Attentiveness Cycles: Synchronized Behavior and Aggregate Fluctuations*

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Contents: 1. Introduction; 2. Inattentive Firms; 3. Sticky-Information Phillips Curves; 4. Macroeconomic Equilibrium; 5. Monetary Policy and Perpetual Motion; 6. Conclusion.

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A sticky-information macroeconomic model is developed in order to analyze the behavior of the time trajectories of the inflation rate and of the output gap, when disturbed by eventual monetary policy shocks. In opposition to what is typical in the literature on this subject, different paces on information updating explicitly lead to a setting with interaction among heterogeneous agents. Specifically, we consider firms with different information updating frequencies whose behavior implies the emergence of attentiveness cycles of possibly large lengths; within these cycles we deduct a differently shaped Phillips curve for each time period. Systematic changes on the form of the aggregate supply relation will be the engine that triggers a sluggish response to shocks and the eventual persistence of business fluctuations.

Um modelo macroeconómico envolvendo rigidez de informação é desenvolvido de forma a analisar o comportamento das trajetórias temporais da taxa de inflação e do hiato do produto, quando perturbadas por eventuais choques de política monetária. Em oposição ao que é comum em literatura sobre este assunto, diferentes ritmos de atualização de informação conduzem explicitamente a um cenário de interação entre agentes heterogéneos. Especificamente, consideram-se empresas com diferentes frequências de atualização de informação cujo comportamento implica a emergência de ciclos de atenção de possivelmente longa duração; no seio destes ciclos deduzem-se curvas de Phillips com diferentes formas para cada período temporal. Alterações sistemáticas na forma da relação de oferta agregada são o

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motor que despoleta uma resposta lenta a choques e a eventual persistência de flutuações cíclicas.

1. INTRODUCTION

Economic theory has been built, along the last few decades, in turn of the concept of representative agent. The representative agent paradigm accounts for the behavior or the beliefs of an "average" individual, leaving no room for the emergence of aggregate phenomena as the result of interaction between agents endowed with different capabilities, desires and preferences. However, one easily recognizes that the whole is certainly something different from the mere sum of the parts and that a deeper understanding of economic phenomena requires a different view, a view that takes into consideration the idiosyncrasies of the elements of a given system and the unique relations that it allows to establish.

The very essence of economic relations is based on the idea of heterogeneity. If we all saw the same costs and the same benefits on everything, we would all act in the same way and would want the same things. As a result, no place would be left for the most fundamental entity that makes the economy work: trade relations. Heterogeneity is being increasingly associated to standard economic models in order to explain meaningful facts, both of a micro and of a macroeconomic nature. The work on heterogeneous interacting agents received its most meaningful pioneer contributions in the 1990s with the work, among others, of Rios-Rull (1995) and Brock and Hommes (1997, 1998), and since then it has been expanded in multiple directions.[1]

Inseparable from the notion that agents are heterogeneous is the need to relax the concept of strict rationality that mainstream macroeconomics adopts. The representative agent is endowed with a capacity to avoid incurring in systematic mistakes and, as a result, she formulates, in every circumstance, fully rational expectations. If rationality is definable as the ability to correctly choose after weighting benefits and costs, the least we can say is that different agents necessarily find different benefits and different costs when faced with similar situations. It is no longer the formal notion of rationality that matters; instead, we must resort to the behavioral foundations of human actions in order to understand how individuals effectively react to stimulus from the surrounding environment.

For a current new strand of thought in economics, heterogeneity and bounded rationality are seen as the main ingredients for the emergence of a new paradigm, based on the notion of systemic complexity.[2] In Simon (1962), we find a definition of complexity that offers a better understanding of the context in which economic issues could be approached; specifically, an economic system is defined as a body composed by a large number of parts that interact with each other in order to produce a result that is necessarily different from the simple sum of the individual results; thus, a complex system is something that generates a new entity or a new reality whenever its components are put together to interact.

The above definition helps in clarifying what an aggregate economic model should be. Some other authors, namely, Arthur et al. (1997), Martin and Sunley (2007) and Fontana (2008), systematize the main features a model fitted to discuss real world events should contain: besides contemplating agent heterogeneity, it must take into consideration that dynamic processes are not immutable in time, because agents are evolving organisms that adapt and learn; furthermore, it should capture the evidence

[1] See, e.g., Barucci (1999), Chiarella and He (2002), Giannitsarou (2003), Negroni (2003), Branch and McGough (2004), Gomes (2005) and Gallegati et al. (2011), on evolutionary and adaptive learning, herding behavior, bubbles and related phenomena, in connection with the presence of belief heterogeneity in the economic system.

