The paradox of concessions in developing countries★

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Abstract
This paper presents a game-theoretic analysis of concessions that explicitly considers both the concession auction and the subsequent operation game. The concession contract requires investment to be made, but the concessionaire may benefit from underinvesting and ex-post renegotiating with the regulator. The paper highlights the "Paradox of Concession": the more successful the auction is, the higher is the probability of underinvestment. We propose a new mechanism, based on benefits for investment rather than punishment for underinvestment. The new mechanism: (i) is efficient; (ii) increases auction bids; (iii) eliminates the "paradox of concessions"; (iv) and can be fine-tuned to reduce the likelihood of underinvestment.

Keywords: game theory, mechanism design, airport concession, privatization in Latin America

JEL Codes: C72, D44, D78, L33

1 Introduction

Throughout the 20$^{\text{th}}$ century, governments implemented and managed the most important infrastructure projects in Latin America (LA). However, most LA countries suffered strong financial restrictions in the 1980's that led to a lack of public investment capabilities. To deal with the compelling need of investment, governments undertook important privatizations and concessions, aiming at fostering private sector participation. However, for a concession strategy to be successful, private operators' incentives need to be in line with the country's

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objectives. This paper finds inspiration in Brazilian airport concessions to analyze incentives in concessions that require significant investments.

Due to the growing relevance of public-private partnerships all-over Latin America in the last decade, studies about the expected and unexpected consequences of concessions became an active field of research. According to Laffont (2005, p.245), less developed countries suffer from incomplete contracts in part due to their institutional weaknesses. Engel, Fischer, and Galetovic (2008) argue that in most developing countries, governments focus on beginning new projects, rather than on assuring compliance of the running contracts, which affects the governments' overall reliability. Furthermore, Guasch, Laffont, and Straub (2003) find empirical evidence that noncompliance and subsequent renegotiation of contracts has been a common feature of late 20^{th} century concessions in Latin America, reaching up to 75% of all concessions in the water and sanitation sector.

Airport concession contracts are complex and require significant investments with deadlines and penalties in case of noncompliance. However, due to the high costs involved, demand shocks and the long-term perspective, the operator may find it optimal to postpone or cancel these investments, which frustrates the very objective of the concession. Therefore, a moral hazard problem arises during the operation period. This, in turn will trigger a response in the part of the government. Naturally, a firm that is competing in the concession auction anticipates this moral hazard issue, the government's reaction, and takes it into account when designing its bid strategy.

The present study explores the interaction between the expectations about contract enforcement of a concession and its initial auction equilibrium. In order to achieve that goal, this study first builds an extensive-form game between a concessionaire and the government, the "Operation Game", based on the signaling games approach of Kreps and Wilson (1982) and Milgrom and Roberts (1982), further explored in the context of subnational debt payments in a Federation in Pires and Bugarin (2002) and Bugarin (2006). The game explores the strategic tradeoffs that the concessionaire faces when deciding whether to comply with the concession contract's investment requirement. Moreover, it also analyses the tradeoffs the government faces in deciding whether to enforce the contractual penalties when the firm defaults. The Perfect Bayesian equilibrium of that game highlights a moral hazard problem whereby the concessionaire tends not to make the investment if the required amounts are high compared to the additional profits it generates, and there is a reduced likelihood that the government will enforce the corresponding penalties.

Next, this paper builds an incomplete-information normal-form auction game between firms that compete for the concession, the "Concession Auction". The main novelty of this approach is that the bidding firms in the Concession Auction anticipate the equilibria in the Operation Game and take that into account when designing their bid strategies. The main equilibrium result is that bidders internalize the expected penalties for (possibly) defaulting so that the stronger the government's reputation as a penalty enforcer is, the lower the equilibrium bids. This is the "Paradox of Concessions" in weak-institutions countries: The more successful a concession auction is in the sense of yielding higher than expected equilibrium bids, the more likely the winning firm will default the required investment.

Finally, to cope with the adverse "Paradox of Concessions", the present paper proposes a new mechanism, alternative to the traditional concession mechanism (TM), the Bonification Mechanism (BM). The BM replaces the punishment to a noncomplying concessionaire with a deduction in the due concession fees if the investment is completed. The theoretic analysis shows that, compared to the TM, the BM: (i) frees the government from the weak-institutions problems; (ii) increases equilibrium bids; (iii) eliminates the "paradox of concessions"; and (iv) can be fine-tuned in order to reduce the likelihood of noninvestment.

In addition to this introduction the paper is organized as follows. As a motivation to the theoretic study, next section presents a very brief history of airport concessions in Brazil. Section 3, then briefly discusses the literature on optimal concession design in Latin America. Section 4 starts solving the integrated concession mechanism model by backwards induction, modeling and solving the operation game. Next, section 5 models and solves the auction game, taking into consideration the operation game equilibrium. Section 6 discusses "the paradox of concessions". Then, section 7 proposes the alternative design of the "Bonification Mechanism" that could be used to offset the paradox of concessions. Finally, section 8 concludes with a discussion on the findings, the limitations, and possible extensions of the present modeling approach.

2 A brief review on airport concessions in Brazil

The Brazilian public airports (i.e., those that cannot deny traffic) network was mainly state run until the beginning of the 2010s. The main airports were operated by Infraero, a federal-government owned company founded in 1973, and some regional airports were operated by states or municipalities' agencies.

Following airlines prices and routes deregulation in the 1990s, and a favorable economic environment, the air traffic experienced a rapid increase in the 2000s, with enplanements growing from 38 million in 2001 to 100 million in 2011. However, investments in airport infrastructure fell behind, which lead to a decrease in the level of services in airports and to apron constraints.

In the years 2000, a series of events triggered a federal decision to concede airports operations.[1] Following these events, the federal government granted the first airport operation concession in Brazil in 2011, a greenfield project to build and operate for 28 years the São Gonçalo do Amarante/RN airport (ASGA), which substituted the previous Natal/RN airport in 2014. Next, followed the concessions of the airports of Guarulhos/SP, Campinas/SP and Brasilia/DF in 2012, Confins/MG and Rio de Janeiro–Galeão/RJ in 2014, and Fortaleza/CE, Salvador/BH, Florianópolis/SC and Porto Alegre in 2017. Last, the concession of three clusters took place in 2019, summing up 12 airports, and an additional three clusters, with 22 airports, in 2021.

Presently, most passengers are enplaned in concession airports. The concessions program is deemed successful due to the increase in the airports' service level, the entrance of internationally experienced operators and the high concession fees the government earned. However, concession airports face now considerable financial hardship. With different allegations, the first six airports of the concession programs applied 80 times to federal government "financial economic rebalance", which sum up to a demanded reimbursement of more than R$ 15 billion (about US$ 37 billion[2]). However, the federal government conceded a mere R$ 300 million (US$ 73 million) of credit, sustaining that the other demands were unfounded. Some of these airports are now facing difficulty paying the concession fees. For example, the Rio de Janeiro–Galeão airport concessionaire committed to an average concession fee of near R$ 800 million per year (in 2014 values), plus 5% of gross revenue, but in 2017 it had revenues of R$ 900 million mostly consumed by costs. Campinas' airport appears to be presently the clearest example of failure, having defaulted both the payments of the concession fees and the public funding loans as well, so that it is officially under bankruptcy risk ("judicial recovery"[3]).

[1] The 2014 FIFA World Cup and 2016 Olympic Games to be held in Brazil; the Gol Airline flight 1907 mid-air collision catastrophe in 2006; the Tam Airline flight 3,054 overrun accident catastrophe at Congonhas airport in 2007; and the air traffic services strike that followed them, leading to an aviation crisis.

[2] According to the average October 2019 exchange rate of US$ 1 = R$ 4.086. See https://economia.acspservicos.com.br/indicadores_iegv/iegv_dolar.html

[3] "Recuperação judicial." To this date, the actual bankruptcy has been successively postponed

Arguing that the economic turndown frustrated the air transport sector's growth, the concessionaires requested the government to reduce the concession fees for some years to come and pay higher values by the end of the concession. The federal government passed in Congress Law nº 13.499 in 2017 to allow concessionaires to postpone the concession fees payments, in response to a formal demand from the Brasilia, Rio de Janeiro, Guarulhos and ASGA airports.

