

# Use of ordinal multi-criteria methods in the analysis of the Formula 1 World Championship

João Carlos Correia Baptista Soares de Mello\*

Luiz Flávio Autran Monteiro Gomes\*\*

Eliane Gonçalves Gomes \*\*\*

Maria Helena Campos Soares de Mello\*\*\*\*

## Abstract

*Sporting activities apply various multi-criteria (and multi-decision maker) methods in their championship regulations. In the majority of cases, the organizers are not aware that they are using them and indeed often apply them incorrectly. Variations of the Borda voting and lexicographic methods are among those most widely used. In the case of the Formula 1 World Championship, a variation of the Borda method is employed. This article shows that some of the problems that occurred during recent seasons are a result of the use of this method with consequent aggravation of the inherent distortions. Furthermore, the use of the Condorcet method is suggested as an alternative. Its limitations are shown, as well as how to overcome them by employing the Copeland method. Lastly, an analysis of the results of the 2002 championship is conducted.*

**Key words:** Borda method – Condorcet method – Copeland method.

## 1. Introduction

A sporting championship is a composite series of games or trials, the results of which are taken as a whole to establish the final result of the competition. At times there is total aggregation, though at others each individual result indicates the shape of the next game. In both cases, if each game is interpreted as a criterion, or a decision-maker, the final result of the championship is a multi-criteria problem, and usually of an ordinal nature.

Multi-criteria Decision Support formally arose as a branch of Operational Research in the 1970s. However, some elementary methods had already been in existence since the French Revolution. The ordinal methods of Borda and Condorcet (Soares de Mello, 2002), which are considered, respectively, the forerunners of the American and French schools of Multi-criteria Decision Support, date from this period. These methods were originally designed to be used by multiple decision-makers, and were applied to problems involving law court judgments. Today, their use is no longer widespread. Nevertheless, some variations of the Borda method serve as a basis for sporting regulations, and can also be applied as parameters in the evaluation of suppliers for an oil company (Rocha & Cavalcanti Netto, 2002). The Condorcet method, on the other hand, has ended up being used implicitly in some problems which use ELECTRE I, when all the criteria have the same weight and neither discordance nor vetos are considered (Soares de Mello et al, 2004). Another elementary method employed in sports is the lexicographic method, principally in the preparation of the Olympic Games medal tables (Gomes et al, 2001, Lins et al, 2003, Soares de Mello et al, 2001).

The scope of this paper is to show that the regulations of the Formula 1 World Championship follow a variation of the Borda method, combined with the lexicographic method. Some of the polemic situations, which have occurred in various races, notably in 2002, are a direct consequence of the properties of this method and of the variation adopted. The disadvantages of the method will be explained, accompanied by an illustration of how

\* Department of Production Engineering – Universidade Federal Fluminense. E-mail: gmajcsm@vm.uff.br.

\*\* Faculdades Ibmecc/RJ. E-mail: autran@ibmeccrj.com.

\*\*\* Embrapa. E-mail: eggomes@yahoo.com.br.

\*\*\*\* Department of Production Engineering – Universidade Federal Fluminense. E-mail: helena\_mello@yahoo.com.br

Artigo recebido em e aceito em junho de 2004 e aceito em agosto de 2004.

such disadvantages can be reduced, in addition to a listing of the potential consequences of the use of the Condorcet method as a replacement to the Borda method. The 2002 championship is analyzed using the Condorcet method, showing a major intransitivity cycle. To control this cycle, the Copeland method will be presented and applied.

## 2. Multi-decision maker methods

According to Arrow (1951), cited by Barba-Romero and Pomerol (1997), just choices do not exist, in other words, there is no "perfect" multi-criteria or multi-decision maker method. A multiple decision-maker selection method could only be considered just if it obeyed the axioms of universality, unanimity, and independence in relation to irrelevant alternatives, transitivity and totality. The Arrow theorem states that, with the exception of dictatorial methods, no choice or decision-aiding method serves all of these axioms simultaneously.