[2] The idea that the economic system should be viewed and analyzed as a complex entity is vigorously emphasized by authors such as Colander et al. (2004), Markose (2005), McCauley (2005), Velupillai (2005), Hommes (2007), Bouchaud (2008), Colander (2008), Rosser (2008), Anufriev and Branch (2009), Puu (2010) and Holt et al. (2011), just to cite some of the more representative.

that interaction determines aggregate outcomes because the economy is an interconnected and self-organized world. Out-of-equilibrium dynamics are often relevant and it is reasonable to question if an equilibrium even exists. In this context, policy results are not time invariant and nonlinear dynamics might emerge from simple policy rules.

In this paper, we propose to analyze a simple macroeconomic model, based on the sticky-information setup of Mankiw and Reis (2002, 2006, 2007), in order to highlight the role of heterogeneity and of departures from complete rationality, and to discuss the corresponding consequences for aggregate economic analysis. The source of agents' heterogeneity resides on the distinct abilities evidenced by firms in what concerns the gathering and processing of information. In the proposed setting, as in the Mankiw-Reis framework, firms will update their sets of relevant information sporadically in time and at distinct time periods. However, the Mankiw-Reis framework is presented in a way such that information updating follows a Poisson process, meaning that each firm has an equal probability of being one of the firms updating its information independently of the date of the previous change. This assumption eliminates heterogeneity on the aggregate and the model can be fully analyzed through a simple Phillips curve (the sticky-information Phillips curve) that will be time invariant (i.e., although individual agents may update or not their information sets at time t, this does not change the kind of aggregate supply relation one will have at t).

Underlying our investigation will be an assumption on synchronized behavior that departs from what is conventional in related literature. Synchronization prevents infrequent information updating to be averaged out when taking the behavior of all firms in simultaneous. With this assumption, we intend to provide a more realistic view than the one that simply indicates that firms select dates to update information in a completely random mode. The idea of synchronized behavior finds support on the evidence that different groups of agents exist and that within groups firms have similar behavior concerning information and price updating. For instance, Pfajfar and Santoro (2010) identify group heterogeneity in information updating.[3] Perhaps more striking are the findings in Alvarez et al. (2006) concerning price setting; these authors find evidence of price adjustments that are heterogeneous across sectors since sectoral conditions (like the cost structure or the degree of competition) differ. Therefore, it is reasonable to conceive an economic system in which there is homogeneity in information and price updating behavior within groups of agents sharing similar conditions (in terms of the activity they develop, the costs they face, the dimension they have, the location they are in, the qualifications of their labor force), but where there is heterogeneity across groups that differ in terms of the cited conditions.[4]

In synthesis, it is logically admissible and empirically defensible the idea that firms updating their information infrequently do not act randomly, but instead choose to collect and process information in well defined time periods. Similar firms should select similar time periods to make such effort; firms in different sectors or with relevant differences in terms of dimension, location or other distinguishing factors, will adopt independent information updating rules across groups.

A second relevant departure relatively to the benchmark setup relates to the formation of expectations. Only the firms that update information in a given period will have the ability to formulate rational expectations from that period to the next (well informed agents are endowed with perfect foresight). All the other firms will have to adopt some proxy mechanism to predict the future. In particular, firms know the steady-state of the economy but they do not know when this state will be

[3]These authors distinguish between a nearly rational group, a static or highly autoregressive group and a group behaving in accordance with adaptive learning and sticky information.

[4]Imagine, for instance, hairdresser services. It appears logical that establishments developing this activity will update information and, eventually, prices at roughly the same time moments: information updating and price changes will be a reaction to changes on the costs of the specific inputs of the activity, on the behavior of its own demand and on particular tax conditions. Thus, it is likely that a large majority of the hairdressers in a given geographical location choose to update their information and their prices in a synchronized mode. This behavior does not necessarily spread to firms in other sectors of activity, that may be subject to different external forces and complementarity relations.

reached. As a rule-of-thumb, they make a forecast according to which relevant variables (namely, the inflation rate and the output gap) will evolve to the next period as if they were in their steady-state (in the framework to be presented, this means that the output gap is not expected to vary, while inflation predictably evolves at a constant rate). This assumption makes sense because, in the setting to be discussed, macro variables tend to remain in their steady-state values unless some kind of disturbance makes them to depart temporarily from such state; when the agents have no possibility to know whether such a disturbance is occurring, then the best they can do is to assume the absence of shocks and to take the evolution of variables as if no exceptional events were occurring.