3 Brief literature review on concessions in Latin America

The airport concession difficulties, unfortunately, are not isolated cases in Latin America's concessions. According to Guasch (2001) *apud* Guasch et al. (2003), "excluding the telecommunications sector, over forty percent of concessions appear to be renegotiated, and sixty percent of those within three years of the award of the concession, when in principle the contract agreement was for a period of 15 to 30 years". Guasch, Benitez, Portables, and Flor (2014) finds that, in the transport sector, the incidence of renegotiation reaches 70% in LA in the past 25 years. This incredibly high level of renegotiation of concessions impacts the credibility of government and affects the performance of the corresponding sectors.

Although this is such an important phenomenon in developing countries, Guasch et al. (2003) argues that this has not traditionally been well studied in the theoretic literature because most of the procurement and regulation literature has focused on developed countries where "[...] the quality of institutions yields a level of enforcement of contracts so high that renegotiations can be considered as secondary [...]".[4]

Several studies in the 2000s have aimed at filling that gap. Most of the theoretic literature, however, is based on Laffont and Tirole's (1986) Principal–Agent regulation model in which the regulator (government) is the Principal and offers a procurement contract to the Agent, a private firm.[5] These models typically derive a socially optimal contract given the typical restrictions, such as the participation and incentive constraints of the firm. The research by Guasch, Laffont and Straub[6] includes shocks and, additionally, imperfect monitoring

by trying to negotiate selling the airport or by successive Court appeals.

[4] Although concession renegotiations are not exclusive to developing country. See, for example, Gagnepain, Ivaldi, and Martimort (2013) for an assessment of the welfare cost of renegotiation in the French Urban Transportation Industry.

[5] See Auriol and Picard (2013), Estache and Quesada (2001), or Wang and Pallis (2014).

[6] Guasch et al. (2003); Guasch, Laffont, and Straub (2006, 2007, 2008).

by means of a probability that the regulator accepts to ex-post renegotiate the contract with the concessionaire. It derives a series of testable hypothesis about the effect of several institutional characteristics on the probability of renegotiation. That research line, however, does not model the initial auction phase of the concession and, therefore, cannot analyze the effect of the probability of renegotiation on the behavior of the initial auction participants. Furthermore, that research does not explain what kind of signal can be extracted from the results of the auction phase as to the likelihood of fulfilment of the concession requirements.

An alternative line of research by Engel, Fischer, and Galetovic (2001; 2008; 2013) advocates the "Least-present-value-of-revenue" (LPVR) auction mechanism. That research also uses a Principal–Agent model where the Social Planner (the government) wishes to propose a socially optimal contract to the firm. Once the optimal contract is derived, the paper proposes the LPVR auction format to implement it. There are basically two limitations in that model. First, it assumes that the auction will lead to a "competitive outcome" where all rents are dissipated away from the auction participants. This strong hypothesis limits the analysis of strategic behavior of the auction participants. Second, since the LPVR model solves the problem of renegotiation, it does not allow to analyze the probability of renegotiation (that is zero in that model) and its relationship with the auction results.

One recent paper that does analyze the effects of renegotiation on the original concession auction is Menezes and Ryan (2015). That paper examines the issue of using cash-finance versus debt-finance for the investment a concessionaire makes. In their model, investment is enforced, and it is realized low demand that triggers renegotiation with the government. Furthermore, renegotiation is solved using the Nash bargaining solution. Their main result is that the concessionaire uses debt finance to force the government into bailing it out in case of low demand. The main effect on the original auction is that bids are lower due to strategic debt finance and the main operation result is that more efficient firms are more likely to require government bailout.

The present paper extends a line of research initiated in Ribeiro (2016), which also models and solves the two-phased concession mechanism. Like the work by Menezes and Ryan, this research aims at filling a gap in the literature by carefully modeling the ex-post renegotiation game as well as the ex-ante auction game and analyzing the interaction of these two phases of the concession mechanism. Unlike that work, this study models the renegotiation as an explicit non-cooperative signaling game, the operation game, and derives the effect of that game's equilibrium on the bids and on the likelihood that the concessionaire will actually perform the required investment. As it will become clear in the following sections, in contrast to Menezes and Ryan's findings, the weaker the government is, the higher are the bids in the auction phase.

4 The Operation Game

The integrated analysis of the concession mechanism is made by backward induction: This paper first analyzes the Operation Game and then, conditional on the solution of the Operation Game, it derives the Auction Game Equilibrium. The Operation Game is modeled following the approach in Bugarin (2006), which is inspired by the seminal works of Kreps and Wilson (1982) and Milgrom and Roberts (1982) on signaling and reputation.

4.1 The extensive form of the Operation Game

The Operation Game starts after conclusion of the Concession Auction. The concession is awarded to the winning firm, say firm i, and that firm is expected to invest a certain capital I to be able to amass the benefits v_i of the modernized facility.[7]

The investment I is clearly established in the concession contract and is, thereby, common knowledge. The firm's benefit of investment, v_i, is her private information; different firms may have different managerial abilities, different cost of capital structure, etc. The value v_i is the firm's type and the higher it is, the more efficient is the firm.

Although the firm knows its type, all the government knows is that the value v_i is distributed in the interval $[v, V]$, where $v > 0$ corresponds to the present value of the concession, before any investment is made (in which case, the investment is completely unproductive to that firm), and V is the maximum value the concession can generate when the firm invests the established amount I.

The firm's main decision in the Operation Game is whether to make the investment I. If the firm makes the investment, its net profit is $v_i - I$. On the other hand, if the firm does not invest, then it is subject to paying the fine p if the government decides to enforce the concession contract. Therefore, when the firm does not make the investment I, its net profit is v if the government

[7] Although the main inspiration of this paper are airport concessions, hereafter we use the term "facility" or "concession" to reflect the fact that the analysis applies to any concession that requires the concessionaire to make significant amounts of investment.

does not enforce the contract and it is $v - p$ if the government enforces the contract and collects the penalty p. It is natural (but not necessary) to expect that $p < I$.

On the government side, if the firm makes the investment, the government receives the benefit $B > 0$ that corresponds to the social gain from a modernized facility. On the other hand, if no investment is made, then the government receives the basic benefit $b < B$ that corresponds to the social benefit of the original, outdated facility.

If the firm does not comply with the investment contract, the government can either enforce the contract charging the penalty p, or revise the concession contract not punishing the firm. If the government does not punish the firm, the government incurs the popularity cost σ that corresponds to society's disappointment with the government's lack of attitude, commitment, responsiveness. Therefore, its resulting utility is $b - \sigma$. On the other hand, if the government does apply the penalty to the concessionaire, it incurs the cost of confronting that firm, φ, that corresponds to the pressure the firm can exert on the government, the loss of campaign finance contributions, etc. Therefore, its net utility in that case is $b + p - \varphi$. Note that a government that strongly cares about social expectations (high σ) tends to apply the penalty, whereas a government that strongly cares about the concessionaire support (high φ) tends not to enforce the contract.

Hereafter, the government is said to be of the "socially responsive type" (σ_s, φ_s), or more simply "strong", if $\sigma_s \geq \varphi_s - p \Leftrightarrow p \geq \varphi_s - \sigma_s$. Conversely, the government is of the "unresponsive type" (σ_w, φ_w), or "weak", if $p < \varphi_w - \sigma_w$. The government knows his type, but the concessionaire only knows the probability $\mu \in [0, 1]$ that it is strong.

Figure 1 depicts the extensive form of the incomplete information Operation Game, where G refers to the government, C refers to the concessionaire and N refers to nature. The game starts with concessionaire C deciding either to make the investment I (strategy i) or not to make that investment (strategy ni), without knowing if it is dealing with a strong government (note t_1) or a weak government (node t_2).

If C makes the contracted investment, the game ends with payoffs B for the government and $v_i - I$ for the concessionaire. If C does not comply with the concession contract, then G decides whether to enforce the established penalty. If G is strong (node t_3) and applies the penalty, the corresponding payoffs are $b + p - \varphi_s$ for the government and $v - p$ for the firm. If it does not enforce the

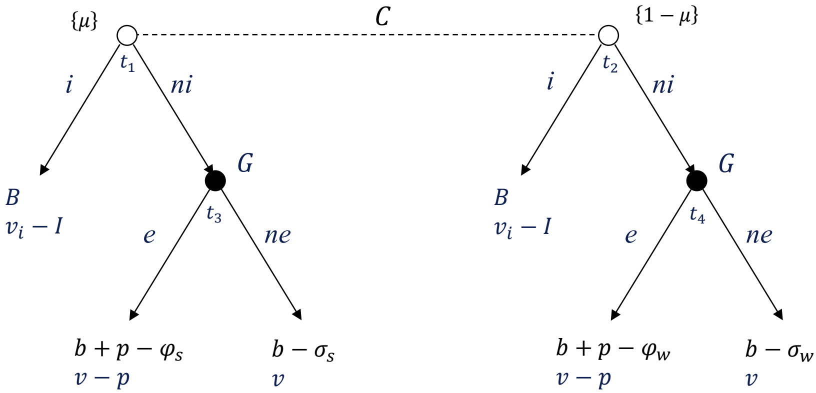


Figure 1. The extensive form of the incomplete-information Operation Game

contract, the corresponding payoffs are $b - \sigma_s$ for the government and v for the firm. An analogous situation occurs in node t_4, where G is weak.

