by the monitored items and industrial products, while the foods and drinks have the lowest and it is not significantly different to zero.

Considering the second level disaggregation, there is also some difference in order between the groups depending on whether the means or aggregated series are observed, but the general picture is the same. The categories that in both the metrics have significant responses and that also mostly have a mean that is considerably above the IPCA, are the semi-durable industrial products, the food services outside the home, and "leisure, tourism, and communication", as well as the monitored items, except the energy category. The categories with non-significant responses in the two metrics are all the food and drinks, the services except the two categories already mentioned, and the monitored energy items. The industrial products present a greater discrepancy between the two measures: in the subitem mean the three categories are significant and close to the IPCA mean, and in the aggregated series the semi-durables have the greatest response among all the second level categories, but the other two are non-significant.

5.1 Summary of sectoral results: magnitude and sign of the responses to monetary and exchange rate shocks

The main sectoral results presented in this section and in the previous one are summarized below.

The food and drinks group is the one that responds most to monetary shocks. The tradable foods, which account for more than 80% of the group's weighting, determine this behavior. The non-tradable foods also respond to monetary policy, but with an intensity that is within the IPCA mean. On the other hand, for the exchange rate shock this is the group with the lowest median response value in the aggregate, as well as its responses not being significantly different to zero in the aggregate, in the two second level categories and in the subitem mean. Nonetheless, it cannot be affirmed that the exchange rate does not affect this group, but rather that there is great uncertainty in the responses, since the median response is higher than zero and the confidence intervals are long. That is, food exports are an important determinant of the Brazilian balance of trade, and consequently the exchange rate, but the effects of the exchange rate on variations in the domestic prices of these goods are very variable.

The effects of monetary and exchange rate shocks on the subitems of industrial products are within the IPCA mean. The same pattern is observed when separating the subitems into the three categories according to durability. Taking the aggregated series, for the whole group the responses to both shocks are intermediate in comparison with the other groups. For the three second level categories there are differences, with higher responses of the semi-durables for the monetary shock and principally for the exchange rate shock, non-significant responses for both shocks in the durables, and non-significant responses for the exchange rate shock in the non-durables. That is, for this group the analysis changes in the second level of disaggregation depending on whether the aggregated series or subitem means are taken, but even for the non-significant responses there is a clear predominance of positive values in the confidence interval. In general, it can be said that the responses of the industrial prices to monetary and exchange rate shocks follow the mean for the IPCA.

In services, the food outside the home category differs from the rest, since it presents high responses both to monetary and exchange rate shocks, even compared with the other IPCA groups. Regarding the monetary shock, the responses are non-significant in all the other categories of the services for the aggregated series and in almost all the subitem means. For the exchange rate shock, among the other categories only that of leisure, communication, and tourism has significant responses, in the aggregate and in the subitem mean. Nonetheless, in the combined group of services the responses are not small: for the mean of the subitems, they are close to the IPCA in both shocks; for the aggregated series, the response to the monetary shock is intermediate and the response to the exchange rate shock is high.

The monitored prices group is the one that responds least to monetary policy, but it is considerably affected by the exchange rate. The responses are non-significant to the monetary shock, both for the subitem means and for the four series of intermediate categories and for the aggregate of the group. For the exchange rate shock, in the subitem mean or in the aggregated series, the responses are non-significant in the monitored energy items, but they are high in the other categories.

6. Variance decomposition

In this section, two variance decompositions of the prediction error in the first 12 months are presented. The first one, which refers to the VAR equation, shows the contributions of the monetary and exchange rate shocks for the variability part of the series that can be explained by the set of macroeconomic variables plus the common factor. In the second decomposition, the variance of each series is decomposed in one part attributed to the macroeconomic aspects (macro variables plus common factor) and another due to the component specific to the series. The decompositions were calculated with confidence intervals, using the following procedure:

i. For each series $k_3 = 1,..., 363$, $j = 1,..., 1000$ decompositions of the variance are calculated for the macroeconomic shocks (monetary, exchange rate, and "others") and 1000 for the specific and common shocks, in each of the first 12 months;

ii. For each series k_3 and decomposition j, the means of the contributions of each shock were calculated: in the 12 months, in the first six months, and in the last six months;

iii. For each series k_3, considering the distribution formed by the 1000 decompositions, the confidence intervals (quantiles: 0.16, 0.5, and 0.84) were calculated for the contribution of each shock, in the first six, last six, and mean for the 12 months;

iv. Tables 5 and 6 present the two decompositions of the variance, only for the 20 aggregated series, $k_2 = 344,..., 363$. The mean of the decompositions of the subitems, $k_1 = 1,..., 343$, will be commented on only for the IPCA total, in Table 7.

In Table 5, related to the decomposition of the variance of the VAR, the contributions attributed to the monetary and exchange rate shocks are presented, omitting the part attributed to the set of the other macroeconomic components. For the 20 aggregated series, the decomposition is shown for the mean for the 12 months, with confidence intervals. As a measure of the persistence and propagation of each shock, for the 20 series the ratio between the medians of the contribution in the last six months over those in the initial six months is also shown.

Regarding the mean of the decomposition in the 12 months, for the IPCA inflation the monetary shock and exchange rate shock together account for approximately 20% of the variability part of the series attributable to macroeconomic shocks, at the median. Considering the confidence interval, however, this contribution fluctuates between 5.8% and almost 50%. Comparing the two shocks, the exchange rate shock has the greatest contribution, of 13.4% at the median, which is approximately double the 6.8% corresponding to the monetary shock.

In the first level disaggregation, there is only a certain deviation from this proportion in the food and drinks, with greater proximity between the contributions of the two shocks. Even in the second level disaggregation, it is noted that there are

Table 5
Variance decomposition in the VAR equation – monetary and exchange rate shocks, mean for the 12 months (%), and ratio between the initial and final six months.