In the sections that follow, we intend to explicitly take the information updating heterogeneity assumption and to study its implications. If firms update information at different paces, this will have consequences for the aggregate outcome that we obtain at each time period, implying that a different Phillips curve can be derived for each consecutive period. Thus, heterogeneity leads to an evolving and ever-changing system that contemplates most of the properties of a complex system, as debated above. By assuming agents that update information with different frequencies, we will be able to attain long-term results according to which different agents will interact to produce nonlinear long-term trajectories for the inflation rate and for the output-gap, that are unique for each different policy and for the same policy applied at different time moments.

The remainder of the paper is organized as follows. In section 2, the economic environment is described, with special focus on the behavior of inattentive firms. Section 3 derives, for each assumed time period, the sticky-information Phillips curve. Section 4 characterizes the macroeconomic equilibrium, after we have briefly described the demand side of the economy. In section 5, we disturb the equilibrium by considering different types of monetary policy; we observe that relatively simple monetary policy rules may imply everlasting endogenous fluctuations. Finally, section 6 concludes, emphasizing that our simple modelling structure is well equipped to discuss short-run economic fluctuations within an interaction scenario.

2. INATTENTIVE FIRMS

Consider an economy populated by a large number of firms, each one producing a different variety of a given tradable good. The single feature that distinguishes firms from each other, besides the production of different varieties, is their ability to update information on the state of the economy. For some firms, it pays to frequently update information in time; other firms will incur in relatively higher costs of collecting and processing information, and thus they will update the corresponding information sets on an infrequent basis.

We assume that firms can be separated into several groups, in which they share two features: a specific information updating periodicity and a completely synchronized behavior, i.e., all firms within the group update their information at exactly the same periods. This occurs, according to the discussion in the introduction, because firms within a group supposedly share a similar organization structure, act in a strategic complementarily way and face a same set of environmental variables, what leads them to select the same periods when updating their information sets.

We will take the following exercise: firms can be separated into n groups of identical dimension. Firms in the first group are fully attentive, i.e., they update information at all periods; firms in group 2 update information every two periods; firms in group 3 update information every three periods, and so forth. Logical arguments to be presented in the paper are developed for four groups of firms with the above characteristics. If we start at a time period where all agents update information, the pattern we obtain is the one displayed in Table 1.

In Table 1, the Xs indicate the time moments in which firms in a given information updating group search for, collect and process costly information. Four groups are considered ($G1$,$G2$,$G3$,$G4$) and each one of these groups contains 25% of the firms in the market. Parameter λ indicates the share of firms

Table 1: Attentiveness cycle for $n = 4$

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
$G1$	X	X	X	X	X	X	X
$G2$	X		X		X		X
$G3$	X			X			X
$G4$	X				X		
λ	1	0.25	0.5	0.5	0.75	0.25	0.75
	$t=7$	$t=8$	$t=9$	$t=10$	$t=11$	$t=12$	...
$G1$	X	X	X	X	X	X	
$G2$		X		X		X	
$G3$			X			X	
$G4$		X				X	
λ	0.25	0.75	0.5	0.5	0.25	1	...

that update information at each time period. The table shows that an attentiveness cycle is generated by the pattern of behavior that was assumed. If all firms are attentive at $t = 0$, they will all be attentive again only at $t = 12$; at this point in time, a new cycle is initiated with the pattern of attentiveness being repeated for the following 12 periods. This process will, then, repeat itself over and over again, as long as firms maintain the information updating periodicities with which they were endowed.

If we generalize the previous exercise maintaining, for simplicity, the assumption that each group contains an equal number of firms, we will understand that the periodicity of the attentiveness cycles depends on the number of groups of firms we consider. Specifically, the amplitude of each cycle (from a full attentiveness outcome to the next) will correspond to the lowest common multiplier between all integers ranging from 1 to n. For $n = 4$, we have encountered a period-12 cycle; this is the lowest common multiplier between 1, 2, 3 and 4.

Table 2 shows what happens when we consider other dimensions. For instance, the last value we consider in the table is $n = 25$. With $n = 25$, there are 4% of firms that are attentive to new information at every time period, 4% of firms that are attentive every two periods and so on. The last group of firms, displaying the lowest degree of attentiveness, corresponds to the 4% of productive units that only update information at each 25 periods.

We will implicitly consider quarterly data, i.e., we attribute the correspondence between 1 period and 1 quarter of a year. Thus, in Table 2 we present a last column with annual data. The case we will work with ($n = 4$) assumes that the same aggregate behavior by firms in terms of information updating is repeated each three years. However, as we consider more groups of a smaller dimension, the amplitude of the cycles will dramatically increase. At the most extreme case displayed in the table, it would be necessary to allow for more than 6 and a half billion years to complete a cycle. Noticing that the age of the earth and of the rest of the solar system is estimated at 4.55 billion years, we conclude that considering this type of heterogeneity implies forming attentiveness cycles of such a length that, in order for the same state of nature to be repeated, we would have to go back in time to a period where our planet did not exist or to go forward towards a period where most probably our planet will no longer exist.