4.2 The solution to the Operation Game

This section derives here the Perfect Bayesian Nash Equilibrium of this game. First, sequential rationality requires the strong-type government to enforce the penalty (e) in node t_3, and the weak-type government not to enforce the contract (ne) in node t_4.

Next, again sequential rationality at information set $\{t_1, t_2\}$ requires the concessionaire to choose to invest in the concession if and only if $v_i - I \geq v - \mu p$ or, again, if and only if $v_i \geq v - \mu p + I$.

Define $w_\mu = v - \mu p + I$. Then, the (pure strategy) Perfect Bayesian Nash Equilibrium of the Operation Game is

$$\Big((i, (e, ne)), \mu\Big) \quad \text{if } v_i \geq w_\mu = v - \mu p + I, \tag{1}$$

$$\Big((ni, (e, ne)), \mu\Big) \quad \text{if } v_i < w_\mu = v - \mu p + I. \tag{2}$$

The equilibria show that, given the investment requirement I and the penalty p, the behavior of the concessionaire depends fundamentally on two parameters: the firm's efficiency or ability to derive profits out of the investment, v_i, and the ex-ante reputation of the government, μ.

If v_i is large enough, then the concessionaire's expected profits are larger than the cutoff value of w_μ, and as a result, the firm will invest to modernize the facility. Similarly, the higher the expectation parameter μ, the higher is the probability that the firm will invest, as there is a larger chance that the government will punish the firm in case of underinvestment.[8] Figure 2 describes the choice of the concessionaire as a function of the threshold parameter w_μ.

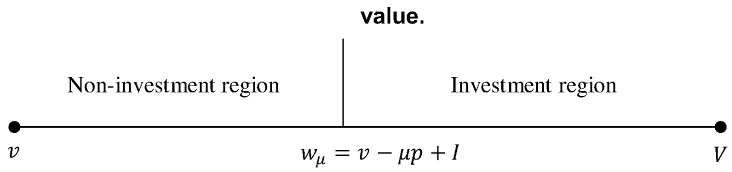


Figure 2. The investment decision of the concessionaire as a function of its value

Several policy conclusions can be drawn from the Operation Game. First, having a reputation of being a socially-responsive-type government increases the interval of firm types in which there will be investment. Therefore, if the government could first build a reputation before the concession begins, it would increase the likelihood of a successful concession. This could be done if the government could have shown strength in renegotiation in other sectors that would affect the firm's beliefs, for example, in other sectors' concessions or even in subnational debt payment negotiations (Bugarin, 2006).

Furthermore, if there are several similar concessions under way, such as several airport concessions, for example, and the government has a good reputation (high value for μ) at the outset, even if, in fact, the government is of the weak type, the government may still profit from acting "tough", applying the penalty if a concessionaire does not invest, in order to avoid other concessionaires to update their belief to $\mu = 0$, in which case the noncompliance region increases. By doing so, the government incurs a utility loss in the present negotiation with the concessionaire, but it is compensated with the disciplining effect it will have on the other concessionaries.[9]

Finally, we may argue that the government can increase the value of p in order to make sure that condition (1) holds for the concessionaire. However, there are limits to how high the punishments can be and higher values of p will probably lead to additional judicialization, which would probably reduce the likelihood of enforcement of the penalty. In other words, it may be the case that

[8] We are grateful to an anonymous referee for suggesting this rewriting of the original text.

[9] For a detailed discussion on the reputational preserving concerns in an environment where the government is playing a larger game with many other firms, please refer to Bugarin (2006), which studies a similar problem in the context of a game of debt payment between a federal government (the lender) and its subnational governments (the borrowers); see also the seminal paper from Selten (1978).

there is an inverse relationship between μ and p, in such a way that a large increase in p brings about, by means of increased judicialization, of a reduction in μ. As a consequence, μp may not change much or even reduce.

5 The Auction Game

5.1 The primitives of the model

For the sake of simplicity, assume there are two competitors, $i = 1, 2$ in the auction for selling a concession. For $i = 1, 2$, player i's type is the value $v_i \in [v, V]$ she obtains from the concession operation if she makes investment I. Recall that if no investment is made, then the value of the concession is v to the concessionaire regardless of her type.

Each player $i = 1, 2$ knows her own value, but the other player only knows that her value is distributed in $[v, V]$ according to the probability distribution function $F(v_i)$ and the probability density function $f(v_i)$.

The investment requirement I and the noncompliance penalty p are common knowledge, as well as the government's reputation parameter $\mu \in [0, 1]$. Furthermore, the model assumes that $v - p > 0$, so that even if there is punishment for sure, the concessionaire firm will still make a profit in the concession phase when she decides not to invest.

The two players play a first price sealed bid auction where players bid the amount that they are willing to pay for the concession. If both players make the same bid, then the winner is selected randomly with equal probability 1/2 for each player. When the players prepare their bids, they are aware of the continuation Operation Game that the winner will play with the government. Therefore, if a player i has value v_i, makes a bid β_i and wins the auction, she pays to the government the bided value β_i, is awarded the concession, and then plays the Concession Game with the government.[10]

5.2 The solution of the Auction Game

Theorem 1 below presents the solution to the auction game. The detailed proof is developed in the Appendix A.

Theorem 1. *Suppose two players $i = 1, 2$, participate in the concession auction. Players have private independent values v_i, $i = 1, 2$, that are identically distributed on the interval $[v, V]$ according to a distribution F. Then, when players*

[10] In practice, the bid β_i may be the present value of the stream of payments the concessionaire makes along the concession period.

take into consideration the operation game, there is a unique, non-decreasing, differentiable (up to a one-value point w_μ) Perfect Bayesian Equilibrium given below:

$$b(v_i) = \begin{cases} v - \mu p & \text{if } v_i \leq w_\mu = v - \mu p + I, \\ (v - \mu p)\frac{F(w_\mu)}{F(v_i)} + \frac{1}{F(v_i)}\int_{w_\mu}^{v_i}(y - I)f(y)\,\mathrm{d}y & \text{if } v_i > w_\mu = v - \mu p + I. \end{cases} \tag{3}$$

See Appendix A for the proof.

Note that this will indeed be the solution to the auction game only if, when one replaces $b^{-1}(\beta_i)$ in the original maximization problem, one obtains a strictly concave function. This can be checked once the ex-ante distribution function is made explicit.

5.3 The role of government's reputation

Consider now the effect of the expectation bidders have on the likelihood they are dealing with a social type of government, i.e., a strong government that will not hesitate to enforce the contract sanctions in case of noncompliance. This is measured by means of the parameter $\mu \in (0,1)$. The higher the value of μ, the higher the probability a noncompliant concessionaire will have to pay the fine p. Furthermore, as shown in the Appendix A, the higher the value of μ, the lower the auction bids.

Theorem 2. *Suppose two players $i = 1,2$, participate in the concession auction. Players have private independent values v_i, $i = 1,2$ that are identically distributed on the interval $[v,V]$ according to a distribution F. Then, in the unique Perfect Bayesian Equilibrium when players take into consideration the operation game, there is an inverse relation between the government enforcing reputation and the bid value.*

See Appendix A for the proof.

The intuition for this result is that when the players believe the government is tough, then the noncompliants internalize the higher expected cost of not investing and, therefore, reduce their bid. But the noncompliants' bids are the lower bounds for the compliants' bids. Therefore, competition reduces even among compliants. Thus compliants too will reduce their bids, which will yield lower revenues for the government in equilibrium.

In summary, the higher the government reputation of being a social, strong type, the lower the bids and the lower the noncompliance region.

Conversely, the less likely the government is strong, the more aggressive bids will show up in the auction phase, but the smaller is the probability that the concessionaire will make the required investments.