Grouping	12 months mean						Ratio (median) Final 6m/initial 6m	
	Monetary			Exchange rate			Monetary	Exchange rate
	inf.	md.	sup.	inf.	md.	sup.		
IPCA	2,4	6,8	15,9	3,4	13,4	33,4	1,46	1,02
First level disaggregation								
1. Food and drink	2,6	7,2	17,6	2,7	10,2	28,7	1,49	1,04
2. Industrial products	2,3	6,9	16,3	2,9	12,3	31,6	1,39	1,00
3. Services	2,3	6,2	13,8	3,3	13,8	35,3	1,54	1,19
4. Monitored	1,9	5,9	14,5	3,5	12,5	32,2	1,22	1,30
Others	2,0	6,0	14,2	5,2	17,0	36,1	1,36	1,03
Second order disaggregation								
Food and drink								
11. Tradables	2,4	7,2	17,1	3,0	10,2	29,1	1,48	1,09
12. Non-Tradables	3,0	8,7	22,8	3,1	11,4	30,0	1,28	1,01
Industrial products								
21. Durables	1,9	6,6	18,0	3,1	12,8	33,1	1,29	1,02
22. Semi-durables	1,8	5,9	15,1	4,6	15,6	35,4	1,26	1,07
23. Non-durables	2,3	6,5	15,8	2,6	9,8	27,3	1,32	1,08
Services								
31. Food outside the home	2,4	6,5	15,1	3,5	13,4	33,8	1,56	1,06
32. Education	2,0	6,9	18,7	2,6	9,1	21,9	1,21	1,33
33. Housing (rent, condominium)	2,1	6,9	18,2	3,0	11,2	26,1	1,25	1,37
34. Personal services	1,8	6,3	19,4	2,5	9,5	26,0	1,12	1,27
35. Leisure, tourism, and communicatio	2,1	6,1	14,3	2,6	12,4	31,8	1,41	1,20
36. Various (maint., transp., health)	2,0	6,5	16,8	2,2	8,9	26,3	1,37	1,24
Monitored								
41. Energy (combustibles and electricit	1,8	6,3	17,3	2,3	8,7	23,3	1,21	1,23
42. Health plans and medication	1,9	5,6	14,1	5,0	15,3	34,7	1,16	1,37
43. Public transport	2,1	6,3	16,2	2,7	10,8	28,8	1,31	1,26
44. Various (telephone, fees, and game)	1,4	5,1	13,6	5,1	16,6	37,4	1,08	1,37

Source: Elaborated by the authors.

few deviations from this proportion and that in none of the cases does the monetary shock contribute more than the exchange rate shock at the median. The participation of the monetary shock is between 6% and 7% in most of the categories, with the greatest deviations above in the two food categories and below in the various monitored and health items. For the contribution of the exchange rate, in relation to the IPCA series the greatest deviations above are in the various monitored and health items, as well as in the semi-durable industrial products, while the greatest deviations below are found in the monitored energy items and in the various, educational, and personal services.

Observing the ratio between the medians of the variance decomposition in the final six months over the initial six months, for the IPCA it can be said that the monetary shock has a slower propagation that the other macro shocks, with a 46%

greater contribution to the variance in the final six months. The exchange rate shock, on the other hand, does not differ in this aspect from the other macro shocks, with only a 2% greater contribution in the final six months. For the first level series of disaggregation, this same pattern is observed for food and drinks and industrial products. The services are a little more persistent than the IPCA in the monetary shock and considerably more persistent in the exchange rate shock. Only in the monitored items is there an inversion and the exchange rate shock propagates a little more slowly than the monetary one.

Among the second level categories, only in two is the lag of the effects of the monetary shock greater than that of the IPCA series: food outside the home and tradable foods. These are precisely the categories in which there is a greater impact from the monetary shock, as seen in Table 4. For the exchange rate shock, there is a greater lag of the effects of the exchange rate shock in the monitored categories, in which the effect of the exchange rate is generally strong, and in the services except food, over which the impact of the exchange rate is lower (Table 4). In food and industrial products the effect of the exchange rate is quicker.

The same results are presented for the variance decomposition between macroeconomic and specific shocks in Table 6. At the median, for the inflation series measured by the IPCA almost 70% of its variance of the prediction error in 12 months is determined by macroeconomic shocks, with extremes of 58% and 75% in the confidence interval, while the specific shocks determine between 25% and 42% of the variance. At the first level disaggregation, more than half of the variance of the series of the four groups is caused by the macro shocks, but in food and drinks and industrial products this contribution is only a little greater than 50% at the median, while in the monitored items it is greater than 60% and in services it exceeds 70%.

For the second level disaggregation, considerable heterogeneity is observed even within the groups. In the tradable foods, macro and specific shocks have the same contribution, but in the non-tradables, the specific shocks predominate with 70% of the variance. In the industrial products, for durables and semi-durables more than 60% of the median variance is caused by macro shocks, a proportion that inverts in favor of specific shocks in the non-durables. Among the services categories, the macro shocks are more prominent for housing and food outside the home, determin-

Table 6
Variance decomposition between macroeconomic and specific shocks, mean for the 12 months (%), and ratio between the initial six and the final six months.

Grouping	12 months mean						Ratio (median) Final 6m/initial 6m	
	Macroeconomic			Specific Shocks			Macroeconomic	Specific Shocks
	inf.	md.	sup.	inf.	md.	sup.		
IPCA	58,0	68,4	75,0	25,0	31,6	42,0	1,14	0,75
First level disaggregation								
1. Foodanddrink	43,4	54,0	62,9	37,1	46,0	56,6	1,12	0,88
2. Industrialproducts	45,0	55,3	62,8	37,2	44,7	55,0	1,25	0,76
3. Services	61,8	72,3	78,7	21,3	27,7	38,2	1,25	0,55
4. Monitored	52,0	62,4	70,8	29,2	37,6	48,0	1,30	0,65
Others	52,7	62,4	70,4	29,6	37,6	47,3	1,26	0,68
Second order disaggregation								
Food and drink								
11. Tradables	39,4	50,1	58,7	41,3	49,9	60,6	1,12	0,89
12. Non-Tradables	22,2	30,5	39,1	60,9	69,5	77,8	1,21	0,92
Industrial products								
21. Durables	53,4	63,4	71,2	28,8	36,6	46,6	1,27	0,66
22. Semi-durables	51,2	61,1	70,1	29,9	38,9	48,8	1,32	0,64
23. Non-durables	27,7	36,8	45,9	54,1	63,2	72,3	1,25	0,88
Services								
31. Food outside the home	49,4	58,9	66,6	33,4	41,1	50,6	1,21	0,76
32. Education	9,7	17,2	27,0	73,0	82,8	90,3	1,39	0,93
33. Housing (rent, condominium)	69,3	77,9	84,2	15,8	22,1	30,7	1,22	0,49
34. Personal services	11,1	20,5	31,9	68,1	79,5	88,9	1,58	0,89
35. Leisure, tourism, and communicatio	27,8	40,7	51,7	48,3	59,3	72,2	1,50	0,76
36. Various (maint., transp., health)	25,1	37,0	49,2	50,8	63,0	74,9	1,56	0,77
Monitored								
41. Energy (combustibles and electricit)	30,9	41,5	50,5	49,5	58,5	69,1	1,41	0,79
42. Health plans and medication	33,5	45,1	55,2	44,8	54,9	66,5	1,40	0,76
43. Public transport	42,2	54,1	63,3	36,7	45,9	57,8	1,38	0,68
44. Various (telephone, fees, and game)	55,0	66,1	76,2	23,8	33,9	45,0	1,36	0,54

Source: Elaborated by the authors.

ing between 59% and 78% of the variance at the median. But for the other categories the specific shock is more important, accounting for 59% and 83%. Finally, in the monitored items the energy and health categories are most influenced by specific shocks, while public transport and various monitored items experience more impact from macroeconomic shocks.