The point we want to highlight is that a relatively simple rule that separates an economic agent into units with different abilities and different opportunities to update information may generate cycles for which a regular pattern cannot be identified under a reasonable time length. We reemphasize that the

Table 2: Attentiveness cycles for different degrees of heterogeneity.

n	Attentiveness cycles	Attentiveness cycles (annual data)
2	2	0.5
3	6	1.5
4	12	3
5	60	15
6	60	15
7	420	105
8	840	210
9	2.520	630
⋮	⋮	⋮
22	232.792.560	58.198.148
23	5.354.228.880	1.338.557.220
24	5.354.228.880	1.338.557.220
25	26.771.144.400	6.692.786.100

analysis we will pursue on fluctuations in settings with information updating heterogeneity will be restricted to a four-agent setting; this will be enough to illustrate the features of complexity we have discussed in the introduction. However, we should keep in mind that the fluctuations we will observe are immensely expanded in time once additional groups are added.

Our inattentiveness setting will be similar to the one in Mankiw and Reis (2002). We assume a monopolistically competitive environment where each existing firm intends to maximize profits. This well known optimization problem will lead to a result under which every firm wants to set a same target price p_t^* such that,

$$p_t^* = p_t + \alpha y_t \tag{1}$$

Variable p_t represents the aggregate price level and y_t respects to the output gap, i.e., to the difference between the effective and the potential output (prices and the output gap are represented in logs). Parameter $\alpha \in (0,1)$ reflects the degree of real rigidities or the degree of substitutability between different varieties of the assumed good. If $\alpha = 0$, varieties are perfect substitutes, and we are at the competitive scenario where the desired price is always the observed price. A relatively large value of α indicates that real rigidities are relevant, meaning that firms will desire to set a price larger than the observed price level in cases of expansion ($y_t > 0$) or a price lower than the market price in scenarios of recession ($y_t < 0$).

Since firms do not necessarily update information at every period, they select a price that corresponds to the expectation on the desired price formulated at the date of the last information updating. A firm that has updated its information for the last time j periods ago will select price $p_t^j = E_{t-j}(p_t^*)$. The aggregate price level will be the average price practiced by the different groups that are considered, i.e.,

$$p_t = \sum_{i=1}^{n} p_t(i) \Big/ n \tag{2}$$

Notation $p_t(i)$ represents the price that is set by firms in group i; if firms in this group have updated their information j periods ago, then $p_t(i) = p_t^j$.

The specific price level one finds for each time period can be withdrawn from Table 1. Take for instance period $t = 2$; in this precise time period, the first two groups have just updated the corresponding information; this corresponds to 50% of the population of firms. The other two groups have accessed novel information only at $t = 0$; thus, 50% of the agents are using, at $t = 2$, information that is two periods old. The price level might, thus, be written as:

$$p_t = \frac{p_t(1) + p_t(2) + p_t(3) + p_t(4)}{4} = \frac{p_t^0 + p_t^0 + p_t^2 + p_t^2}{4} = \frac{1}{2}p_t^0 + \frac{1}{2}p_t^2$$

If we repeat the previous analysis for the twelve periods, we obtain different price levels for each one of the time moments:

$$t = 0{,}12{,}24{,}\ldots : p_t = p_t^0$$

$$t = 1{,}13{,}25{,}\ldots : p_t = \frac{1}{4}p_t^0 + \frac{3}{4}p_t^1$$

$$t = 2{,}14{,}26{,}\ldots : p_t = \frac{1}{2}p_t^0 + \frac{1}{2}p_t^2$$

$$t = 3{,}15{,}27{,}\ldots : p_t = \frac{1}{2}p_t^0 + \frac{1}{4}p_t^1 + \frac{1}{4}p_t^3$$

$$t = 4{,}16{,}28{,}\ldots : p_t = \frac{3}{4}p_t^0 + \frac{1}{4}p_t^1$$

$$t = 5{,}17{,}29{,}\ldots : p_t = \frac{1}{4}p_t^0 + \frac{1}{2}p_t^1 + \frac{1}{4}p_t^2$$

$$t = 6{,}18{,}30{,}\ldots : p_t = \frac{3}{4}p_t^0 + \frac{1}{4}p_t^2$$

$$t = 7{,}19{,}31{,}\ldots : p_t = \frac{1}{4}p_t^0 + \frac{1}{2}p_t^1 + \frac{1}{4}p_t^3$$