6 The paradox of concessions: The better they appear, the worse they may be

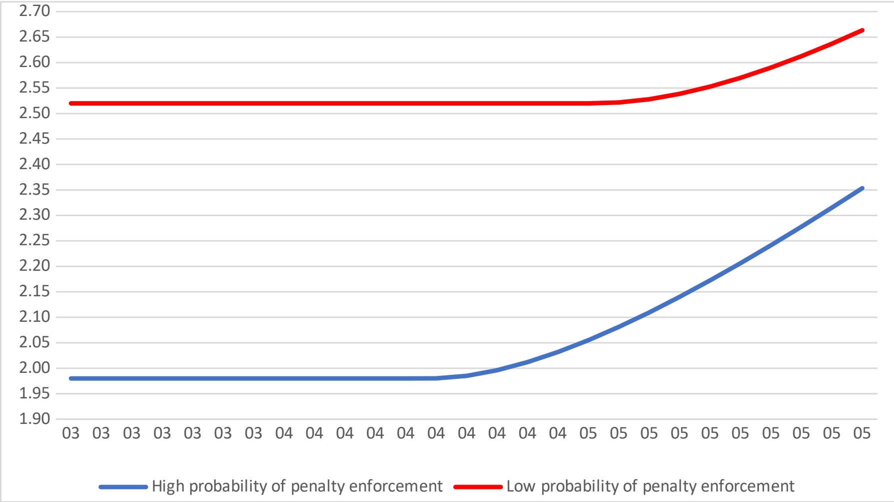
The results presented in the previous section show clearly the tragic expected long-term implementation of concessions that require important investment on the part of the concessionaire. Indeed, when we observe higher than expected competition with high bids and a high selling price for a concession, this is exactly the situation one should also expect that the likelihood of noncompliance with the required investments is the highest.

Therefore, an initially successful concession contract is a red light suggesting the government should carefully follow the investment schedule of the concessionaire, as that firm may more likely have decided not to invest already at the outset, when it participated in the auction.

Figure 3 presents two simulations for the auction-operation sequential games. This simulation assumes that the players' types are uniformly distributed between 2.7 and 6 billion reals (the Brazilian currency);[11] the required investment is $I = 2$ billion reals and the penalty fine is $p = 0.9$ billion reals. This numerical example considered two values for the probability of the government type being strong; the blue line corresponds to the high probability $\mu_{\text{high}} = 0.8$ whereas the red line corresponds to the low probability $\mu_{\text{low}} = 0.2$. With these parameters, the noncompliance threshold values are 3.98 for the case of high-punishment probability and 4.52 for the case of low-punishment probability. The simulation makes it clear that the worse the ex-ante reputation of the government, the higher the bids in equilibrium and the more successful the auction will appear, but also the higher noncompliance region.

In this specific simulation, the probability of having a winning concessionaire

[11] In January 2020, one US dollar corresponded to approximately 4 Brazilian reals. The present simulation parametrization is hypothetical. The choices were loosely inspired by the selling of Brasilia airport that took place in 2012. The winning bid was 4.51 billion reals, slightly below the noncompliance threshold in this simulation when the government is weak, whereas the expected investment was 2.8 billion reals.



Note: The red line corresponds to the bids in the low punishment probability $\mu_{\text{low}} = 0.2$ environment and the blue line corresponds to the bids in the high punishment probability $\mu_{\text{high}} = 0.8$ environment. The blue dashed line corresponds to the threshold of the concessionaire value from which there will be investment in the high punishment probability case and the red dashed line to the corresponding threshold in the low punishment probability case. Parametrization: $[v, V] = [2.7, 5.3]$, $I = 2$, $p = 0.9$.

Figure 3. The equilibrium bid functions for different levels of penalty enforcement probability: a simulation

that will not make the required investments is

$$F[w_{\mu_{\text{low}}}]^2 = \left[\frac{w_{\mu_{\text{low}}} - v}{V - v}\right]^2 = 49,$$

for $\mu = \mu_{\text{low}}$, and

$$F\left[w_{\mu_{\text{high}}}\right]^2 = \left[\frac{w_{\mu_{\text{high}}} - v}{V - v}\right]^2 = 24,$$

for $\mu = \mu_{\text{high}}$. Figure 4 presents the regions of noncompliance in equilibrium for the above simulation.

This is the unfortunate consequence of a weak institutional environment where a firm may be able to break a contract and not being punished for it. Note that the present model focusses on the type of the government, arguing that the responsive (strong) type government cares more about the lack of popularity that will come from not punishing a noncompliant firm, whereas the unresponsive (weak) type government cares more about losing the support of the concessionaire. In addition to the government itself, there may be other constraints to the punishment of a noncompliant firm. In Brazil, the Judiciary

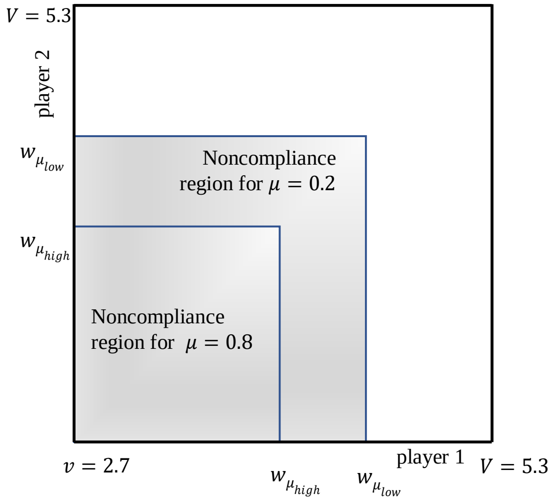


Figure 4. The region of values of the players where, in equilibrium, the winning concessionaire firm will not comply with the investment requirement: a simulation.

may be the utmost source of such institutional weakness.[12]

In the present model, these additional institutional issues can be modeled by introducing an addition parameter, say $\delta \in [0,1]$ in the expected punishment when the government decides to apply the penalty that is specified in the concession contract. In that case, the government still must bear the cost of losing the firm's support, σ, in the concession game, but the penalty will only be applied with probability δ because, for example, the firm will use all institution mechanisms available to avoid having to pay that penalty, such as appealing all the way to the Brazilian Supreme Court.

The consequence is that, in the Operation Game, the payoff when the firm does not comply, and the government decides to apply the fine increases from $v - p$ to $v - \delta p$. This, in turn, makes the government less likely to be strong, because the benefit of punishing the noncompliant concessionaire reduces to $\sigma \geq \varphi - \delta p$, which is less likely to happen. Thus, the threshold $w_\mu = v - \delta\mu p + I$ increases. Therefore, we will observe: i) still higher bids in the auction phase; and ii) higher noncompliance in the concession phase.[13]

[12] See, on this subject, the vote of Supreme Court Justice Luís Roberto Barroso available online at https://www.youtube.com/watch?v=JhxrUHL_6fY

[13] The detailed calculations mimic the ones presented here and can be provided by the authors upon demand.

7 The Bonification Mechanism

The main motivation for the government to offer the concession of public enterprises in developing countries is the government's lack of investment capabilities. Therefore, one of the main goals of any concession mechanism in such countries is to create the conditions for the concessionaire to decide to comply with the investment requirements. However, an important characteristic of many developing countries is the lack of credibility of the government and, in general, of the countries' institutions (Guasch et al., 2003). This suggests that the firms in a concession auction may most likely believe the likelihood of the government really being able to enforce the due penalties is low, which implies a high likelihood that the concessionaire will not invest, precisely in countries that are the most in need of private investment.

One way to deal with this adverse incentive of the institutional and reputation environment in developing countries is getting rid of the need for the government to have to decide whether to enforce the noncompliance penalties. This needs to be done while preserving the incentives for the concessionaire to invest. The present section proposes an alternative mechanism aiming precisely at aligning the investment incentives of the firm while, at the same time, freeing the government from having to decide whether to punish the firm.

A central concern in the traditional mechanism is the credibility of the government threat to punish a noncomplying firm. The punishment requires a government initiative starting action, that could, for the reasons exposed, not be an equilibrium choice. If the government does not need to appeal to punishment in order to enforce the investment, the credibility issue vanishes.

Suppose that in the operation game, rather than punishing a noncompliant firm, the government awards a reduction in the concessionaire payments if it does make the expected investment.[14] This is especially implementable because the concession fee's payment is typically spread over a long period of time after the auction determines who is the concessionaire firm. Furthermore, the firm must pay regular license fees as well. The deduction can be made from the entire due payment in case of proper investment. Call that mechanism the "Bonification Mechanism" and denote it BM and, for the sake of comparison, call the original mechanism the "Traditional Mechanism" and denote it by TM.