Regarding the ratio between the medians of the contributions in the final six months over the initial six months, it is noted that for all the series the macroeconomic shocks are more persistent than the specific shocks, with ratios higher than the unit in the first case and lower in the second case. Nonetheless, for each one of the two shocks the categories can be compared regarding relative persistence. In the first level disaggregation, the monitored items have the greatest lag in the effects of the macro shocks and the food and drinks have the lowest one, while for

Table 7
Variance decomposition between the macroeconomic and specific shocks, mean for the 12 months (%), and ratio between the initial six months and final six, IPCA, and mean for the subitems

IPCA	12 months mean		Ratio fin 6m/ini 6m	
	Macro	Specific	Macro	Specific
Aggregated series	68	32	1,14	0,75
Mean of the weighted subitems	38	62	1,36	0,83
Mean of the subitems without weighting	31	69	1,37	0,87

Source: Elaborated by the authors.

the specific shocks the lag is relatively greater in the food and drinks and lower in the services. At the second level, the greatest relative slowness in the propagation of the macro shocks is among the services and monitored items, while the category with the quickest propagation is that of the tradable foods. For the specific shock, the propagation is relatively slower in the two food categories, in the non-durable industrials, and in the personal and educational services, but it is relatively quicker in the housing services, the various monitored items, and the durable and semi-durable industrial goods.

Finally, in Table 7, the same information presented in Table 6 (at the median) is shown for the aggregated IPCA inflation series and for the mean of the price variation rates for all the subitems, with and without weighting.

From the 12 month mean of the variance decomposition, the same result obtained by Boivin, Giannoni, and Mihov (2009) is observed, where the aggregate inflation responds more to macroeconomic shocks, but most of the variance of the disaggregated price series is explained by sector specific shocks. For the mean of the subitems without weighting, almost 70% of the variance is caused by the specific shocks, a proportion that inverts in the aggregated index. That is, aggregation cancels out sectoral effects that point in different directions, while it enhances the impacts of macroeconomic shocks.

The measure of persistence given by the ratio of the contributions in the final six months over the initial six indicates that the macro shocks are more persistent than the specific ones not only for the aggregated series, as seen in Table 6, but also for the mean of the subitems series, which is indicated by the ratio greater than one for macro shocks and lower than one for specific shocks. It is also noted that since in the two columns the ratio is greater for the subitem mean than for the aggregate, both shocks have more persistence over the mean for the series than over the aggregated series, at least at the median.

7. Conclusion

Using a FAVAR model, the effects of monetary and exchange rate shocks on inflation measured by the IPCA were calculated at the subitem level, with series from 1999 to 2011. The results found are coherent with the literature and shed light on important aspects of the effects of monetary policy and the behavior of disaggregated prices in Brazil.

As in the study from Boivin, Giannoni, and Mihov (2009) for the United States, it was observed that adopting the FAVAR methodology drastically reduces the occurrence of responses with the opposite sign to expected, that is, the so-called price puzzle. This result contrasts with the one presented by Guillén and Garcia (2011), who estimate for each subitem of the IPCA, from 1999 to 2006, a system composed of a VAR with macroeconomic variables and the equation of the price of the subitem. Ignoring confidence intervals of the responses, the authors obtain responses with the price puzzle for 58% of the number of subitems in the monetary shocks and 40% in the exchange rate shocks. In this study, considering the significance of the responses, different signs from expected were obtained only in 0.3% of the number of subitems for monetary shocks and 4.7% for exchange rate shocks, which respectively equate to 0.1% and 2.5% of the weighting of the IPCA in December of 2011. It is worth remembering that although the identification procedure in this study uses sign restriction, this is imposed only on the responses of the macroeconomic variables to the shocks, and not on the responses of the subitem prices, therefore the low occurrence of price puzzles in the individual prices is not derived from the identification using sign restriction.

From the analysis of the variance decomposition of the prediction error in 12 months, it was verified that the series of subitems are on average more affected by the specific shocks than by the macroeconomic shocks, in a proportion of approximately 70%/30%. The opposite proportion was observed for the inflation series measured by the IPCA, with almost 70% of the variance determined by the contribution from the macroeconomic shocks. This evidence is similar to that obtained by Boivin, Giannoni, and Mihov (2009), who using other procedures obtain an 85% participation of specific shocks in the mean for the variance decomposition in the disaggregated prices. Also from the variance decomposition, it was found that the effects of the specific shocks are relatively more concentrated in the first six months, while the contributions from the macro shocks are relatively more intense in the final six months. Therefore, as in Boivin, Giannoni, and Mihov (2009), it is observed that macro shocks are more persistent than sector-specific ones.

Together, these results support the hypothesis that disaggregated prices respond quickly and more intensely to specific shocks, but slowly and in lower magnitude to macro shocks, and that the annulment of specific effects in opposite directions in the aggregation means that aggregated inflation fluctuates less. This hypothesis conciliates the evidence regarding the slow response of macroeconomic variables to shocks with the high variability of prices observed in microdata by Bils and Klenow (2004) and others, which is also observed for Brazil by Gouvea (2007), Lopes (2008), and Barros and Matos (2009).

The study also enabled the sectoral heterogeneity of the responses to monetary and exchange rate, macroeconomic, and sector-specific shocks to be captured. In general, for the monetary shock, the responses with the greatest magnitude are for the food and drinks group and for food services outside the home, in industrial products they are intermediate, and in the monitored items and other services they are typically non-significant. For the exchange rate shock, the most expressive responses are those of the monitored items (except energy) and of the food services outside the home, they are intermediate in the industrialized products and "leisure, tourism, and communication" services, and lowest or non-significant in the food and drinks group, in the other services, and in the monitored energy items. Regarding persistence, taking the period of one year after a shock, in all the sectoral series the propagation of the exchange rate shock is greater in the initial six months, while the propagation of the monetary shock is greater in the final six months.

In the variance decomposition between macroeconomic and specific shocks, taking the period of one year after the shock, for the IPCA aggregate and in the four main groups there is a preponderance of macroeconomic shocks, especially in the services and monitored items. At the second level of aggregation, this result is not maintained, since there is a preponderance of specific shocks in various categories of all four groups. It is noted that the greater the disaggregation, the role of the specific shocks grows, which is reasonable if we consider that a good amount of these shocks cancel each other out in the aggregation. Regarding persistence, in all the cases the effects of specific shocks are greater in the initial six months, while the effects of macroeconomic shocks are greater in the final six months.