$$t = 8{,}20{,}32{,}\ldots : p_t = \frac{3}{4}p_t^0 + \frac{1}{4}p_t^2$$

$$t = 9{,}21{,}33{,}\ldots : p_t = \frac{1}{2}p_t^0 + \frac{1}{2}p_t^1$$

$$t = 10{,}22{,}34{,}\ldots : p_t = \frac{1}{2}p_t^0 + \frac{1}{4}p_t^1 + \frac{1}{4}p_t^2$$

$$t = 11{,}23{,}35{,}\ldots : p_t = \frac{1}{4}p_t^0 + \frac{1}{4}p_t^1 + \frac{1}{4}p_t^2 + \frac{1}{4}p_t^3$$

The above expressions are presentable in a different form, after considering expectations (note that $p_t^0 = p_t^*$),

$$t = 0{,}12{,}24{,}\ldots : y_t = 0$$

$$t = 1{,}13{,}25{,}\ldots : p_t = \frac{1}{3}\alpha y_t + E_{t-1}(p_t + \alpha y_t)$$

$$t = 2{,}14{,}26{,}\ldots : p_t = \alpha y_t + E_{t-2}(p_t + \alpha y_t)$$

$$t = 3{,}15{,}27{,}\ldots : p_t = \alpha y_t + \frac{1}{2}E_{t-1}(p_t + \alpha y_t) + \frac{1}{2}E_{t-3}(p_t + \alpha y_t)$$

$$t = 4{,}16{,}28{,}\ldots : p_t = 3\alpha y_t + E_{t-1}(p_t + \alpha y_t)$$

$$t = 5{,}17{,}29{,}\ldots : p_t = \frac{1}{3}\alpha y_t + \frac{2}{3}E_{t-1}(p_t + \alpha y_t) + \frac{1}{3}E_{t-2}(p_t + \alpha y_t)$$

$$t = 6{,}18{,}30{,}\ldots : p_t = 3\alpha y_t + E_{t-2}(p_t + \alpha y_t)$$

$$t = 7{,}19{,}31{,}\ldots : p_t = \frac{1}{3}\alpha y_t + \frac{2}{3}E_{t-1}(p_t + \alpha y_t) + \frac{1}{3}E_{t-3}(p_t + \alpha y_t)$$

$$t = 8{,}20{,}32{,}\ldots : p_t = 3\alpha y_t + E_{t-2}(p_t + \alpha y_t)$$

$$t = 9{,}21{,}33{,}\ldots : p_t = \alpha y_t + E_{t-1}(p_t + \alpha y_t)$$

$$t = 10{,}22{,}34{,}\ldots : p_t = \alpha y_t + \frac{1}{2}E_{t-1}(p_t + \alpha y_t) + \frac{1}{2}E_{t-2}(p_t + \alpha y_t)$$

$$t = 11{,}23{,}35{,}\ldots : p_t = \frac{1}{3}\alpha y_t + \frac{1}{3}E_{t-1}(p_t + \alpha y_t) + \frac{1}{3}E_{t-2}(p_t + \alpha y_t) + \frac{1}{3}E_{t-3}(p_t + \alpha y_t)$$

Observe, in the above list, that the same expression for the price level is obtained, for each 12-period cycle, at moments $t = 6$ and $t = 8$; all the others are different from these and different from each other.

3. STICKY-INFORMATION PHILLIPS CURVES

To arrive to Phillips curve expressions (i.e., expressions relating the output gap and the inflation rate) from the relations on the last section, we will need to establish an explicit assumption about how expectations are formed. Our assumption will be based on aggregate information availability as presented in Table 1. The following rule is considered: agents who have updated information at a given time period will be able to predict, at that period, with accuracy, the next period values of the macroeconomic variables, i.e., perfect foresight will hold. If agents have not updated information at a given time moment, they will be unable to perfectly forecast the following period values of the variables and will act as if the economy remained in its steady-state. As mentioned in the introduction, this assumption makes sense because we are analyzing the long-term equilibrium of a macro system, which can be disturbed by policy shocks. If agents are not endowed with an ability to accurately predict the future, they will guess that no shock will occur and, thus, that the economy continues to evolve as if the steady-state suffered no perturbation.

We define the steady-state as the scenario in which the output gap is equal to zero, $y^* = 0$ (i.e., target price and price level coincide). Prices will grow in the steady-state at a constant rate π^*; later on, when characterizing the demand side of the economy, we will be able to present an explicit expression for π^*. Note that the inflation rate is defined simply as the difference between the logarithms of prices at two consecutive periods: $\pi_t := p_t - p_{t-1}$.