A discussion on this mechanism is in order. Its main assumption is that it is easier for a weak institutions' government not to award a discount than to enforce a punishment, in case of noncompliance. If there is doubt about this assumption,

[14] Wang and Pallis (2014) also propose rewards to compliance in the context of port concessions.

one may add to the mechanism additional institutions that have interest in carefully assessing compliance. Suppose, for example, that in addition to the regulator, a private audit company is hired to assess whether the investment was fully done. Suppose, moreover, that in case of noncompliance, the company receives a percentage of the discount that is not given away. Then, that private company will have strong incentives to expose noncompliance.[15] Also, the grant of the bonification could also be assured by the use of an escrow account, with a bank receiving previously the full payment—or a guarantee—from the concessionary and giving the bonification back automatically when the audit company confirms the compliance.

These are institutional features that could imitate an ex-ante irreversible government decision to behave as a strong one, preventing him from the burden of a strategic choice about punishing.

7.1 The new Operation "Game"

Let $d < I$ be a discount or deduction in the price the concessionaire has to pay—which is the auction winning bid—that can be reduced from the firm's concession payments to the government, in case the convened investments are concluded as expected. Then, the original concession game becomes a simple decision problem for the firm. If she does not make the expected investments, then she will pay the full original concession price. On the other hand, if she does make the investment, she will receive the additional discount d. Therefore, the concessionaire will decide to invest if and only if $v_i \geq v - d + I$.

Note that the discount d in the present mechanism plays the same role as the expected punishment μp in the original mechanism. However, now it does not depend on the type of the government. This is an objective, crystal clear, legal rule that is to be applied if and only if the investment has been made regardless of how strong or weak the government may be. Furthermore, as d is a parameter of the mechanism, it can be chosen by the government strategically according to its interest, as it will become clearer next.

7.2 The new Auction Game

Consider now the auction game induced by this mechanism and define $w_d = v - d + I$.

It is again interesting to separate the set of types of a player into two subsets. If $v_i < w_d$, then if player i wins, she will find not to her interest making

[15] The authors are especially grateful to Michael Gilbert for highlighting this point.

the investment in spite of the benefit she would receive if she did invest: the investment is too expensive to her compared to the discount benefit. Call this type of player a “noninvestor” in analogy to the noncompliant type in the other model.

On the other hand, if $v_i \geq w_d$, then if that player wins, she will find in her interest to make the investment and receive the corresponding discount. Call this type of player an “investor”.

A calculation analogous to the one developed in section 6 allows us to determine the symmetric Bayesian Nash equilibrium of this game.[16]

Theorem 3. *Suppose two players $i = 1, 2$, participate in the concession auction in a Bonification Mechanism. Players have private independent values v_i, $i = 1, 2$, that are identically distributed on the interval $[v, V]$ according to a distribution F. Then, when players take into consideration the operation game, there is a unique, non-decreasing, differentiable (up to a one-value point) Perfect Bayesian Equilibrium given below:*

$$b(v_i) = \begin{cases} v & \text{if } \; v_i \leq w_d = v - d + I, \\ v\frac{F(w_d)}{F(v_i)} + \frac{1}{F(v_i)}\int_{w_d}^{v_i}(y - I + d)f(y)\mathrm{d}y & \text{if } \; v_i > w_d = v - d + I. \end{cases} \tag{4}$$

Therefore, the threshold that separates the noninvestors and the investors is the cutoff point $w_d = v - d + I$.

See Appendix A for the proof.

7.3 Comparison of mechanisms when $d = \mu p$

To better compare the two mechanisms, let us start assuming that the government sets the discount to $d = \mu p$. Then, $w_d = w_\mu$ and the cutoff point that separates noninvestors from investors in the bonification mechanism is the same as the cutoff point that separates noncompliants from complaints in the traditional concession mechanism.

In that case the noninvestment region and, therefore, the probability of the concessionaire not investing remains the same as in the traditional concession mechanism. This remark yields the following corollary.

Corolary 1. *If $d = \mu p$, i.e. $w_d = w_\mu$, the probability of the concessionaire not investing is the same in both mechanisms.*

[16] The calculations’ details can be made available upon demand to the authors.

However, the two mechanisms do not yield the same bids. Indeed, comparing expression (3) with expression (4) it is straightforward to check that all bids, including those corresponding to the types in the noninvestment region, increase in the Bonification Mechanism by precisely the amount d. Therefore, the expected revenue of the government in the auction increases by d.

Therefore, in terms of expected auction revenue, the bonification mechanism is clearly superior to the traditional concession mechanism, as long as the government chooses the discount benefit to equal the expected penalty that the noncompliant concessionaire pays in the traditional mechanism.

However, the government wishes to maximize its entire payoffs including the social benefit of investment and the net financial return of the concession. Recall G receives social benefit B when the investment is completed and $b < B$ when it is not made. Since when $d = \mu p$ the noncompliant types in the TM are precisely the noninvestors in the BM, the expected social benefit is the same under both mechanisms. Furthermore, the additional payment of the investors in the BM, d, is discounted by that precise amount d, so that the investors pay the exact same net amount as the investors in the TM. Finally, the noncompliants in the TM make the lower bid $v - \mu p$, but they also pay the expected fine μp when G realizes it has not made the required investment. Thus, the expected net payments of the noncompliants are precisely that same amount v that the payments of the noninvestors in the BM.

Hence, in ex ante terms, i.e., before the government learns his own type, both mechanisms are completely equivalents. If, however, the government knows his type when the selling mechanism is decided upon, the strong government knows that he will be able to enforce the full penalty payment p in case of noncompliance in the TM, in which case, he prefers that mechanism. Conversely, if the government knows that he is of a weak type, he prefers the BM, since he will not be able enforce the penalty payment in case of noncompliance in the TM.

7.4 Comparison of mechanisms when $d \neq \mu p$

Since $w_d = v - d + I$ is a decreasing function of d, the higher the deduction offered for the investment, the lower the noninvestment region. Moreover, from expression (4), it can easily be checked that the investors' bid functions are strictly increasing in d. More precisely, it can be shown that:[17]

$$\frac{\partial b(v_i)}{\partial d} = \frac{F(v_i) - F(w_d)}{F(v_i)} \in (0,1), \qquad \forall v_i \in (w_d, V] \tag{5}$$

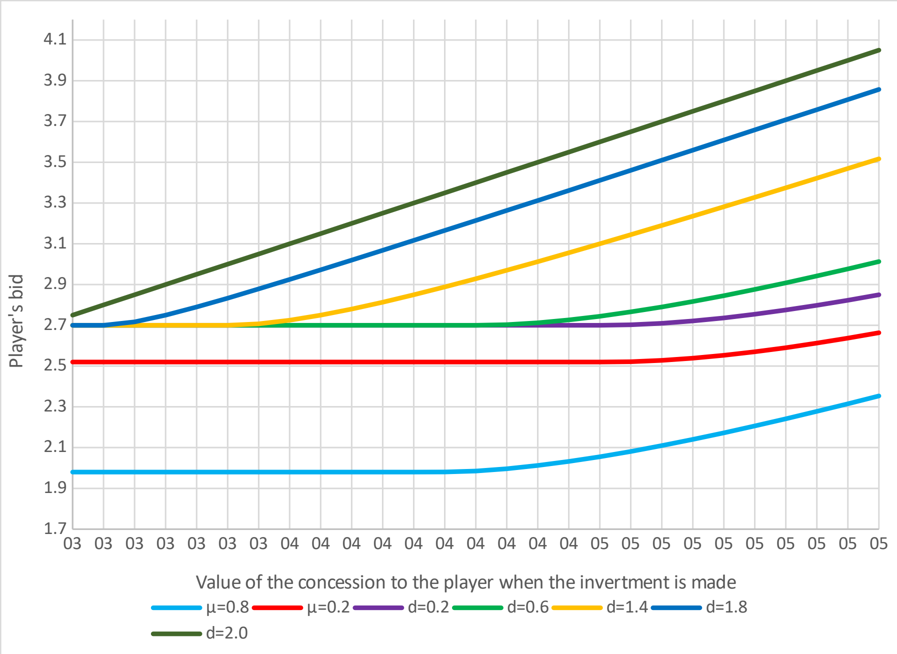
Figure 5 adds to the strategies of the TM simulated in Figure 3, the equilibrium strategies for the BM for different values of the deduction parameter d, ranging from 0.1 to its maximal value $d = I$. The figure shows clearly the strong positive effect on the bids, that increase strongly with d, but most importantly, on the reduction of the noninvestment area, that corresponds to about two-thirds of the type interval for the TM with $\mu = 0.2$ and is reduced to about half of that interval with $d = 0.6$ in the BM and to about 25% of the interval with $d = 1.4$ and to investment for all types if there is total discount of the investment, i.e., $d = 2$. Note that the threshold for investment is virtually the same for $\mu = 0.2$ in the TM and for $d = 2$ in the BM (about 4.5) because $\mu p = 1.8$ which is quite close to $d = 2$. Had we plotted the corresponding bid strategies for $d = 1.8$, the threshold would be identical. Then, if the government is mainly concerned with reducing the noninvestment area and increasing the auction equilibrium bids, it should set d to its maximum possible value, i.e., $d = I$.