The axioms of independence in relation to irrelevant alternatives of transitivity and of universality are of special interest to this study. The first states that the order of preference between two alternatives must not depend on their preferences in relation to a third alternative. The transitivity axiom states that if one alternative is preferable to a second, and this one to a third, then the first must be preferable to the third (the fact that in the results of football matches this property is not confirmed is the reason for the popular saying that "football has no logic"). The universality axiom, meanwhile, requires the method to function, respecting all the other axioms, for any group of preferences of the decision-makers. Thus, a method that respects the axioms in some particular cases, does not respect universality.

## 3. The Borda and Condorcet methods

For the use of the Borda method each decision-maker must order the alternatives according to his/her preferences. The alternative of highest preference is assigned one point, the second two points and so on successively. In the end, the points assigned by the decision-makers to each alternative are added up and the alternative that has obtained the lowest score will be chosen (Dias et al, 1996). All of the alternatives are ordered in a decreasing order of points, (which guarantees adherence to the totality axiom). In sport, variations of the Borda method are widely used, with each competition considered a decision-maker, indicating its preferences in the final classification of the competition. It is common to make an inversion of the method, assigning a greater number of points to the preferred alternative (the victor in the competition). This modification is of little importance, but others, which generate distortions, are also employed and will be described below. It is interesting to note that one of the few examples of the use of the original Borda method is in the Olympic Games yachting competitions.

In spite of its simplicity and the widespread use of its variations, the Borda method does not respect one of Arrow's axioms, namely that the final classification of two alternatives is not independent in relation to irrelevant alternatives. This fact can create undesirable situations, such as a vote where the last voter knows the preferences of the previous voters and alters his/her preferences so as to give greater chances to his/her preferred alternative. It can also, which is of specific interest in this case, encourage the unsporting inversion of positions in a competition to benefit a given competitor.

In the Condorcet method, each decision-maker is also required to order all the alternatives according to his/her preferences. However, instead of assigning points to each alternative, the method establishes relations of superiority. For each pair of alternatives one should identify which alternative is preferred by the majority of decision-makers. It is then said that this alternative is preferred with respect to the other alternative. Graphs can be traced representing these preference relationships, in which the arc  $(u, v)$  belongs to the graph if, and only if, the number of decision-makers who preferred  $u$  to  $v$  is greater than, or equal to, the number of those who preferred  $v$  to  $u$ . These results are analogous to those obtained using the ELECTRE I method (Roy & Bouyssou, 1993), provided there is no veto or discordance or indifference thresholds.

The determination of dominant and dominated alternatives (when they exist) is made substantially easier by representation of the preference relationship on a graph. When there is one and only one dominant alternative,

it is chosen. The Condorcet method, considered more equitable than the Borda method, has the great disadvantage of leading to situations of intransitivity, in turn leading to the celebrated “Condorcet paradox”. This occurs when *A* is preferable to *B*, *B* is preferable to *C* and *C* is preferable to *A* (a situation known as the “Condorcet Triplet”, illustrated in Figure 1). This means that the Condorcet method does not always induce a pre-order in the set of alternatives. However, there are situations in which cycles of intransitivity do not occur, as for example in political voting in which electors have a collective coherence, ordering the alternatives from the most leftwing to the right, or vice-versa. In situations similar to this, the Condorcet method should be preferred over the Borda method. Nevertheless, it is worthwhile remembering that a multi-decision maker method must respect the axiom of universality, and this does not occur with the Condorcet method.

In short, the choice between the Borda method and the Condorcet method is a choice between permitting situations where results might be manipulated or experiencing difficulties in obtaining a complete final result.

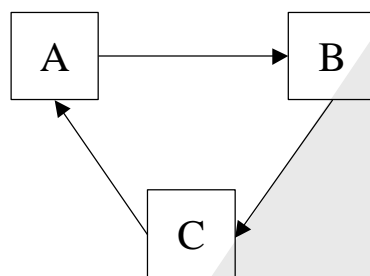


Figure 1 – Condorcet Triplet.

#### 4. The Formula 1 World Championship

Prior to 2002, the Formula 1 World Championship rules established that the champion of the season was the driver accumulating the highest number of points at the end of the 17 races of the season. The other drivers were classified in the championship according to the total number of points they had scored, and only the first six drivers to finish were awarded points in each race. The points awarded for each place are presented in Table 1.