The set of results obtained elicits additional reflections to be developed in subsequent studies, given the large volume of information gathered. Variations in the econometric methodology and in the organization of the data could reveal empirical evidence that is complementary to the evidence here. Other relevant implications for conducting economic policy in Brazil can be more deeply examined in more detailed comparisons with elements from the theoretical literature, such as in the question of the optimal monetary policy reaction to changes in relative prices (Aoki, 2001; Wolman, 2011), in which the recommendations are conditional on the differences between groups of prices with regard to the reaction to monetary shocks.

References

Ahn, S. C. and Horenstein, A. R. (2013). Eigenvalue ratio test for the number of factors. *Econometrica*, v. 81, n. 3, p. 1203-1227.

Almeida, G., Alves, P. and Lima, E. *Impactos dos choques na política monetária e no câmbio no Brasil*: um modelo de autorregressão vetorial estrutural aumentada por fatores dinâmicos. Brasília: Ipea, 2012. (Discussion text, n. 1711).

Aoki, K. (2001). Optimal monetary policy responses to relative-price changes. *Journal of Monetary Economics*, v. 48, n. 1, p. 55-80.

Balke, N. and Wynne, M. (2007). The Relative Price Effects of Monetary Shocks. *Journal of Macroeconomics*, v. 29, n. 1, p. 19-36.

Barros, R. and Matos, S. (2009). Comportamento dos preços no Brasil: evidências utilizando microdados de preços ao consumidor. *Pesquisa e Planejamento Econômico*, v. 39, n. 3.

Bernanke, B., Boivin, J. and Eliasz, P. (2005). Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach. *The Quarterly Journal of Economics*, v. 120, n. 1, p. 387-422.

Bils, M. and Klenow, P. (2004). Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy*, v. 112, n. 5, p. 947-985.

Boivin, J., Giannoni, M. and Mihov, I. (2009). Sticky prices and monetary policy: evidence from disaggregated U.S. data. *American Economic Review*, v. 99, n. 1, p. 350-384.

Carter, C.K. and Kohn, P. (1994). On Gibbs sampling for state space models. Biometrika, v.81, n. 3, p.541-553.

Gamerman, D. (1996). *Simulação estocástica via cadeias de Markov.* Caxambú: Associação Brasileira de Estatística. 196p.

Goodfriend, M. and King, R. G. (1997). "The New Neoclassical Synthesis and the Role of Monetary Policy". In: *NBER Macroeconomics Annual 1997*, edited by Ben S. Bernanke and Julio J. Rotemberg. Cambridge, Mass.: MIT Press.

Gouvea, S. *Nominal price rigidity in Brazil*: a micro evidence approach. Brasília: Banco Central do Brasil, 2007. (Working Paper Series, n. 143).

Guillén, D. and e Garcia, M. (2011). Dispersão na fixação de preços no Brasil. *Revista Brasileira de Economia*, v. 65, n. 1, p. 47-69.

Instituto Brasileiro de Geografia e Estatística - IBGE. *Tradutor das estruturas de ponderação*: POF 1996/ 2002. Rio de Janeiro: IBGE, 2006. Available from: http://servicodados.ibge.gov.br/Download/Download.ashx?u=ftp.ibge.gov.br/Precos_Indices_de_Precos_ao_Consumidor/IPCA/Atualizacao_das_Estruturas_POF2002-2003/Tradutor_Estruturas.zip.

Klenow, P. J. and Malin, B. A. *Microeconomic evidence on price-setting.* National Bureau of Economic Research, 2010. (Working Paper, n. 15826).

Lima, E., Maka, A. and Alves, P. (2011). Monetary Policy and Exchange Rate Shocks in Brazil: Sign Restrictions versus A New Hybrid Identification Approach. *Brazilian Review of Econometrics*, v. 31, n. 1.

Litterman, R. B. (1986). Forecasting with Bayesian vector autoregressions - five years of experience. *Journal of Business & Economic Statistics*, v. 4, n. 1, p. 25-38.

Lopes, L. T. (2008). *A rigidez nominal de preços na cidade de São Paulo*: evidências baseadas em microdados do índice de preços ao consumidor da FIPE. Dissertação de Mestrado em EconomiaMaster's in Economics Dissertation. Universidade de São Paulo.

Martinez, T. S. (2014). Preços relativos e política monetária no Brasil. In: SOUZA JÚNIOR, J. R. S. (Org). *Evolução recente das políticas monetária e cambial e do mercado de crédito no Brasil.* Rio de Janeiro: Ipea, p. 45-81.

Sims, C. A. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review*, v. 36, n. 5, p. 975-1000.

Stock, J. H. and Watson, M. W. Forecasting inflation. *Journal of Monetary Economics*, v. 44, n. 2, p. 293-335, 1999.

Waggoner, D. and Zha, T. (2003). A Gibb's Sampler for Structural VARs. *Journal of Economic Dynamics and Control*, v.28, n. 2, p. 349–366.

Wolman, A. L. (2011). The optimal rate of inflation with trending relative prices. *Journal of Money, Credit and Banking*, v. 43, n. 2-3, p. 355-384.