However, as condition (5) makes it clear, a one dollar increase in the discount d yields an increase inferior to one dollar in the investor's bid. Therefore, one must include the opportunity cost for the government of discounting the amount d from the bids when there is investment.

Therefore, the net return of a winning bid must deduct the amount the government will not receive when there is investment in the BM. Similarly, when there is noncompliance in the TM one should add to the return to the government the expected penalty payment μp. Finally, in order to really be able to compare the two mechanisms, one must include the value to the government of having the investment done, the parameters B, and the analogous value of having the facility functioning without the investment, the parameter b.

Figure 6 replicates Figure 5 but now it shows the net utility of the government when each bid function is the winning function of the auction. Therefore, it takes into account the complete extent of the concession mechanism, including the payment after the auction, the penalties (for the TM), the discounts (for the BM) and the social benefits of the investing or not in the facility (B and b, respectively). In the corresponding simulation, set $B = V$, the maximum possible value of the firm's profits when there is investment and $b = v$ the

[17] Calculations available upon request to the authors.



Parametrization: $[v, V] = [2.7, 5.3]$, $I = 2$, $p = 0.9$, $\mu_{\text{low}} = 0.2$, $\mu_{\text{high}} = 0.8$, $d \in \{0.2j, j = 1, 3, 7, 9, 10\}$.

Figure 5. The equilibrium bid functions for different levels of punishment probability in the Traditional Mechanism and for different deduction parameters in the Bonification Mechanism: a simulation.

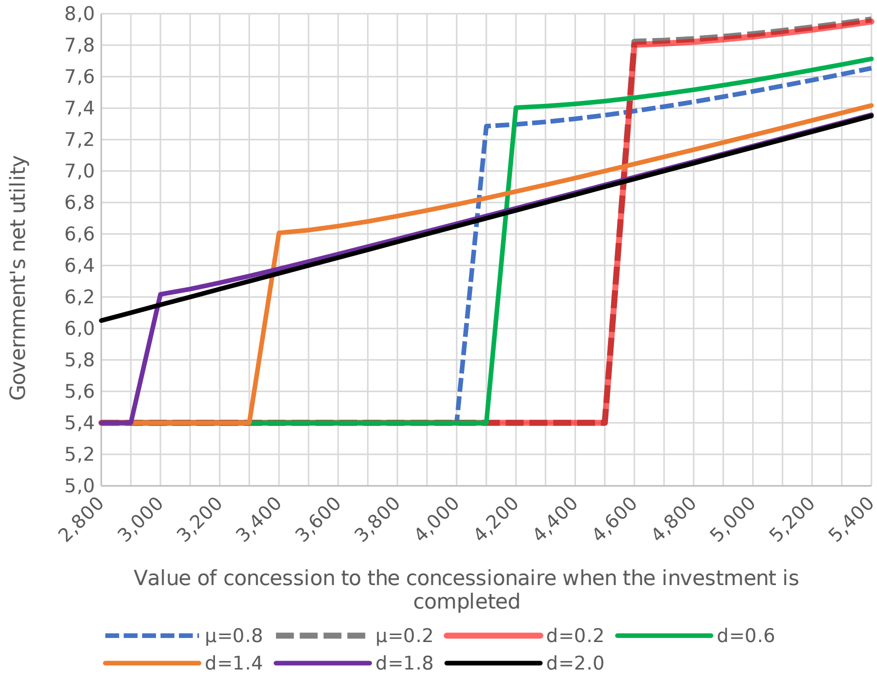
corresponding minimal value when there is no investment at all.[18]

The graph shows clearly, first, the equivalence between TM and the BM when $\mu p = d$. Indeed, the ex-post utility of the government is essentially the same for $\mu = 0.2$ with the TM and for $d = 0.2$ for the BM. Note, however, that the TM involves the complex issue of signal extraction (to determine μ) and may be further jeopardized by institutional instability such as the ones related to judicial instability, whereas rules are clearer in the BM.

In addition, the graphs in Figure 6 highlight two important points for comparing TM and BM and about the choice of the deduction level.

First, the BM mechanism allows a more significant reduction of the noninvestment area. Indeed, with the TM even when the government has a very high reputation of $\mu = 0.8$, i.e., there is an 80% probability that the penalty will be

[18] Note that the chosen parameters are conservative, as the returns are likely to be much higher when one considers the additional benefit to consumers.



Parametrization: $[v, V] = [2.7, 5.3]$, $I = 2$, $p = 0.9$, $\mu_{\text{low}} = 0.2$, $\mu_{\text{high}} = 0.8$, $d \in \{0.2j, j = 1, 3, 7, 9, 10\}$.

Figure 6. The net utility of government associated to the equilibrium bid functions for different levels of punishment probability in the Traditional Mechanism and for different deduction parameters in the Bonification Mechanism: a simulation.

enforced in case of noncompliance, about half of the types ($v_i < 4.2$) will still not invest. The BM, on the other hand, allows the government to fine tune the noninvestment region, all the way to emptying it (all concessionaires will invest) by properly adjusting the deduction parameter ($d = 2 = I$).[19]

Second, Figure 6 highlights the trade-offs between the choice of different deduction values in the BM.

Indeed, the higher is d, the greater is the probability of investment, but the lower will be the ex-post payment if firms decide to invest. In other words,

[19] It is true that the TM can also induce a higher compliance area by increasing the penalty p. However, there are two limitations to the amount of penalty that could be chosen. First, the concessionaire may argue, ex post, and in Court, that the penalty is economically abusive; this means that, in fact, the parameter μ, when also be interpreted as to include the institutions, may be as a decreasing function of the penalty p, which blocks the ability of the Government to increase μp significantly. Second, if the government is able to impose very high expected penalties, depending on the parameters of the problem, some firms may find it ex ante optimal not to participate in the concession auction at all, which may reduce competition overall.

the higher d, the larger is the set of complying firm types, which increases ex post utility to the government. On the other hand, a higher value of d lowers the ex-post return of an investing firm. Therefore, the optimal choice of d will depend on the actual parameters of the model and will be an intermediate value between 0 and I and will still involve some level of noninvestment. The determination of the explicit general solution to the optimal level of deduction in the BM is left here as a suggestion for future research.

8 Conclusion

This paper used the recent history of airport concessions in Brazil as a motivation to analyze, on a theoretic point of view, the concession mechanism as two sequential integrated strategic interactions. The first is the auction game where several companies compete for the concession. The second is the operation game, where the strategic interaction occurs between the concessionaire, who decides whether to make the investments required in the concession contract, and the government, who decides whether to enforce the contract penalties if the concessionaire defaults.

The equilibrium behavior of the players was derived by solving the two games in reverse order. First, the operation game solution showed that the lower the credibility of government, i.e., the lower the likelihood that the government will effectively enforce the contract penalties for default, the higher the probability that the concessionaire will not make the agreed upon investments. Second, the auction game solution showed that the lower that credibility, the higher the equilibrium bids in the concession auction.

These results identify a new phenomenon, the "paradox of concession", which states that the more successful the initial auction may look, with higher bids in equilibrium, the more likely it is that the concessionaire will not comply with the required investment, jeopardizing the main motivation for the concession itself.

To cope with this adverse equilibrium, the paper proposes an alternative design, the Bonification Mechanism that replaces the penalty for default (that may not be enforced) with an ex-post deduction in the concession fee in case the investment is indeed realized. This makes the deduction an objective part of the contract, not subject to the decision of the government and, thereby, not affected by the government's reputation.

The use of the Bonification Mechanism, in addition to making the contract more objective and judicially secure, increases overall bids in the auction and can be fine-tuned to reduce the probability of noncompliance.

