Inflation level and uncertainty: evidence using Brazilian data*

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Durante várias décadas, economistas suspeitaram da existência de uma relação positiva entre a média e a variância da inflação. Essa crença congrega nomes díspares como Milton Friedman e Arthur Okun. Testando essa hipótese para os EUA, Robert Engle não consegue rejeitar a hipótese de que o nível da inflação não afeta a variância condicional da inflação usando um modelo ARCH (*Autoregressive Conditional Heteroskedasticity*). Essa evidência, se verdadeira, remete economistas ao marco zero em termos de explicar os custos associados à inflação. O presente *paper* propõe testar a existência de uma relação positiva entre o nível e a variância condicional da inflação, usando um modelo onde a variância condicional da inflação depende de séries temporais incluindo o nível da inflação. Nesse modelo flexível, encontrou-se evidência a favor do postulado de Friedman e Okun. Um possível problema com o teste usado diz respeito ao fato de que a variância condicional da inflação se comporta como um processo integrado de ordem um, *i.e.*, I(1).

1. Introduction and motivation

Most economists would agree that inflation is costly. However, inflation costs are usually associated with the deadweight loss from "inflation tax". The problem with this view is that this loss is small if an economy is not under hyperinflation. Also, since perfect indexation is theoretically possible, it seems hard to explain under that framework why a society should prefer, say, a 1% to a 10% a *month* inflation rate.

One way around that puzzle is to postulate that there is a positive relashionship between the mean and variance of inflation. Clearly, for a given mean, an increase in the variance of inflation corresponds to a Mean-Preserving spread on the density function of inflation. Of course,

* The author thanks Robert F. Engle for his suggestions on the estimation results and assumes the responsability for all possible errors.

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risk-averse individuals dislike such changes in the p. d. f. of inflation. If the mean of inflation is itself changing over time and the variance (uncertainty) of inflation is positively related to the mean of inflation then risk-averse individuals would prefer low levels of inflation since these correspond to low uncertainty about inflation and therefore to small welfare losses. That kind of argument is presented by Okun (1971) and Friedman (1977) to defend the view that individuals prefer low inflation *vis-à-vis* high inflation. The problem with this argument is that the empirical evidence in Engle (1983) shows no link between the conditional variance of inflation and the inflation level for the U.S. economy. Testing was conducted using and Autoregressive Conditional Heteroskedasticity (Arch) model.[1] This allows for testing the hypothesis that the conditional variance of inflation depends on the mean of inflation under an already flexible structure for the former. This is an advantage for this testing procedure when compared to a simple White's test using a constant conditional variance specification.

The empirical drawback of Okun and Friedman's proposition is serious, since it sends theoreticians back to square one in explaining inflation costs. The objective of this paper is to put their hypothesis to another empirical test, using data from a country that expecienced a high variation in mean inflation by world standards. This choice seems appropriate since for the U.S. economy the mean of inflation does not change widly over time. The country of choice is Brazil, where inflation varied from 12% to 220% a year over the period 1972-85.

2. The model and the data

The model used is a simple reduced form for inflation ($\Delta \ln P_t$) under the assumption that the money supply (M_t) is weakly exogenous.[2] Then, inflation will depend on its own lags and also on the money growth and its lags. Since we will test for the effect of mean inflation on the conditional variance of inflation, a Generalized Arch (Garch) specification for the latter is used, augmented by the inclusion of lagged inflation. In terms of regression equations this model would have the following general specification:

$$\Delta \ln P_t = \delta_0 + \sum_{i=1}^{r} \delta_i \Delta \ln P_{t-i} + \sum_{i=0}^{s} \gamma_i \Delta \ln M_{t-i} + \varepsilon_t \qquad (1)$$

[1] Arch models were first introduced by Engle (1982). Generalized Arch (Garch) models were proposed by Bollerslev (1986). Engle, Lilien and Robins (1987) later introduced the Arch in mean model (Arch-m).

[2] This allows conditioning on current money supply growth when modelling the inflation level. See Engle, Hendry and Richard (1983).