A. Appendix

Table A.1
Classification of the disaggregated categories of the IPCA

Free Prices		
Free goods		
1	**Food and drink**	
11	***Tradables***	1101002 Rice, 1102 Flow er, starch, and dough (except 1102023 Manioc flour), 1104 Sugar and derivatives, 1107 Meat, 1109 Industrialized meat and fish, 1110009/12/14 Chicken (varieties), 1111 Milk and derivatives, 1112 Bakery, 1113 Oils and fats, 114 Drinks and herbal teas, 1115 Canned food, 1116 Salt and seasonings
12	***Non-Tradables***	1101051/52/73 Beans (varieties), 1102023 Manioc flour, 1103 Root vegetables and legumes, 1105 Green vegetables, 1106 Fruit, 1108 Fish, 1110044 Hen egg
2	**Industrial products**	
21	**Durables**	3101 Furniture, 32. Electrodomestics (except 3201012 Liquidifier, 3201013 Fan, and 3201027 Lamp), 4301 Jew elry, 5102001 New automobile, 5102020 Used automobile, 5102053 Motorcycle, 6102 Glasses and lenses (except 6101003 Glasses w ithout lenses), 7201010 Musical instrument, 7201019 Bicycle, 7203001 Camera, 9101019 Telephone
22	**Semi-durables**	3102 Utensils and ornaments (except 3102035 Fresh flow ers), 3103 Bed, table, and bathtub, 3201012 Liquidifier, 3201013 Fan, 4 Clothing (except 4301 Jew elry), 5102009 Accessories , 5102010 Tire, 6102003 Glasses w ithout lenses, 6201005 Orthodontic appliance, 7201002 CD and DVD, 7201008 Laser disk, 7201023 Toys, 7201083 Sports material, 7203002 Disposable film, 8103014 Stationary
23	**Non-durables**	2103 Repairs, 2104 Cleaning articles, 2201003 Charcoal, 3102035 Fresh flow ers, 3201027 Lamp, 5102007 Lubricating oil, 5104002 Ethanol, 6301 Personal hygiene, 7201020 Food for animals, 7202041 Cigarette, 8102 Reading, 8103001 Notepad
Free services		
3	**Services**	
31	**Food outside the home**	12 Food outside the home, 1117 Readymade food (up to 2006)
32	**Education**	8101 Courses, 8104 Various courses (after 2006), 8101014 Various courses (up to 2006)
33	**Housing**	2101001 Residential rent, 2101002 Condominium, 2101012 Moving
34	**Personal Services**	7101 Personal services (except 7101034 Notary)
35	**35 Leisure, tourism, and communication**	5101010 Airfare, 7201001 Cinema, 7201003 Ticket for game, 7201006 Club, 7201052 DVD rental, 72010543 Nightclub and danceclub, 7201068 Motel, 7201090 Hotel, 7201095 Excursion, 7203003 Developing and copying, 8103002 Photocopy, 9101008 Cellphone, 9101010 Cable TV
36	**Various services (maintenance, transport, and health)**	33 Repairs and maintenance, 5101026 School transport, 5102005 Voluntary vehicle insurance, 6201002 Doctor, 6201003 Dentist, 6201008 Psychological treatment and physiotherapy, 6202 Laboratory and hospital services
Monitored or Managed Prices		
4	**Monitored**	
41	**Energy**	2201004 Gas cylinder, 2201005 Piped gas, 2202 Residential electrical energy, 5104001 Gasoline, 5104003 Diesel oil
42	**Health**	6101. Pharmaceutical products, 6203 Health plans
43	**Public Transport**	5101 Public transport (except 5101010 Airfares and 5101026 School transport)
44	**Various monitored items (telephone, fees, and games)**	2101004 Water rates, 5102004 Plaque and licence, 5102015 Tolls, 7101034 Notary, 7201063 Games of luck, 9101001 Mail, 9101002 Landline, and 9101003 Public telephone

Source: Adapted from Martinez (2014). Note: mentions at levels of disaggregation above the subitems mean that all the subitems of the item or subgroup form part of the category, considering only those with series without discontinuity according to the IBGE translator (2006).

Table A.2
Analysis of the sign of the responses in the first 12 months to monetary shocks for different numbers of factors.

N. of Factors	Monetary				
	neg	zero	pp	neg/pp	Total
1	177	165	1	0	343
2	224	111	7	1	343
3	88	253	2	0	343

Source: Elaborated by the authors.

Table A.3
Analysis of the sign of the responses in the first 12 months to exchange rate shocks for different numbers of factors

N. of Factors	Exchange rate				
	pos	zero	pp	neg/pp	Total
1	124	203	11	5	343
2	166	162	12	3	343
3	108	232	3	0	343

Source: Elaborated by the authors.

B. Appendix

The macroeconomic variables used in the study were: Selic interest rate, nominal exchange rate, IPCA, 180 day (pre DI) swap, M1, and industrial production index and a commodities price index. All these series were used in natural logarithm. The FMI commodities price index (combustibles and non-combustibles), in dollars, was deseasonalized using the X12-ARIMA.

For the Y_t vector the ADF test with constant and lag selection using the Schwartz criterion and Phillips-Perron (PP) presented results that indicate that the all variables are integrated in 1st order.

Table B.4
ADF test with constant and lag selection using the Schwartz criterion

Series	Prob.	Lag	Order of integration
EXCHRATE	0,4040	2	1
IPCA	0,4945	1	1
M1AJ	0,2712	0	1
OUTPINDAJ	0,5184	0	1
PCOM	0,7936	1	1
SELIC	0,1572	1	1
SWAP	0,5200	1	1

Source: Elaborated by the authors.

Table B.5
PP test series of the Yt vector

Series	Prob (level)	Prob (1st difference)
EXCHRATE	0,4539	0,0000
IPCA	0,3373	0,0000
M1AJ	0,3442	0,0000
OUTPINDAJ	0,4977	0,0000
PCOM	0,7586	0,0000
SELIC	0,3113	0,0103
SWAP	0,3478	0,0000

Source: Elaborated by the authors.

The Johansen cointegration test with constant indicated the presence of **3** and **1** cointegration relationships between the variables using the trace and maximum eigenvalue methods, respectively.

C. Appendix - Methodology

The FAVAR model in state-space, reduced and structural forms is described, in summarized form, by the following equations:

Measurement Equation:

$$Z_t = \Lambda G_t + \varepsilon_t \tag{C.1}$$

$$e_t \sim N(0, R)(R\ diagonal); \varepsilon_t = (e_t, 0), P = cov(\varepsilon_t).$$

Transition Equation (Reduced-Form VAR)

$$G_t = \Phi(L) G_{t-1} + DJ + \nu_t \tag{C.2}$$

$$\nu_t \sim N(0, \tau), e_t\ independent\ of\ \nu_t;$$

Transition Equation (Structural VAR)

$$HG_t = H\Phi(L) G_{t-1} + HDJ + H\nu_t; \tag{C.3}$$

H = lower triangular with positive numbers in the main diagonal; $u_t = H\nu_t$, with $\mathrm{cov}(\mathrm{u_t}) = \mathrm{I}$.

Where: $\Lambda = \begin{bmatrix} \Lambda^f & \Lambda^y \\ 0 & I \end{bmatrix}$; $Z_t = (X_t,\ Y_t)$; $G_t = (F_t,\ Y_t)$; Y_t is a vector with a small number of macroeconomic variables that are I(1) but cointegrate; F_t is a vector with a small number K of unobserved dynamic factors extracted from the disaggregated price series; X_t is a vector of time series with price variations of subitems and different aggregations of the IPCA; $\Phi(L)$ is a lag polynomial of order p; Λ^f and Λ^y are factor loadings matrices that contain a prior restrictions that allows for the identification of the parameters; D is a matix $(M + K) \times h$ of exogenous variable parameters; J is a vector $h \times l$ of exogenous variables; the matrix H is lower triangular with positive numbers in the main diagonal and is normalized so that $u_t = H\nu_t$ and $cov(u_t) = I$.