Table 1 – Awarding of points in Formula 1 prior to 2002.

Place	Points
1 <sup>st</sup> Place	10 points
2 <sup>nd</sup> Place	6 points
3 <sup>rd</sup> Place	4 points
4 <sup>th</sup> Place	3 points
5 <sup>th</sup> Place	2 points
6 <sup>th</sup> Place	1 point

This rule is, in fact, a variation of the Borda method. The most obvious difference in relation to the traditional Borda method is that those placed first are awarded more points, while in the original method they were awarded less. This is justified by the fact that not all competitors finish, or even participate, in all of the races. A driver who did not participate would not score any points, which would put him in a better situation than the driver placed first. It is, therefore, an inversion which brings an improvement to the operationalization of the method, without creating any disastrous consequences.

The other difference is more important: while in the original method the difference between any two places is the same, as it deals with an ordinal method, in Formula 1, the difference between the first and second places is greater than that between the second and third (4 and 2 points respectively). From the third to the seventh place the difference is only one point and for positions worse than seventh there is no difference, as no competitors

score any points. The intention is to boost the value of the victory without giving undue attention to the dispute for the last places.

This difference results in severe distortions, although these were instituted with the best of intentions. In theoretical terms, the first distortion is caused by trying to deal with an ordinal method in a cardinal way. The effect of this is that, if the first two drivers finish with only a small difference between them, there will nevertheless be a greater difference in points scored than between any other two drivers finishing lower down in the order, even if they cross the line with a large margin between them.

The second consequence is more serious. As the difference in points awarded between two drivers with immediate classifications is different according to the position, the lack of independence in relation to irrelevant alternatives is aggravated. This occurs both in the first places and in the last positions. As regards these, which are less observed by the media, it could happen that a driver always finishes in front of another in positions lower than sixth. However, in the last race of the season, the situation is inverted and the driver who habitually finished behind gains a place scoring a point, due to various race leaders abandoning the race, for example. This driver finishes the championship obtaining a better classification than the other driver who beat him in nearly all of the races. In other words, a driver who habitually finished among the last places in races, only needs to avoid dropping out of a race in which a large number of drivers fail to finish in order to obtain a reasonable classification in the championship. By the same token, an average driver with race results between 7<sup>th</sup> and 10<sup>th</sup> place, might not have the good fortune to remain on the track in a race during which a high number of drivers drop out and would consequently finish without scoring any points.

In relation to the first places, various situations have already been reported. If two drivers from the same team occupy the top two positions, they can trade positions to allow one of them to benefit from the greater difference in points assigned to the first place to improve his situation in the championship. This situation was widely reported in 2002 when, in the Austrian Grand Prix, Rubens Barrichello relinquished first place to Michael Schumacher close to the finishing line. This maneuver was a direct consequence of the rules: without Schumacher changing his position in relation to his main rival (Juan Pablo Montoya), his advantage in the championship increased significantly, thanks to Barrichello (who, in this case was the irrelevant alternative) finishing between the two of them. Curiously, it is worth noting that at the end of the championship, when the dispute was for second place in the championship, the inverse strategy was used on various occasions. Naturally, even if the original Borda method were used, the inversion would reap benefits for the Ferrari team though, with a smaller difference in points, the benefits accrued would also be smaller.

There is also a situation which affects the best teams as much as the worst and intermediate ones. This is the case of a well-placed driver being disqualified. The situation alters the point-scoring relationship among the first-placed drivers and means that a driver who had not initially scored any points receives some points. It can alter the final constructors' classification, with possible financial consequences.

In order to avoid this situation, where there has already been an instance of a driver being disqualified after a race, the other drivers in lower positions have not had their positions altered. Or, there have been even more extreme cases, such as that which occurred in 1997, when, after the last race, Michael Schumacher was punished for a repeat offence involving unsporting behavior for the way in which he tried to stop Jacques Villeneuve from overtaking in the Spanish Grand Prix. He was stripped of all his points and second place in the championship, but his classification in all the races was maintained. If he had been disqualified from all of the races of the season, there would have been serious alterations to the results of the championship.