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=0}^{p} \beta_i h_{t-i} + \sum_{i=1}^{m} \lambda_i \Delta \ln P_{t-i} \qquad (2)$$

where $\varepsilon_t \mid \Psi_t \sim N(0, h_t)$, and Ψ_t is the information set containing lagged $(\Delta P_t, \Delta M_t)$ and current ΔM_t.

The data used is provided by the International Monetary Fund (IMF). The price level is the seasonally adjusted wholesale price index for the Brazilian economy and the money supply is a measure of M_2, with time and saving deposits added to M_1. This latter data series is not seasonally adjusted. The data is available on a monthly basis from 1971:1 to 1985:12. The absence of the last five years of the 1980's from the data set is due to the fact that the IMF stopped collecting data on the Brazilian money supply in 1985. Since the data will be used in log differences, a first step was to identify and remove any possible deterministic seasonal component from $\ln P$ and $\ln M$. This was accomplished by running each univariate series on monthly seasonal dummies and testing their individual and joint significance. For both series, the joint significance of the dummies produced an F-test that was virtually zero and individually the highest T-statistic found on both was around 0.8. Therefore, no deterministic component was removed and inflation and money growth were calculated using first differences $(1 - B)$, where B is the backshift operator.[3] Thus, the dependent and explanatory variables are respectively monthly inflation and monthly money growth.

The plot of $(1 - B)\ln P_t$ is presented in figure 1 in the appendix. This series exhibits two interesting features: first it appears to be non-stationary and probably an integrated process of order one [I(1)]. The sample mean rises steadily from about 2% a month in the early seventies to about 10% in the mid eighties. Second it also shows some Arch structure, with some clusters of high variability followed by some periods of low variability. Moreover, it seems that the variability of the final sample periods is greater than that of the early ones, suggesting a possible positive link between the mean and variance of inflation.[4]

[3] Another approach was also used to differentiate data but it proved unsatisfactory. It consisted in using the $(1 - B^{12})$ operator on $\ln P_t$ and $\ln M_t$. The final regression results using this alternative technique showed a high lag 12 partial correlation coefficient for the residual of the estimated Garch models. For this reason it was dropped in favor of the $(1 - B)$ operator, which showed no such problem. The author thanks Robert Engle for pointing this out to him.

[4] Note that the standard deviation of inflation from 1980-85 is twice as big as that for the period 1971-79. See the appendix.

To investigate if inflation is $I(1)$, the Augmented Dickey Fuller (ADF) test was performed on $(1 - B)\ln P_t$. The computed T-statistic is -0.69, which accepts the null that this variable is $I(1)$ whith very high confidence. The same test performed on the money growth produced similar results. The next step was to check whether these two variables are cointegrated, *i.e.*, if there is a long run relationship between them. The Engle-Granger[5] two steps technique was used and the T-statistic found for the ADF test in the second step was -6.13, whick rejects non-cointegration at the 1% confidence level.[6] Under cointegration, the estimation of equation (1) will be done more efficiently by using an Error Correction Model (ECM) *vis-à-vis* using a Vector Autoregression (VAR). ECM estimation allows working en $I(0)$ space taking into account the long run relationship between money growth and inflation. Thus, multi-step ahead forecast errors of the ECM are not $I(0)$, whereas those of the VAR are.

Prior to the complete estimation of the model, the mean ECM was estimated by OLS, assuming a constant conditional variance. After some experimentation a preferred model included as explanatory variables: $\Delta^2 \ln P_{t-1}$, $\Delta^2 \ln P_{t-2}$, $\Delta^2 \ln M_t$ and the lagged Error Correction (EC) term. The results are presented in table 1 in the appendix.

The results in table 1 confirm the mis-specification of the constant conditional variance assumption, since the model fails both the Arch and White's test for heteroskedasticity. In the conditional mean, however, the lagged EC term coefficient has the right sign, showing that whenever inflation is high *vis-à-vis* money growth it will decline in the future *ceteris paribus*.

The same type of EC model was estimated using a Garch structure. After some experimentation the preferred model was:

$$\Delta^2 \ln P_t = \delta_0 + \delta_1 \Delta^2 \ln P_{t-1} + \delta_2 \Delta^2 \ln P_{t-2} + \gamma_0 \Delta^2 \ln M_t + \Theta EC_{t-1} + \varepsilon_t \quad (1')$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h_{t-1} + \lambda_1 \Delta \ln P_{t-1} \quad (2')$$

i.e., a Garch (1,1) with the variance equation augmented by lagged inflation.