The FAVAR in reduced form is composed by equations C.1 and C.2. In the structural form it is composed by equations C.1 and C.3

Letting $H\Phi(L) = C_l L + ... + C_p L^p, \Psi = [C_1 ... C_p D]$, and $W_t = [G_{t-1} G_{t-2} ... G_{t-p} J]$ then equation (C.3) can be summarized in the following way:

$$G_t' H' = W_t' \Psi' + u_t' \tag{C.4}$$

Next we will explain the methodology used in the estimation of the FAVAR equations.

Let $\theta = (H, \Phi\ (L)\ ,\ D,\ \Lambda^f\ ,\ \Lambda^y\ ,\ R,\ \tau)$ and let $\tilde{X}_T = (X_1,\ X_2, ...,\ X_T$ be the history of X between period 1 and period T. The Gibbs sampling procedure adopted consists of the following steps:

Step 01: Choose a set of initial values for the parameters $\theta = (H, \Phi(L), D, \Lambda^f, \Lambda^y, R, \tau)$, called $\theta^{(0)}$;

Step 02: Conditioned on $\theta^{(0)}$ and on the data $\tilde{Z}_T = (\tilde{X}_T, \tilde{Y}_T)$, draw a value for $\tilde{G}_T = (\tilde{F}_T, \tilde{Y}_T), \tilde{G}_T^{(1)}$, from the conditional density $p(\tilde{G}_T | \tilde{Z}_T, \theta^{(0)})$;

Step 03: Conditional on the values of the sample and on the simulation "s" for $\tilde{G}_T, \tilde{G}_T^{(s)}$, draw a value for the parameter θ, say $\theta^{(s+1)}$, based on the conditional distribution $p(\theta | \tilde{Z}_T, \tilde{G}_T^{(s)})$ and obtain a draw of the impulse-response function and of the variance decomposition that satisfies the sign restrictions imposed by economic theory. The last two steps constitute an iteration and are repeated until the empirical distributions of $\tilde{G}_T^{(s)}$ and $\theta^{(s)}$ converge towards the theoretical ones. Where s is the index of the iteration. Geman and Geman (1984, p.731) demonstrate that as the number of iterations "s" converges towards infinite ($s \to \infty$), the marginal distributions and the set of sampled values of $\tilde{G}_T^{(s)}$ and $\theta^{(s)}$ converge to the true distributions at an exponential rate.

Next, we will present each step described previously in more detail. As before, let $\tilde{X}_T = (X_1,\ X_2,\ ...,\ X_T)$ be the history of X between period 1 and period T. The following can thus be defined: $\tilde{F}_T = (F_1,\ F_2, ...,\ F_T)$

Step 0: Choice of $\theta^{(0)}$

As in Bernanke, Boivin, and Eliasz (2004, p.30), the principal component of $\tilde{X}_T$ was used as an initial estimate of $\tilde{F}_T$. Given initial values of $\tilde{F}_T$ the factor loadings matrix was estimated based on equation (C.1) by imposing, the identification restriction, that its upper block $k \times (k+M)$ satisfies $[I_k, 0_{K\times M}]$. The structural VAR parameters (C.3) were estimated in two steps: i) in the first step the VAR parameters in the reduced form (C.2) were estimated via ordinary least squares (OLS); ii) in the second step the H matrix was estimated as the transpose of the inverse of the Cholesky decomposition of the covariance matrix of the residuals, of the reduced-form VAR (τ matrix). The estimates described previously were used as initial values for θ in the recursions described below.

Step 1: Draw from the conditional distribution $p(\tilde{F}_T^{(s)}/\tilde{Z}_T, \theta^{(s)})$

With the model in state-space form [equations (C.1) and (C.2) and given $\theta^{(s)}$, $\tilde{Z}_T$, plus adopting as a prior, for each element of the state vector, an independent normal distribution with zero mean and variance equal to 4, the vector F_t (a subset of the state vector G_t), in each period t, has, as its posterior distribution, a multivariate normal with mean and variance given by its smoothed values obtained by the Kalman filter (Harvey, 1994; Kim and Nelson, 1999). Therefore, $\tilde{F}_T^{(s)}$ is obtained by a draw of the normal multivariate distribution $p(\tilde{F}_T^{(s)}/\tilde{Z}_T, \theta^{(s)})$, and $\tilde{G}_T^{(s)}$ is also obtained (since it is composed by $\tilde{F}_T^{(s)}$ and by $\tilde{Y}_T^{(s)}$.

Step 2: Draw from the conditional distribution $p(\theta/\tilde{Z}_T, \tilde{F}_T^{(s)})$

Conditional on the observed data, and on $\tilde{F}_T^{(s)}$, obtained in the previous iteration, and on the prior adopted, a new draw of the parameter $\theta, \theta^{(s)}$ is carried out. This draw is carried out considering the stages below:

Stage 01: Draws of $\tilde{\Lambda}$ and P:

Considering equation (C.1) presented in this section and the joint prior density adopted, we can obtain the conditional distributions that will enable draws of $\tilde{\Lambda} = [\Lambda^f \Lambda^y]$ and R, remembering that the covariance matrix of the residuals of this system of equations, called R (the non-fixed part of P), is diagonal and, therefore, that the system can be estimated equation by equation (we are not in the context of SUR). Let $\tilde{\Lambda}_i$ be a line of the matrix $\tilde{\Lambda}$ and X_j a column of $\tilde{X}_j'$. We adopt as a joint prior density function for R_{ii} and $\tilde{\Lambda}_i$ an inverse normal-gama2 density,

$$\varphi(\tilde{\Lambda}_i, R_{ii}) = f_{ngi}(\tilde{\Lambda}_i, R_{ii} | \beta, \xi, s, \nu)$$

Where:

$$\beta = 0;\ \xi = I_{K+M} * (1/4);\ s = 0,02\ e\ \nu = 0,02.$$

Adopting the previously described joint prior density function, we obtain that the conditional posterior density function $P(R_{ii}^{-1}/\tilde{X}_T, \tilde{G}_T^{(s)}, \beta, \xi, s, \nu)$ is given by (Bauwens, Lubrano, and Richard, 1999):

$$P(R_{ii}^{-1}/\tilde{X}_T, \tilde{G}_T^{(s)}, \beta, \xi, s, \nu) = G(\bar{R}_{ii}/2, (2/(0,02 + T)))$$

Where:
$\hat{\Lambda}_i$ is the estimate of $\tilde{\Lambda}_i$ obtained via OLS, using the specification of equation (C.1) and given $\tilde{G}_T^{(s)}$ and $\tilde{X}_T$.