The rules also foresee the possibility of a tie in the final point totals, establishing successive criteria to break the tie and decide the winner. In this way, the rules, in fact, use the lexicographic method, with the most important criterion (and therefore the first to be employed) being the points obtained using the modified Borda method. When there are two or more alternatives with the same number of points at the end of the championship, the number of race victories of each driver is considered to break the tie. If the alternatives are still tied, the second criterion is the number of races in which each driver finished in second place and so on successively.

Since a victory depends on the performance of other drivers, the use of the number of victories as the second criteria in the lexicographic method aggravates the lack of independence in relation to the irrelevant alternatives even further.

## 5. Analysis of the 2002 Championship

The 2002 championship was characterized by the absolute domination of the Ferrari team, with the Williams and McLaren teams disputing the following positions. Due to this consistency in the results it was easy to make an analysis by ordinal methods. This classification eliminates the distortions explained earlier, caused by the way in which ordinal methods have been used.

The Condorcet method was used in order to obtain a classification without these distortions. Table 2 shows the matrix of adjacency obtained by the Condorcet graph related to the 2002 Formula 1 championship. The number 1 signifies that the driver shown in the line obtained a better classification more times than the driver whose initials are shown in the column. In order to construct this matrix it was decided that when two drivers failed to finish a race the one who completed more laps achieved a better classification. In addition to this, a driver who did not finish a race scored better than a driver who did not achieve a sufficient lap time to qualify for the grid, and, in turn, this driver was ranked higher than another who did not participate in the time trials. The spaces left blank are equivalent to zeros. The order in which the drivers appear in the matrix is the same as their official classification in the championship.

The final classifications of all the races were obtained from the site://www.insidef1.com.

In order to extract an ordering from the matrix, one starts with a descending distillation (Dias et al., 1996). For this, one observes if any driver beat all the others, in other words, if there is any line whose only zero is in the main diagonal. This driver is removed and the procedure repeated. With this distillation it is possible to select 5 drivers, namely M. Schumacher, R. Barrichello, J. P. Montoya, D. Coulthard and R. Schumacher. It should be noted that this ordering of the five best-placed drivers is the same as the official classification, albeit with an inversion in the fourth and fifth places.

For positions below R. Schumacher, the distillation cannot be continued, as there is no dominant driver among those remaining. The inverse procedure, an ascending distillation, was therefore performed. The last placed drivers are, thus, starting from the lowest, A. Davidson, E. Bernoldi, A. Yoong, H. H. Frentzen, A. McNish, P. de la Rosa, M. Webber and O. Panis. Above O. Panis the distillation cannot be continued, as there is no dominated driver. These drivers, M. Webber, O. Panis and H. H. Frentzen obtained points in the official classification, being amongst the top 18 drivers. They were the major beneficiaries of the distortions of the Borda method, aggravated by the changes in its application.

After removing the drivers ordered by the two distillations, 10 drivers are still left. Among them there must be cycles of intransitivity. It remains to be checked if there are several cycles, with one of them dominated by the others. An analysis of the relationships between the remaining drivers shows a strongly connected graph (Boaventura Netto, 2002), in other words, a graph where, for any pair of drivers, there is a path (formed by one or more arcs) connecting the representative vertices of the two drivers. The remaining drivers therefore form a great cycle of intransitivity, resulting in it not being possible to establish an ordering with the Condorcet method.

Table 2 – Matrix of adjacency of the Condorcet graph for the 2002 championship.

	MS	RB	JPM	RS	DC	KR	JB	JT	EI	NH	GF	FM	JV	OP	MW	MSA	HHF	TS	PLR	AY	AM	EB	AD
M. SCHUMACHER	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
R. BARRICHELLO			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
J. P. MONTOYA				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
R. SCHUMACHER						1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D. COULTHARD				1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
K. RAIKKONEN								1			1	1		1	1		1		1	1	1	1	1
J. BUTTON						1			1	1		1	1	1	1	1	1	1	1	1	1	1	1
J. TRULLI							1		1		1	1	1	1	1	1	1	1	1	1	1	1	1
E. IRVINE						1				1	1	1	1	1	1	1	1	1	1	1	1	1	1
N. HEIDFELD						1		1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
G. FISICHELLA							1				1		1	1			1		1	1	1	1	1
F. MASSA												1	1		1	1	1	1	1	1	1	1	1
J. VILLENEUVE						1				1	1		1	1			1	1	1	1	1	1	1
O. PANIS															1		1		1	1	1	1	1
M. WEBBER																	1		1	1	1	1	1
M. SALO						1				1		1	1	1			1	1	1	1	1	1	1
H. H. FRENTZEN																				1		1	1
T. SATO						1				1			1	1			1		1	1	1	1	1
P. DE LA ROSA																	1			1	1	1	1
A. YOONG																						1	1
A. McNISH																	1			1		1	1
E. BERNOLDI																							1
A. DAVIDSON																							1