5 See Engle and Granger (1987).

6 The cointegrating vector found was (1,-0.63), which is far from long-run proportionality.

The results of the estimation using Maximum Likelihood are:

Dependent variable: $\Delta^2 \ln P_t$		
Coefficient	Estimated value	T-statistic
δ_0	0.0003	0.27
δ_1	-0.1315	-2.69
δ_2	-0.2329	-2.79
γ_0	0.0925	3.26
Θ	-0.0938	-1.24
α_0	-0.00003	-2.63
α_1	0.0318	0.29
β_1	0.4407	1.92
λ_1	0.0035	2.63

Log likelihood = 517.9958 LM(1) for adding ε^2_{t-2} = 1.92

LR(7) for HO: constant mean and var.: 151.41

Skewness: 0.491 Kurtosis: 3.22 Ljung-box(12) $\hat{\varepsilon}$ = 9.80

Ljung-box(12) $\hat{\varepsilon}^2$ = 18.99

The results of the estimated augmented Garch(1,1) model seem satisfactory. Even though the EC term lagged is insignificant it is included due to its theoretical importance. The estimated variance parameters seem well behaved. The only unexpected result is the insignificant coefficient of $\hat{\alpha}_1$. Clearly the solution for $1 - (\hat{\alpha}_1 + \hat{\beta}_1) Z = 0$ lies outside the unit circle. Also, even though α_0 is negative and significant this is not necessarily a bad sign, since the Garch(1,1) specification is augmented with lagged inflation. The lagged inflation term is 0.0035 times the lagged monthly inflation rate. This number will lie in the interval [0.00002,0.00058]. Therefore, the inflation effect on the conditional variance will, apart from the initial sample observations, be enough to offset the negative value of α_0. Given the results above, it seems that the conditional variance of inflation is explained solely by lagged inflation, since α_1 is statistically zero. If β_1 is significant, the conditional variance of inflation will display some persistence. Thus, a rise in inflation will affect uncertainty way into the future, carried forward by the autoregressive nature of h_t.

The test whether inflation mean affects positively the conditional variance of inflation is simply a test of H_0: $\lambda_1 = 0$ *versus* $H_1 : \lambda_1 > 0$. Even though it is tempting to say that the T-statistic of $\hat{\lambda}_1$ is asymptotically normally distributed, some care is necessary. As seen before, inflation is $I(1)$. Even though we are working in $I(0)$ space in the mean, h_t is a function of an $I(1)$ variable with a significant coefficient. In general, a linear combination of $I(1)$ and $I(0)$ variables is $I(1)$, which may imply that the asymptotic distribution of the T-statistic of $\hat{\lambda}_1$ will be non-standard. Thus, we may not be able to say whether the T-statistic of 2.63 is enough to reject H_0 with reasonable confidence. One thing to keep in mind is that we are using the Maximum Likelihood method in estimation, therefore asymptotic normality may be achieved.

Subject to the above caveat, inflation mean affects positively the conditional variance of inflation for the Brazilian economy. Figure 2 in the appendix illustrates the relationship between $\hat{h}_t$, the estimated value of h_t, and ΔP_{t-1}. As noted before, it seems that h_t is being explained exclusively by lagged inflation (notice that ranges of the two variables are matched in figure 2). Some details of the search process for the preferred model are worth special mentioning: several Garch specifications were tried without the term in $\Delta l_n P_{t-1}$ up to Garch(1,4). None presented well behaved estimates, which reinforces the belief that inflation uncertainty increases with inflation mean. In most cases, the test for exclusion of a higher order ε^2_{t-i} term on the variance was significant.[7] Therefore, it seems that a Garch structure without including inflation lagged will be mis-specified.[8]

To investigate possible problems with the preferred model, sample autocorrelations of $\hat{\varepsilon}_t$ were examined (see table 2 in the appendix). The results show only a moderate tendency for ε_t to be a low order MA. Using the MA(1) specification in addition to the preferred model showed no significant improvement under the LR test. Thus, the preferred model was maintained.