$$e_i^{(s)} = X_i - \tilde{G}_T^{(s)\prime}\hat{\Lambda}_i'$$

$$\bar{R}_{ii} = 0,02 + (e_i^{(s)\prime} e_i^{(s)}) + \hat{\Lambda}_i(\xi^{-1} + (\tilde{G}_T^{(s)}\tilde{G}_T^{(s)\prime})^{-1})\hat{\Lambda}_i'.$$

The first K lines of $\tilde{\Lambda}$ are fixed due to the identification hypotheses mentioned in step 1. Adopting the previously described joint prior density function we obtain the conditional posterior density function $p(\tilde{\Lambda}_i/R_{ii}^{(s)}, \tilde{X}_T, \tilde{G}_T^{(s)}, \beta, \xi, s, \nu)$, for i > K. It is given by (Bauwens, Lubrano, and Richard, 1999):

$$p(\tilde{\Lambda}_i/R_{ii}^{(s)}, \tilde{X}_T, \tilde{G}_T^{(s)}, \beta, \xi, s, \nu) = N[(\xi + \tilde{F}_T^{(s)}\tilde{F}_T^{(s)\prime})^{-1} * \tilde{F}_T^{(s)}X_j, R_{ii}(\xi + \tilde{F}_T^{(s)}\tilde{F}_T^{(s)\prime})^{-1}]$$

Stage 02: Draws of ψ, H and of the Impulse-Response Function that Satisfies the Sign Restrictions

Considering $\tilde{F}_T^{(s)}$ and $\tilde{Z}_T$ as given and equation (C.4), we can use the algorithms proposed by Waggoner and Zha (2003 and 2007) to obtain, via a Bayesian procedure with similar priors to that of Litterman (1986), a draw "s" for Ψ and H, $\Psi^{(s)}$ and $H^{(s)}$, and for the impulse-response function and the variance decomposition that satisfies the sign restrictions.

As already mentioned, in the FAVAR model the equation of the structural VAR is given by:

$$G_t'H' = W_t'\Psi' + u_t'$$

Where: $\Psi = [C_1 ... C_p D]$ and $W_t = [G_{t-1} G_{t-2} ... G_{t-p} J]'$ and $N = (M + K) * p + h$. We will refer to Ψ', of dimension N x (M+K), as the matrix of lagged parameters, although it also contains parameters of exogenous variables.

For $1 \leq i \leq M + K$, let a_i be the i-th column of H' and let f_i be the i-th column of Ψ', let Q_i be a matrix m x n of post q_i. The linear restrictions of interest, to be imposed on H, can be summarized as follows (the Q_i matrices, for example, can be chosen so that H is lower triangular):

$$Q_i a_i = 0, \quad i = 1, ..., n. \tag{C.5}$$

The restrictions given by (C.5) are non-degenerated if there is at least one H non-singular matrix that satisfies them. It was imposed, to identify the parameters of H, that it is a lower triangular matrix with numbers that can be different from zero in the main diagonal. However, it is worth highlighting that as the identification of the structural VAR part of the FAVAR is carried out, via sign restrictions in the impulse-response functions, it can be verified in the presentation below, that the ordering of the variables, used in the estimation of H, does not affect the results found for the impulse-response functions.

Sims and Zha (1998a) introduced a Litterman (1986) type of prior for a structural VAR, by specifying a Guassian prior distributions for a_i and f_i with zero mean and covariance matrix that will be presented below.

The distribution of the prior can be obtained as follows:

$$a_i \sim N(0, \bar{S}_i) \ and \ f_i | a_i \sim N(\bar{P}_i a_i, \bar{\Sigma}_i). \tag{C.6}$$

Where $\bar{\Sigma}_i$ is a symmetric and positive definite matrix of dimension N x N, given by:

$$\bar{\Sigma}_i = \begin{bmatrix} \begin{matrix} \frac{\lambda_0\lambda_1}{\sigma_i l^{\lambda_3}} & 0 & \cdots & 0 \\ 0 & \frac{\lambda_0\lambda_1}{\sigma_i l^{\lambda_3}} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\lambda_0\lambda_1}{\sigma_i l^{\lambda_3}} \end{matrix} & 0 \\ 0 & \begin{matrix} \frac{\lambda_0\lambda_4}{\sigma_i} & 0 & \cdots & 0 \\ 0 & \frac{\lambda_0\lambda_4}{\sigma_i} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \vdots & 0 & \frac{\lambda_0\lambda_4}{\sigma_i} \end{matrix} \end{bmatrix}$$

The matrix that contains the conditional standard deviation of the coefficient of variable j with lag l in equation i is given by:

$$\frac{\lambda_0\lambda_1}{\sigma_i l^{\lambda_3}}$$

Where: the hyperparameter λ_0 controls for the precision (tightness) of the beliefs about H'; λ_1 controls for what Litterman calls precision of the general beliefs around the random walk prior; λ_3 controls for the rate according to which the variance of the prior is reduced as the lag increases; the hyperparameter λ_4 corresponds to the constant term. For the constant term, a conditional prior with mean equal to zero and standard deviation controlled by $\lambda_0\lambda_4$ is adopted. The vector of parameters $\sigma_1, ...\sigma_m$ are scale factors since the units of measure, or the scales of variation of the variables, may not be uniform. Each σ_1 is equal to the standard deviation of the residuals of a univariate autoregression of the i variable.

The definite positive and symmetric matrix S, of dimension (M+K) x (M+K), is composed of independent individual elements (the standard deviations of the priors) set equal to $\lambda_0/\hat{\sigma}_i$ in the i-th line:

$$\bar{S}_{i\ ((M+K)\times(M+K))} = \begin{bmatrix} \frac{\lambda_0}{\hat{\sigma}_1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{\lambda_0}{\hat{\sigma}_2} & 0 & \cdots & \vdots \\ \vdots & 0 & \frac{\lambda_0}{\hat{\sigma}_3} & & \\ & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \frac{\lambda_0}{\hat{\sigma}_{M+K}} \end{bmatrix}$$

The values of the hyperparameters were set at the following values:

Hyperparameter	Value
λ_0	0.5
λ_1	0.25
λ_3	1
λ_4	0.5

$\bar{P}_i$ is a matrix $N \times (M + K)$ defined by:

$$\bar{P}_i = \begin{bmatrix} I_{M+K} \\ 0 \end{bmatrix}$$

The form of the prior, summarized above, belongs to a class of Bayesian priors that have been widely used in structural VAR models. Combining the form of the prior in (C.6) with the restrictions in (C.5), we obtain the functional form of the conditional prior distribution.

$$q(a_i, f_i | T_i a_i = 0) \tag{C.7}$$

Let U_i be a matrix $n \times q_i$ whose columns form an orthonormal basis for the null space of T_i. The column vector a_i will satisfy the restrictions (C.5) if and only if there is a vector b_i of dimension $q_i \times 1$ so that:

$$a_i = U_i b_i$$

Where b_i represents the free parameters of column i of the matrix H'. The distribution of b_i and f_i is given by:

$$b_i \sim N(0, \bar{S}_i) \;\; e \;\; f_i|b_i \sim N(\tilde{P}_i b_i, \bar{\Sigma}_i) \tag{C.8}$$

Where: $\tilde{P}_i = \bar{P}_i U_i$, and $\tilde{S}_i = (U_i' \bar{S}_i^{-1} U_i)^{-1}$.