Analytically, the characterized rule for expectations takes the form:

$$E_{t-j}(p_t + \alpha y_t) = \lambda_{t-j}\left[p_{t-j+1} + \alpha y_{t-j+1} + (j-1)\pi^*\right] + (1-\lambda_{t-j})\left(p_{t-j} + \alpha y_{t-j} + j\pi^*\right) \quad (3)$$

with λ_{t-j} the share of attentive firms at $t-j$.

Consider equation (3) for $j = 1$,

$$E_{t-1}(p_t + \alpha y_t) = \lambda_{t-1}(p_t + \alpha y_t) + (1-\lambda_{t-1})\left(p_{t-1} + \alpha y_{t-1} + \pi^*\right)$$

The above expression translates the idea that well informed agents at $t-1$ will be able to predict the target price at t, while non-informed agents will formulate a forecast such that the target price at $t-1$ comes augmented by the steady-state growth rate of that price. For $j = 2$, equation (3) is

$$E_{t-2}(p_t + \alpha y_t) = \lambda_{t-2}\left(p_{t-1} + \alpha y_{t-1} + \pi^*\right) + (1-\lambda_{t-2})\left(p_{t-2} + \alpha y_{t-2} + 2\pi^*\right)$$

In this case, agents who update information at $t-2$ will be able to predict the target price at $t-1$; then, to arrive to t they add to their forecast the steady-state growth rate of prices. If agents do not update information at $t-2$, they simply form an expectation according to which the desired price at t is equal to the desired price at $t-2$ plus the growth prices would suffer in a two-period interval if the steady-state had been already accomplished.[5]

Applying expectations formation rule (3) to our list of price level expressions, we obtain a series of Phillips curves. These Phillips curves all involve a positive contemporaneous relation between the output gap and inflation, but each one attributes a different weight to the impact of past inflation and output gap values over the current inflation rate. The exception for the mentioned shape will occur for time periods $t = 0$ and $t = 1$. At $t = 0$, information is fully flexible and, as a result, the output gap is zero. At $t = 1$, decisions are taken resorting to expectations formed a period earlier, when it was possible to formulate accurate or perfect forecasts given the full information setting; again, a Phillips curve relation does not exist and the output gap is zero. Thus, finding Phillips curves will imply the presence of departures relatively to the full information / perfect foresight benchmark apparatus.

As in the Mankiw-Reis framework, we can call the obtained expressions Sticky-Information Phillips Curves (SIPC). Some algebra allows to calculate the following relationships:

$$t = 0{,}12{,}24{,}\ldots : y_t = 0$$

$$t = 1{,}13{,}25{,}\ldots : y_t = 0$$

$$t = 2{,}14{,}26{,}\ldots : \pi_t = \alpha y_t + \alpha y_{t-1} + \pi^*$$

$$t = 3{,}15{,}27{,}\ldots : \pi_t = \frac{5}{3}\alpha y_t - \frac{2}{3}\pi_{t-1} + \frac{1}{3}\alpha y_{t-1} + \frac{2}{3}\alpha y_{t-2} + \frac{5}{3}\pi^*$$

$$t = 4{,}16{,}28{,}\ldots : \pi_t = 7\alpha y_t + \alpha y_{t-1} + \pi^*$$

[5] One may wonder whether agents will, sooner or later, understand that the economy works at a 12-period cycle and, accordingly, gain the precise conscience of what the values of the aggregate variables will be in the future. Our implicit understanding is that the cycles are too long in order for agents to perceive the existence of a circular functioning of the economy. Recall that we are using a simple 4-agent setting, but that this can be much more complicated once we allow for a larger degree of inattentiveness heterogeneity.

$$t = 5{,}17{,}29{,}... : \pi_t = \frac{5}{3}\alpha y_t - \frac{1}{3}\pi_{t-1} + \frac{2}{3}\alpha y_{t-1} + \frac{1}{3}\alpha y_{t-2} + \frac{4}{3}\pi^*$$

$$t = 6{,}18{,}30{,}... : \pi_t = 3\alpha y_t - \frac{1}{4}\pi_{t-1} + \frac{3}{4}\alpha y_{t-1} + \frac{1}{4}\alpha y_{t-2} + \frac{5}{4}\pi^*$$

$$t = 7{,}19{,}31{,}... : \pi_t = \frac{5}{3}\alpha y_t - \frac{2}{3}\pi_{t-1} - \frac{1}{6}\pi_{t-2} + \frac{1}{3}\alpha y_{t-1} + \frac{1}{2}\alpha y_{t-2} + \frac{1}{6}\alpha y_{t-3} + \frac{11}{6}\pi^*$$