There are some interesting properties in this graph. In the first place, it is a non-planar graph, in fact an asymmetric clique of 10<sup>th</sup> degree. It has therefore 45 arcs, which makes it problematic to extract relevant information from its topological representation. Figure 2 shows the partial topographic representation of the graph, in which, for the sake of clarity, only 10 of the 45 arcs are shown, enough to visualize a cycle of maximum intransitivity. In second place, there is low vertex connectivity. It is possible to extract only two vertices, and, of the 8 remaining drivers, at least one must be dominant or dominated. In other words, the extremely irregular classifications of a small number of competitors provoked intransitivity between 10 drivers!

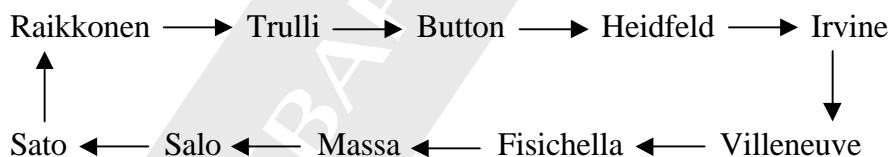


Figure 2 – Representation of a cycle of maximum intransitivity.

To order these 10 drivers, and thus obtain a final championship classification, another method needs to be used. In some cases it is possible to use relaxations of the Condorcet conditions (Rodríguez and Pérez, 2003). However, the choice of the majority qualified for rejection introduces an element of undesired subjectivity. One alternative is to use the Copeland method (Barba-Romero and Pomerol, 1997). This method consists of assessing how each alternative is preferable in relation to the many available alternatives, in other words, adding up the elements of each line of the adjacency matrix. The alternatives are ordered by the result of this sum. The Copeland method allies the advantage of supplying a total ordering to the fact of giving the same result as Condorcet, when this does not present any cycle of intransitivity. When these cycles exist, the Copeland method allows the ordering to be performed and maintains the classification of the alternatives (drivers) who do not belong to any cycle of intransitivity.

The classification obtained by the Copeland method, compared with the official classification, is presented in Table 3. In this table, tie-breaking criteria were not considered.

Table 3 – Final Results for 2002 (official and Copeland)

Copeland Classification	Driver	Copeland Points	Official Classification
1	M SCHUMACHER	22	1
2	R BARRICHELLO	21	2
3	J P MONTOYA	20	3
4	D COULTHARD	19	5
5	R SCHUMACHER	18	4
6	N HEIDFELD	16	10
7	J BUTTON	15	7
7	J TRULLI	15	8
9	E IRVINE	14	9
10	J VILLENEUVE	12	12
10	M SALO	12	16
12	K RAIKKONEN	11	6
13	G FISICHELLA	10	10
13	F MASSA	10	12
13	T SATO	10	18
16	O PANIS	7	14
17	M WEBBER	6	15
18	P DE LA ROSA	5	-
19	A McNISH	4	-
20	H H FRENTZEN	3	17
21	A YOONG	2	-
22	E BERNOLDI	1	-
23	A DAVIDSON	0	-

## 6. Conclusions

During the course of this article, by the formal analogy between the Formula 1 World Championship and a multi-decision maker process, it has been shown that there is no rule which can be considered fully just.

However, the rule in force prior to 2002 aggravated the defects of the Borda method upon which it was based. The Condorcet method makes it possible to overcome the distortions of the variation of the Borda method, but it does not always permit a complete ordering, owing to the existence of cycles of intransitivity. In addition to this, it is an extremely technical method that is not easily understood by the general public.