Note that $\tilde{S}_i$ is a symmetric and positive definite (SPD) matrix of dimension $q_i \times q_i$, $\bar{\Sigma}_i$ is a SPD matrix of dimension $r_i \times r_i$, and $\tilde{P}_i$ is a matrix of dimension $r_i \times q_i$. As the linear transformation $\mathrm{U_i}$ enables $\mathrm{a_i}$ to be easily recovered based on $\mathrm{b_i}$ we will go on to present only the deductions for $\mathrm{b_i}$.

Let $b = [b_i' ... b_n']', \; X = [X_1 ... X_T]', \; and \; G = [G_1 ... G_T]'$.

The likelihood for b and $f(l(b_1 f_1, ..., b_n f_n|X, G))$ is proportional to

$$|det[U_i b_i| \cdots |U_n b_n]|^T \; exp\left(-\frac{1}{2}\sum_{i=1}^{n} b_i' U_i' G' G U_i b_i - 2 f_i' X' G U_i b_i + f_i' X' X f_i\right). \tag{C.9}$$

Combining the priors for b and f given by (C.8) with the likelihood function given by (C.9) enables us to reach the combined posterior for b and f:

$$p(b_1, ..., b_n|X, G \prod_{i=1}^{n} p(f_i|b_i, X, G),$$

where

$$p(b_1, ..., b_n|X, G) \propto |det[U_i b_i| \cdots |U_n b_n]|^T \; exp\left(-\frac{T}{2}\sum_{i=1}^{n} b_i' S_i^{-1} b_i\right), \tag{C.10}$$

As (C.10) is not a standard distribution it is necessary to use Gibbs sampling to obtain draws from it. Based on the draws of (C.10) it is possible to estimate the posterior distribution for $\mathrm{f_i}$. The distribution for f, conditional on b, is Gaussian and given by:

$$p(f_i|b_i, X, G) = \varphi(P_i b_i, \Sigma_i), \tag{C.11}$$

with

$$\Sigma_i = (X'X + \bar{\Sigma}_i^{-1})^{-1}, \tag{C.12}$$

The notation $\varphi(P_i b_i, \Sigma_i)$ in (C.10) denotes that the density is Gaussian with a mean $\mathrm{P_i b_i}$ and covariance matrix Σ_i. To obtain the inference for b and f, or functions of their values, it is necessary to obtain draws from the joint posterior distribution of b and f. This can be done by following two consecutive steps. First, obtain draws of b from the marginal posterior distribution (C.10). Second, given each draw of b, obtain a draw of f from the conditional posterior distribution (C.11). The second step is easy since it only requires draws from a joint normal distribution. The first step is complex but can be implemented using an algorithm based on the Gibbs sampling, developed by Waggoner and Zha (2002), which enables draws of b to be obtained, even when there are identification restrictions in the matrix of coefficients of the contemporaneous relationships of the VAR, making the model over-identified.

Algorithm 1. *The following steps belong to algorithm 1, for draw from the posterior distribution of b.*

(1) *Choose arbitrary values* $b_1^{(0)}, ..., b_n^{(0)}$ *(typically the estimate at the peak of the posterior density function, if available).*

(2) *For* $s = 1, ..., N_i + N_2$ *and given* $b_i^{(s-1)}, ..., b_n^{(s-1)}$ *obtain* $b_i^{(s)}, ..., b_n^{(s)}$ *in the following way:*

   (a) *simulating* $b_1^{(s)}$ *of the distribution* $b_1 | b_2^{(s-1)}, ..., b_n^{(s-1)}$,

   (b) *simulating* $b_2^{(s)}$ *of the distribution* $b_2 | b_1^{(s)}, b_3^{(s-1)}, ..., b_n^{(s-1)}$,

   (c) *simulating* $b_n^{(s)}$ *of the distribution* $b_n | b_1^{(s)}, ..., b_{n-1}^{(s)}$,

(3) *Collect the sequence* $\{b_1^{(0)}, ..., b_n^{(0)}, ..., b_1^{(N_1+N_2)}, ..., b_n^{(N_1+N_2)}\}$ *and keep only* N_2 *values of the sequence.*

In the algorithm described above all the simulations of the second step are carried out according to Theorem 1 of Waggoner and Zha (2002). In step 3 of the algorithm it is necessary to choose N_1 and N_2. This choice is arbitrary. If the initial values $b_1^{(0)}, ..., b_n^{(0)}$ are stochastic and are extracted from the target distribution the first N_1 draws are removed in order to avoid a very improbable initial draw.

Theorem 11 proposed by Waggoner and Zha (2007) provides a convenient way of implementing a stochastic selection of an orthogonal matrix that can generate impulse-response functions that satisfy the sign restrictions. The procedure consists of finding a stochastic matrix

in which each element has an independent standard normal distribution. Let $\tilde{\Upsilon} = \tilde{Q}\tilde{R}$ be the decomposition QR of $\tilde{\Upsilon}$ with the diagonal of $\tilde{R}$ normalized to be positive and $\tilde{Q}$ uniformly distributed. Algorithm 2 that we use and that is presented below follows Theorem 11.

Algorithm 2

Step 1. Draw an independent standard normal matrix $\tilde{\Upsilon}$*, n x n, so that* $\tilde{\Upsilon} = \tilde{Q}\tilde{R}$ *is the decomposition QR of* $\tilde{\Upsilon}$ *with diagonal* $\tilde{R}$ *normalized to be positive.*

Step 2. Generate an impulse-response function based on the structural VAR, the transition equation of the FAVAR, substituting H' for $\tilde{Q}'$ *H'.*

Step 3. If the impulse-response function does not satisfy the sign restrictions, return to step 1. If it does satisfy, save the result.

If (H', Ψ') is the "unrestricted" estimation of the parameters of the structural VAR (imposing only the restriction that H is lower triangular with positive numbers in the diagonal), then $(\tilde{Q}'\tilde{H}', \tilde{Q}'\tilde{\Psi}')$ obtained via Algorithm 2 will be an estimation of the structural parameters, of the structural VAR part of the FAVAR, that satisfies the sign restrictions, if it generates an impulse-response function that satisfies the sign restrictions.