This is the main original contribution of this paper. This paper is a first attempt to carefully model the incentives that arise in concession mechanism that need significant investments, in an integrated framework that links the operation phase to the initial auction. The basic model, naturally, does not include several additional features of the real-world interaction, especially in the operation game. For example, although in the real-world contracts the investments are well specified in terms of their outcomes, such as building a new airport terminal with a specified capacity by a certain date, the noncompliance may be partial in the sense that a smaller terminal is built, or the terminal is not completed on time. A more general model would consider the possibility of partial compliance in the traditional mechanism. Note that, in the BM this is not really an issue as the deduction would only take place once the investment is completed as required in the contract.

Another simple extension would be to explicitly include in the operation game the role of institutions in addition to the role of the government, as discussed in the text, to better disentangle these two factors, and being able to characterize the role of weak institutions on the likelihood the government will be of a strong type. The main insight here is that the cost of confronting the concessionaire for the government remain the same, but the financial return, in terms of expected revenue from enforcing the penalty, reduces. Therefore, the government is less likely to apply the penalty in weak-institutions countries.

A more significant extension would consider the possibility of the firm herself not knowing exactly her true value of the concession. The model would follow without much change if the concessionaire took her expected value into consideration in the auction phase. However, depending on the timing of real value discovery, the concessionaire may start making the investments and later find out that she is a low-value type and, therefore, stop the investment after having initiated it, or even go bankrupt. This more general model would allow to discuss the role of investment risk and risk sharing between the concessionaire and the government, an important issue in real-world concession.

The extension of the original model to include these additional frictions is left here as a suggestion for further research.

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Appendix A Algorithm for stationary equilibrium computation

A.1 Proof of Theorem 1

This section derives the equilibrium strategies $b_1(\cdot), b_2(\cdot)$ of the players as symmetric non-decreasing functions of the players' types v_i, $i = 1, 2$, for the traditional mechanism.

Considering the result of the Operation Game, it is possible to separate a player's type in basically two sets of types, one including the types who will make the investment I in the Operation Game and the other one including those types who will not invest, the noncompliants.

A player i of type v_i will be a noncompliant if $v_i < w_\mu$ and will invest if $v_i \geq w_\mu$. Let us analyze the behavior of a player in each one of these categories separately.

Consider first a noncompliant player i's bid. If he wins, he will have expected profit $v - \mu p$ in the concession period. Therefore, he will never choose a bid higher than that profit. Suppose he makes a lower bid $\beta_i < v - \mu p$. Then his opponent, when he is also a noncompliant type, can make a bid β_{-i}, $\beta_i < \beta_{-i} < v - \mu p$ and win. Therefore, $\beta_i < v - \mu p$ cannot be a best response for player i. Hence, a Bertrand-type analysis implies that noncompliant types will all choose bid $\beta_i = v - \mu p$, regardless of their value $v_i < w_\mu$:

$$b_i(v_i) = v - \mu p, \qquad \forall v_i < w_\mu \,.$$

Consider now a compliant player i's bid. For that player, $v_i \geq w_\mu$.

Suppose first that $v_i = w_\mu$. Then, if she chooses a bid $\beta_i < v - \mu p$, then she will surely loose. On the other hand, if she chooses a bid $\beta_i > v - \mu p$, her utility when she wins will be $v_i - \beta_i = (v - \mu p) - \beta_i < 0$. Therefore, her best response is to set $b_i(w_\mu) = v - \mu p$.

Suppose next that $v_i > w_\mu$. Then, she will choose a bid higher than $v - \mu p$ and win for sure if her opponent is a noncompliant or of type $v_i = w_\mu$, which occurs with probability $F(w_\mu)$. Therefore, if she chooses bid $\beta_i > v - \mu p$, her expected utility is

$$(v_i - I - \beta_i)F(w_\mu) + \frac{v_i - I - \beta_i}{2}\Pr[\beta_i = b_{-i}(v_{-i})] \\ + (v_i - I - \beta_i)\Pr[\beta_i > b_{-i}(v_{-i}) > v - \mu p].$$

Therefore, the best response of a compliant player of type $v_i > w_\mu$ is the solution

β_i to the following maximization problem:[20]

$$\max_{\beta_i}(v_i - I - \beta_i)F(w_\mu) + \frac{v_i - I - \beta_i}{2}\Pr[\beta_i = b_{-i}(v_{-i})] \\ + (v_i - I - \beta_i)\Pr[\beta_i > b_{-i}(v_{-i}) > v - \mu p].$$

Let us look for a Bayesian Nash equilibrium (b_1, b_2) where the strategy of the compliant-type player is strictly increasing, i.e., for $v_i,\ v_i' > w_\mu,\ v_i > v_i' \Rightarrow \beta_i(v_i) > \beta_i(v_i'),\ i = 1, 2$.

Then, b_{-i} is strictly increasing on $[w_\mu, V]$ and, thereby, invertible. Therefore,

$$\Pr[\beta_i = b_{-i}(v_{-i})] = 0$$

and

$$\Pr[\beta_i > b_{-i}(v_{-i}) > v - \mu p] = \Pr\left[b_{-i}^{-1}(\beta_i) > v_{-i} > b_{-i}^{-1}(v - \mu p)\right] \\ = \int_{w_\mu}^{b_{-i}^{-1}(\beta_i)} f(v_{-i})\,\mathrm{d}v_{-i} = F\left(b_{-i}^{-1}(\beta_i)\right) - F(w_\mu).$$

Hence, her maximization problem can be reduced to

$$\max_{\beta_i}(v_i - I - \beta_i)F(w_\mu) + (v_i - I - \beta_i)\left[F\left(b_{-i}^{-1}(\beta_i)\right) - F(w_\mu)\right]. \tag{6}$$

Or, equivalently, $\max_{\beta_i}(v_i - I - \beta_i)F\left(b_{-i}^{-1}(\beta_i)\right)$.

Assuming that the objective function is strictly concave, the solution to this maximization problem is obtained by calculating its first order condition (FOC):

$$\frac{\mathrm{d}}{\mathrm{d}\beta_i}(v_i - I - \beta_i)F\left(b_{-i}^{-1}(\beta_i)\right) \\ = -F\left(b_{-i}^{-1}(\beta_i)\right) + (v_i - I - \beta_i)f\left(b_{-i}^{-1}(\beta_i)\right)\left(b_{-i}^{-1}\right)'(\beta_i) = 0.$$

In a symmetric equilibrium, all bidders choose the same bid function, i.e., $b_1(v) = b_2(v)$. Denote by that common bid function.

Note that the solution to the player's problem is that player's bid, therefore, $\beta_i = b(v_i)$ and, since b is invertible (for $v_i > w_\mu$), $v_i = b^{-1}(\beta_i)$. Therefore, the

[20] We ignore the additional condition $\beta_i > v - \mu p$ and check that the solution indeed satisfies that condition.

above FOC can be rewritten as

$$-F(v_i) + (v_i - I - \beta_i) f(v_i) \big(\beta^{-1}\big)(\beta_i) = 0. \tag{7}$$

Now recall that if b is an invertible, differentiable function, then its inverse is also differentiable and $\big(b^{-1}\big)'(\beta_i) = [b'(v_i)]^{-1}$. Hence, the FOC can be written as $F(v_i)b'(v_i) + b(v_i)f(v_i) = (v_i - I)f(v_i)$.

Therefore, from the Fundamental Theorem of Calculus, for every $w \in (w_\mu, v_i]$, $b(v_i)F(v_i) - b(w)F(w) = \int_w^{v_i} (y - I) f(y) \mathrm{d}y$.

Now, by continuity, since $b(w_\mu) = v - \mu p$, it follows that

$$b(v_i) = (v - \mu p)\frac{F(w_\mu)}{F(v_i)} + \frac{1}{F(v_i)} \int_{w_\mu}^{v_i} (y - I) f(y) \mathrm{d}y.$$

In summary, the solution to the auction game can de written as

$$b(v_i) = \begin{cases} v - \mu p & \text{if } \; v_i \leq w_\mu = v - \mu p + I, \\ (v - \mu p)\frac{F(w_\mu)}{F(v_i)} + \frac{1}{F(v_i)} \int_{w_\mu}^{v_i} (y - I) f(y) \mathrm{d}y & \text{if } \; v_i > w_\mu = v - \mu p + I. \end{cases} \tag{3}$$

Note that $b(v_i)$ is indeed bigger than $v - \mu p$ for $v_i > w_\mu$. Furthermore, this will indeed be the solution to the auction game only if, when one replaces $b^{-1}(\beta_i)$ in the original maximization problem, one obtains a strictly concave function. This can be checked once the ex-ante distribution function is made explicit.