$$t = 8{,}20{,}32{,}... : \pi_t = 3\alpha y_t - \frac{1}{4}\pi_{t-1} + \frac{3}{4}\alpha y_{t-1} + \frac{1}{4}\alpha y_{t-2} + \frac{5}{4}\pi^*$$

$$t = 9{,}21{,}33{,}... : \pi_t = 7\alpha y_t + \alpha y_{t-1} + \pi^*$$

$$t = 10{,}22{,}34{,}... : \pi_t = \frac{5}{3}\alpha y_t - \frac{1}{6}\pi_{t-1} + \frac{5}{6}\alpha y_{t-1} + \frac{1}{6}\alpha y_{t-2} + \frac{7}{6}\pi^*$$

$$t = 11{,}23{,}35{,}... : \pi_t = \frac{3}{5}\alpha y_t - \frac{3}{5}\pi_{t-1} - \frac{1}{10}\pi_{t-2} + \frac{2}{5}\alpha y_{t-1} + \frac{1}{2}\alpha y_{t-2} + \frac{1}{10}\alpha y_{t-3} + \frac{17}{10}\pi^*$$

For the same reason already pointed out, the Phillips curves of periods $t = 6{,}18{,}30{,}...$ and $t = 8{,}20{,}32{,}...$ are identical. After replacing expectations, also the SIPCs of periods $t = 4{,}16{,}28{,}...$ and $t = 9{,}21{,}33{,}...$ possess the same expressions. Looking at the set of displayed equations one realizes that, under the specified setup, there is not a unique timeless rule capable of describing mechanically the aggregate supply relation of the economy; for each time moment, a different Phillips curve will characterize the output gap - inflation rate relation, as the result of distinct information updating scenarios available each period.

The perfect information results for $t = 0{,}12{,}24{,}...$ and $t = 1{,}13{,}25{,}...$, in which the Phillips curve gives place to a zero output gap and desired prices that coincide with the price level are useful in order to guarantee the stability of the system: no matter how unstable some of the other SIPC equations might be, implying an eventual departure of the output gap and of the inflation rate from their steady-state levels, the first two periods of each cycle always guarantee a return to the steady-state result.

4. MACROECONOMIC EQUILIBRIUM

We now proceed to the characterization of the demand side of the economy. We assume, in order to simplify the analysis, the absence of any kind of inattentiveness concerning consumption plans. Households solve a trivial intertemporal consumption utility maximization problem. As a result, consumption will evolve in time as follows:

$$E_t(c_{t+1}) = c_t + \theta r_t \tag{4}$$

Variable c_t represents the difference between the logarithms of effective and potential consumption.[6] Since we are not considering capital accumulation, the market clearing condition is simply

[6] Potential consumption is understood as the level of consumption that one would obtain under the absence of any information constraints. Effective consumption corresponds to the level of consumption computable under the information stickiness assumption. These notions find correspondence on the ones established for the output variable and that served to define the output gap as the difference between effective and potential output.

$y_t = c_t$. Parameter $\theta > 0$ is the intertemporal elasticity of substitution between consumption in two consecutive time periods and r_t is the real interest rate.

The real interest rate is given by the Fisher equation $r_t = i_t - E_t(\pi_{t+1})$, with i_t the nominal interest rate. The value of i_t is established by the central bank given the goal of maintaining price stability. The following Taylor rule characterizes how monetary policy is conducted:

$$i_t = \widehat{i} + \phi \left[E_t(\pi_{t+1}) - \overline{\pi}\right] \tag{5}$$

In expression (5), ϕ is a policy parameter and $\overline{\pi}$ represents the inflation rate target selected by the central bank. The value $\widehat{i}$ is the real interest rate for an expected inflation rate that equals the corresponding target value. The monetary authority chooses a path for the nominal interest rate such that this responds to the extent of the deviation between the expected inflation rate and the target value. Typically, Taylor rules consider, as well, a real stabilization term; we concentrate only on price stability in order to maintain the analysis relatively simple on the demand side. It is well known that determinacy requires an active interest rate policy, i.e., the nominal interest rate should respond aggressively (by more than one to one) to a change on the expected inflation rate; analytically, this implies imposing the constraint $\phi > 1$.