By applying the Copeland method to the results of the 2002 championship it was seen that it is possible to supersede the cycle of intransitivity. This method provides the same result as the Condorcet method in the absence of intransitivity and is easier to explain to the general public. When intransitivity cycles arise, the solution which it supplies is less sensitive to irrelevant alternatives than the Borda method.

The comparison of the official result with that obtained by the Copeland method shows very similar results in the top places, with greater discordance in the lower positions. This is a natural result, as the variation of the Borda method used only allocated points to the first six of twenty competitors. The most marked differences in classification occurred with the drivers Raikkonen and Heidfeld. The first obtained several very low finishing positions along with a few very good results. As the official classification made no distinction between any position below 6<sup>th</sup> place, Raikkonen ended up obtaining a reasonable official score. As the Copeland method takes into account all the results, this approach lowers the final position of the driver.

The opposite occurred with Nick Heidfeld: he obtained various classifications close to 6<sup>th</sup> place, none of which scored any points, and a few classifications scoring a couple of points. This regularity was penalized in the official classification and recognized in the Copeland method. It is worth noting that this analysis shows the pertinence of the complaint lodged by Peter Sauber (owner of the team for which Heidfeld races) concerning

the injustice that his team was suffering, as results after sixth place were not taken into consideration. In fact, although it did not officially accept the complaint, the FIA changed the point scoring scheme for 2003. This new system attributes points to 8 drivers, and the differences between the first positions and the following places is reduced. Thus, the distortions mentioned above are reduced and the 2003 championship produced a result closer than that which would have been obtained using the Copeland method. To a greater extent than the innocuous prohibition of "team driving", while not yet the ideal solution, this change contributes to the eradication of unsporting attitudes.

## References

- Arrow, K.J. *Social Choice and Individual Values*. Wiley, New York, EUA, 1951
- Barba-Romero, S. & Pomerol, J.C. *Decisiones Multicriterio: Fundamentos Teóricos e Utilización Práctica*. Colección de Economía, Universidad de Alcalá, Espanha, 1997.
- Boaventura Netto, P.O. *Grafos: Teoria, Modelos, Algoritmos*. Edgard Blücher, São Paulo, 2ª edição, 2002.
- Dias, L.M.C., Almeida, L.M.A.T. & Climaco, J.C.N. *Apoio Multicritério à Decisão*. Faculdade de Economia da Universidade de Coimbra, Coimbra, Portugal, 1996.
- Gomes, E.G., Soares de Mello, J.C.C.B. & Lins, M.P.E. (2001). Uso de Análise de Envoltória de Dados e Auxílio Multicritério à Decisão na análise de dados das Olimpíadas 2000. *Anais do XXI ENEGEP*, Salvador, Brasil.
- Lins, M.P.E., Gomes, E.G., Soares de Mello, J.C.C.B. & Soares de Mello, A.J.R. (2003). Olympic ranking based on a Zero Sum Gains DEA model. *European Journal of Operational Research*, 148 (2), 85-95.
- Rocha, R.B. & Cavalcanti Netto, M.A. (2002). A data envelopment analysis model for rank ordering suppliers in the oil industry. *Pesquisa Operacional*, 22 (2), 123-132.
- Rodríguez, C.M.C. & Pérez, J.A.M. (2003). Relaxation of the Condorcet and Simpson conditions in voting location. *European Journal of Operational Research*, 145 (3), 673-683.
- Soares de Mello, J.C.C.B., Gomes, E.G., Lins, M.P.E. & Soares de Mello, A.J.R. (2001). Uso da Pesquisa Operacional em esportes: o caso das Olimpíadas. *Boletim da SOBRAPO*, 19, 5-6.
- Soares de Mello, M.H.C. *Avaliação de Desempenho nas Engenharias: Estudo de Caso UFF*. Tese de Mestrado, Engenharia de Produção, Universidade Federal Fluminense, Niterói, 2002.
- Soares de Mello, M.H.C., Quintella, H.L.M.M., Soares de Mello, J.C.C.B., Avaliação do desempenho de alunos considerando classificações obtidas e opiniões dos docentes, *Investigação Operacional* 24 (2004) (no prelo)