The results obtained here are not inconsistent with other results of empirical studies about Brazilian inflation. One interesting such study is done by Loyola (1987), which provides some evidence that relative price dispersion depends on inflation mean. Starting with the definition of a price

[7] The Garch(1,1) and Garch(1,2) had convergence problems. The Garch(1,3) and Garch(1,4) had both a LM(1) statistic for inclusion of a higher order ε^2 term bigger than 15.

[8] Probably h_t is $I(1)$. Trying to explain it with ε_t^2 alone will not work, since ε_t^2 is $I(0)$.

index, we have: $\Delta P_t = \sum_{i=1}^{n} \alpha_i \Delta P_{it}$, where ΔP_{it} is the inflation observed in the ith good of the basket composing the index and $\sum_{i=1}^{n} \alpha_i = 1$. Using the law of iterative expectations in the linear model (2') we have:

$$E[h_t] = \mathrm{VAR}[\Delta P_t]$$

or

$$\frac{\alpha_0 + \alpha_1 \sigma_\varepsilon^2}{1 - \beta_1} + \lambda_1 \sum_{i=0}^{\infty} \beta_1^i \mu_{t-i} = \sum_{i=1}^{n} \alpha_i^2 \sigma_{\Delta P_{it}} + \tag{3}$$

$$\sum_{i=1}^{n} = \sum_{j \neq i} \alpha_i \alpha_j \mathrm{COV}(\Delta P_{it}, \Delta P_{jt})$$

$\sigma_\varepsilon^2 = \mathrm{VAR}(\varepsilon_t)$, $\mu_t = \mathrm{E}(\Delta P_t)$ and $\sigma^2_{\Delta P_{it}} = \mathrm{VAR}(\Delta P_{it})$. The evidence presented here implies that $\frac{\partial E[h_t]}{\partial \mu_t} = \lambda_1 > 0$. If the effect μ_t on the covariances is negligible, then, we should have:

$$\sum_{i=1}^{n} \alpha_1^2 \frac{\partial \sigma^2_{\Delta P_{it}}}{\partial \mu_t} > 0 \tag{4}$$

which requires that the variance of the heavily weighted goods in the basket be an increasing function of inflation mean. This final result is likely to yield a positive relation between relative price dispersion and inflation mean, as important individual prices distributions have their dispersion increased with inflation mean.

3. Conclusions

It seems that there is some evidence that inflation level affects positively the uncertainty about inflation measured by its conditional variance. This suggests that risk averse Brazilians would definitely prefer low inflation levels *vis-à-vis* high inflation levels, since the latter will imply higher inflation uncertainty.[9]

[9] If there were complete futures and insurance markets, risk-averse individuals would be willing to spend more in buying insurance against future inflation when its level is high *vis-à-vis* when it is low.

Further research should investigate how general this result is using the same methodology. Maybe some high and low inflation countries should be included. Some evidence favoring the results attained here is presented in Ball and Cecchetti (1990), although they used a different methodology. Regarding the caveat on our result, a Monte Carlo experiment may help in checking the correct critical region for the T-test of $\hat{\lambda}_t$.

Appendix

Table 1
Model estimation without Garch structure

Dependent variable: $\Delta^2 \ln P_t$			
Expl. var.	Est. coeff.	H.C.S.E.	T-estat.
$\Delta^2 \ln P_{t-1}$	-0.443	0.132	-5.43
$\Delta^2 \ln P_{t-2}$	-0.319	0.125	-4.10
$\Delta^2 \ln M_t$	0.092	0.453	2.11
EC_{t-1}	-0.169	0.086	-2.84
Const.	0.001	0.001	0.73

R^2 = 0.322 Arch test for residuals = 52.233 ~ χ_4^2

DW = 1.892 White's heteroskedasticity test = 19.179 ~ χ_9^2

Table 2
Autocorrelations of $\hat{\varepsilon}_t$ from the estimation of (1') and (2')