A.2 Proof of Theorem 2

Note, first, that $w_\mu = v - \mu p + I$ decreases with μ, i.e., the noncompliance region decreases. This happens because the opportunity cost of compliance increases: when a firm decides not to comply it will pay a higher expected cost.

Next, if $v_i \leq w_\mu$, then $b(v_i) = v - \mu p$, which decreases with μ, i.e., the higher μ, the lower the bid a noncompliant concessionaire will set. This happens because the expected cost of noncompliance increases and, therefore, the noncompliant revises downwards his bid to compensate that expected cost.

Consider now a compliant firm ($v_i > w_\mu$) for which $b(v_i) = (v - \mu p)\frac{F(w_\mu)}{F(v_i)} + \frac{1}{F(v_i)} \int_{w_\mu}^{v_i} (y - I) f(y) \mathrm{d}y$, and let $G(y)$ be a primitive of $(y - I)f(y)$. Then, $b(v_i)$ can be rewritten as $b(v_i; \mu) = (v - \mu p)\frac{F(w_\mu)}{F(v_i)} + \frac{1}{F(v_i)}[G(v_i) - G(w_\mu)]$. But then

$$\frac{\mathrm{d}w_\mu}{\mathrm{d}\mu} = -p \Rightarrow$$

$$\frac{\mathrm{d}b(v_i;\mu)}{\mathrm{d}\mu} = -p\frac{F(w_\mu)}{F(v_i)} + (v-\mu p)\frac{f(w_\mu)}{F(v_i)}(-p) - \frac{1}{F(v_i)}G'(w_\mu)(-p) = -p\frac{F(w_\mu)}{F(v_i)}.$$

Thus, $\frac{\mathrm{d}w_\mu}{\mathrm{d}\mu} < 0$.

Therefore, the compliant firms reduce theirs bids as well, when μ increases. This is a consequence of the fact that there are less noncompliant types and these noncompliants choose lower bids.

A.3 Proof of Theorem 3

This section derives the equilibrium strategies $b_1(\cdot), b_2(\cdot)$ of the players as symmetric non-decreasing functions of the players' types, v_i, $i = 1, 2$, for the bonification mechanism.

Considering the decisions of the concessionaire in the Operation phase, it is possible to separate a player's type in basically two sets of types, one including the types who will make the investment I in the Operation phase, which we call the investors, and the other one including those types who will not invest, the noninvestors.

A player i of type v_i will be a noninvestor if $v_i < w_d = v - d + I$ and will invest if $v_i \geq w_d$. Let us analyze the behavior of a player in each one of these categories separately.

Consider first a noninvestor player i's bid. If he wins, he will have expected profit v in the concession period. Therefore, he will never choose a bid higher than that profit. Suppose he makes a lower bid $\beta_i < v$. Then his opponent, when she is also a noninvestor type, can make a bid β_{-i}, $\beta_i < \beta_{-i} < v$ and win. Therefore, $\beta_i < v$ cannot be a best response for player i. Hence, a Bertrand-type analysis implies that noninvestor types will all choose bid $\beta_i = v$, regardless of their value $v_i < w_d$:

$$b_i(v_i) = v, \qquad \forall v_i < w_d.$$

Consider now an investor player i's bid. For that player, $v_i \geq w_d$.

Suppose first that $v_i = w_d$. Then, if she chooses a bid $\beta_i < v$, then she will surely loose. On the other hand, if she chooses a bid $\beta_i > v$, her utility when she wins will be $v_i - (I + d) - \beta_i = v - \beta_i < 0$. Therefore, a best response is to set $b_i(w_d) = v$.

Suppose next that $v_i > w_d$. Then, she will choose a bid slightly higher than v and win for sure if her opponent is a noncompliant or of type $v_i = w_d$, which occurs with probability $F(w_d)$. Therefore, if she chooses bid $\beta_i > v$, her expected utility is

$$(v_i - I + d - \beta_i)F(w_d) + \frac{v_i - I + d - \beta_i}{2}\Pr[\beta_i = b_{-i}(v_{-i})] \\ + (v_i - I + d - \beta_i)\Pr[\beta_i > b_{-i}(v_{-i}) > v].$$

Therefore, the best response of a compliant player of type $v_i > w_d$ is the solution β_i to the following maximization problem:[21]

$$\max_{\beta_i}(v_i - I + d - \beta_i)F(w_d) + \frac{v_i - I + d - \beta_i}{2}\Pr[\beta_i = b_{-i}(v_{-i})] \\ + (v_i - I + d - \beta_i)\Pr[\beta_i > b_{-i}(v_{-i}) > v].$$

Let us look for a Bayesian Nash equilibrium (b_1, b_2) where the strategy of the compliant-type player is strictly increasing, i.e., for $v_i, v_i' > w_d$, $v_i > v_i' \Rightarrow \beta_i(v_i) > \beta_i(v_i')$, $i = 1, 2$.

Then, b_{-i} is strictly increasing on $[w_d, V]$ and, thereby, invertible. Therefore, $\Pr[\beta_i = b_{-i}(v_{-i})] = 0$, and

$$\Pr[\beta_i > b_{-i}(v_{-i}) > v] = \Pr\left[b_{-i}^{-1}(\beta_i) > v_{-i} > b_{-i}^{-1}(v)\right] \\ = \int_{w_d}^{b_{-i}^{-1}(\beta_i)} f(v_{-i})\mathrm{d}v_{-i} = F\left(b_{-i}^{-1}(\beta_i)\right) - F(w_d).$$

Hence, her maximization problem can be reduced to

$$\max_{\beta_i}(v_i - I + d - \beta_i)F(w_d) + (v_i - I + d - \beta_i)\left[F\left(b_{-i}^{-1}(\beta_i)\right) - F(w_d)\right].$$

Or, equivalently,

$$\max_{\beta_i}(v_i - I + d - \beta_i)F\left(b_{-i}^{-1}(\beta_i)\right).$$

Assuming that the objective function is strictly concave, the solution to this maximization problem is obtained by calculating its first order condition (FOC).

$$\frac{\mathrm{d}}{d\beta_i}(v_i - I + d - \beta_i)F\left(b_{-i}^{-1}(\beta_i)\right) \\ = -F\left(b_{-i}^{-1}(\beta_i)\right) + (v_i - I + d - \beta_i)f\left(b_{-i}^{-1}(\beta_i)\right)\left(b_{-i}^{-1}\right)'(\beta_i) = 0.$$

In a symmetric equilibrium, all bidders choose the same bid function, i.e.,

[21] We ignore the additional condition $\beta_i > v$ and check that the solution indeed satisfies that condition.

$b_1(v) = b_2(v)$. Denote by $b(v)$ that common bid function.

Note that the solution β_i to the player's problem is that player's bid, therefore, $\beta_i = b(v_i)$ and, since b is invertible (for $v_i > w_d$), $v_i = b^{-1}(\beta_i)$. Therefore, the above FOC can be rewritten as

$$-F(v_i) + (v_i - I + d - \beta_i)f(v_i)\big(b^{-1}\big)'(\beta_i) = 0.$$

Now recall that if b is an invertible, differentiable function, then its inverse is also differentiable and $\big(b^{-1}\big)'(\beta_i) = [b'(v_i)]^{-1}$. Hence, the FOC can be written as

$$F(v_i)b'(v_i) + b(v_i)f(v_i) = (v_i - I + d)f(v_i)$$

Therefore, from the Fundamental Theorem of Calculus, for every $w \in (w_d, v_i]$, $b(v_i)F(v_i) - b(w)F(w) = \int_w^{v_i}(y - I + d)f(y)\mathrm{d}y$.

Now, by continuity, since $b(w_d) = v$, it follows that

$$b(v_i) = v\frac{F(w_d)}{F(v_i)} + \frac{1}{Fv_i}\int_{w_d}^{v_i}(y - I + d)f(y)\mathrm{d}y. \tag{8}$$

In summary, the solution to the auction game can de written as

$$b(v_i) = \begin{cases} v & \text{if } \; v_i \leq w_d = v - d + I, \\ v\frac{F(w_d)}{F(v_i)} + \frac{1}{F(v_i)}\int_{w_d}^{v_i}(y - I + d)f(y)\mathrm{d}y & \text{if } \; v_i > w_d = v - d + I. \end{cases} \tag{4}$$

Note that $b(v_i)$ is indeed bigger than v for $v_i > w_d$. Furthermore, this will indeed be the solution to the Bonification Mechanism's auction game only if, when one replaces $b^{-1}(\beta_i)$ in the original maximization problem, one obtains a strictly concave function. This can be checked once the ex-ante distribution function is made explicit.