	Lags					
Autocorrelations		-.208	-.122	.184	-.219	.538e-01
Standard errors	1 - 5	.754E-01	.786E-01	.796E-01	.820E-01	.853E-01
Q-statistics		7.75	10.5	16.7	25.5	26.2
Autocorrelations		-.117E-01	-149E-01	.313E-01	-.609E-01	.125
Standard errors	6 - 10	.855E-01	-855E-01	.855E-01	.856E-01	.858E-01
Q-statistics		26.4	26.6	26.9	27.8	30.9
Autocorrelations		-.124	.107E-01	.105	-.219E-01	.361E-01
Standard errors	11 - 15	.868E-01	.878E-01	.879E-01	.886E-01	.886E-01
Q-statistics		34.0	34.2	36.6	36.9	37.4

Figure 1

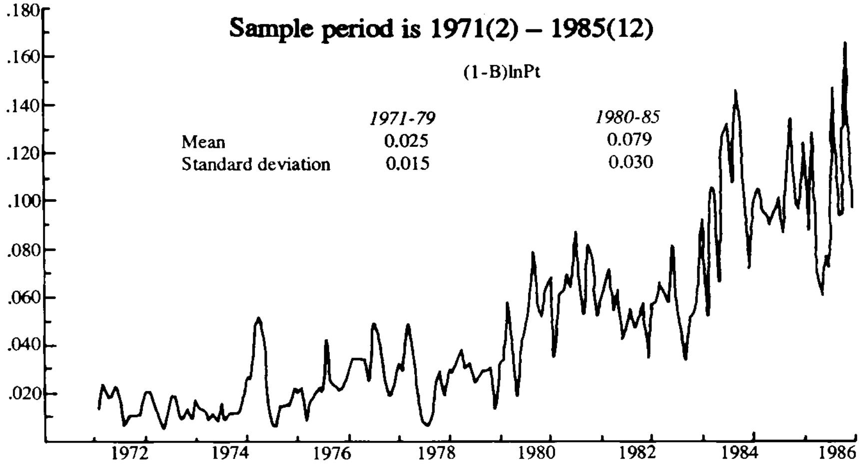
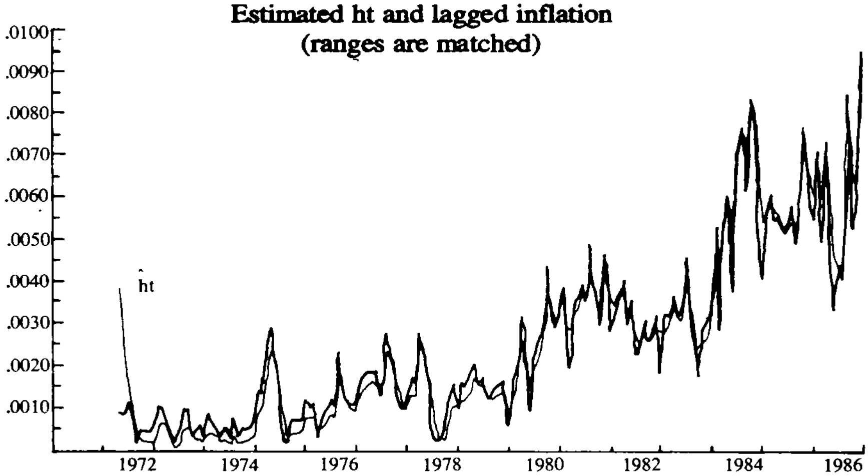


Figure 2
Estimated ht and lagged inflation
(ranges are matched)



Abstract

It has been suspected for a long time that there exists a positive relation between inflation level and its associated forecast variance. This belief congregates names like Arthur Okun and Milton Friedman. The empirical evidence for the U.S. economy however seems to reject this view. Robert Engle's (1983) result using and Autoregressive Conditional Heteroskeda-

sticity (Arch) model was very influential in changing researchers' beliefs about this possible positive relation. The objective of this paper is to revive this controversy using data from a country that has experienced high levels of inflation. A test is undertaken of the hypothesis that inflation level is positively related to the conditional variance of inflation. The test is carried out using a Generalized Arch (Garch) model, which provides a flexible theoretical structure for the conditional variance of inflation. The result of the test shows some evidence in favor of the proposed relationship, however a caveat on the result is the fact that the conditional variance of inflation behaves like an integrated process of order one, *i.e.*, I(1).

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