Replacing the Fisher equation, the Taylor rule and the market clearing condition into (4), the following expression is obtained:

$$E_t(y_{t+1}) = y_t + \theta(\phi - 1)E_t(\pi_{t+1}) - \theta\left(\phi\overline{\pi} - \widehat{i}\right) \tag{6}$$

As already mentioned, households are endowed with full information and perfect foresight; therefore, the above expression will be equivalent, in the present context, to

$$y_{t+1} = y_t + \frac{\omega}{\alpha}(\pi_{t+1} - \pi^*) \tag{7}$$

with $\pi^* := \frac{\phi\overline{\pi} - \widehat{i}}{\phi - 1}$ and $\omega := \alpha\theta(\phi - 1)$. The value π^* corresponds to the steady-state inflation rate; this value is straightforward to obtain once we apply condition $E_t(y_{t+1}) = y_t = 0$ to (6) (the output gap and the expected output gap are zero at the steady-state). Note that $\pi^* \neq \overline{\pi}$ for any $\phi > 1$; this occurs because the interest rate rule is not an optimal rule; however, the more aggressive monetary policy is, the more π^* will approach $\overline{\pi}$.

Equation (7) is the result of an IS relation and characterizes the demand side of the economy once we know the policy rule that the monetary authority follows. This is a unique relation; on the contrary, on the supply side we have a different relation for each time period in the 12-period cycle. An equilibrium result is obtainable for each time period by considering both relations together, and by proceeding with the necessary computation. The pairs of equilibrium values (π_t,y_t) are the following:

$$t = 0{,}12{,}24{,}\ldots : \begin{cases} \pi_t = -\frac{\alpha}{\omega}y_{t-1} + \pi^* \\ y_t = 0 \end{cases}$$

$$t = 1{,}13{,}25{,}\ldots : \begin{cases} \pi_t = -\frac{\alpha}{\omega}y_{t-1} + \pi^* \\ y_t = 0 \end{cases}$$

$$t = 2{,}14{,}26{,}\ldots : \begin{cases} \pi_t = \frac{2\alpha}{1-\omega}y_{t-1} + \pi^* \\ y_t = \frac{1+\omega}{1-\omega}y_{t-1} \end{cases}$$

$$t = 3{,}15{,}27{,}\ldots :$$

$$\begin{cases} \pi_t = \frac{6\alpha}{3-5\omega}y_{t-1} - \frac{2}{3-5\omega}\pi_{t-1} + \frac{2\alpha}{3-5\omega}y_{t-2} + \frac{5-5\omega}{3-5\omega}\pi^* \\ y_t = \frac{3+\omega}{3-5\omega}y_{t-1} - \frac{2\omega}{3-5\omega}\frac{1}{\alpha}\pi_{t-1} + \frac{2\omega}{3-5\omega}y_{t-2} + \frac{2\omega}{3-5\omega}\frac{1}{\alpha}\pi^* \end{cases}$$

$$t=4{,}16{,}28{,}...:\begin{cases}\pi_t=\frac{8\alpha}{1-7\omega}y_{t-1}+\pi^*\\ y_t=\frac{1+\omega}{1-7\omega}y_{t-1}\end{cases}$$

$$t=5{,}17{,}29{,}...:$$

$$\begin{cases}\pi_t=\frac{7\alpha}{3-5\omega}y_{t-1}-\frac{1}{3-5\omega}\pi_{t-1}+\frac{\alpha}{3-5\omega}y_{t-2}+\frac{4-5\omega}{3-5\omega}\pi^*\\ y_t=\frac{3+2\omega}{3-5\omega}y_{t-1}-\frac{\omega}{3-5\omega}\frac{1}{\alpha}\pi_{t-1}+\frac{\omega}{3-5\omega}y_{t-2}+\frac{\omega}{3-5\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

$$t=6{,}18{,}30{,}...:$$

$$\begin{cases}\pi_t=\frac{13\alpha}{4-12\omega}y_{t-1}-\frac{1}{4-12\omega}\pi_{t-1}+\frac{\alpha}{4-12\omega}y_{t-2}+\frac{5-12\omega}{4-12\omega}\pi^*\\ y_t=\frac{4+\omega}{4-12\omega}y_{t-1}-\frac{\omega}{4-12\omega}\frac{1}{\alpha}\pi_{t-1}+\frac{\omega}{4-12\omega}y_{t-2}+\frac{\omega}{4-12\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

$$t=7{,}19{,}31{,}...:$$

$$\begin{cases}\pi_t=\frac{6\alpha}{3-5\omega}y_{t-1}-\frac{2}{3-5\omega}\pi_{t-1}-\frac{1}{6-10\omega}\pi_{t-2}+\frac{3\alpha}{6-10\omega}y_{t-2}+\frac{\alpha}{6-10\omega}y_{t-3}+\frac{11-10\omega}{6-10\omega}\pi^*\\ y_t=\frac{3+\omega}{3-5\omega}y_{t-1}-\frac{2\omega}{3-5\omega}\frac{1}{\alpha}\pi_{t-1}-\frac{\omega}{6-10\omega}\frac{1}{\alpha}\pi_{t-2}+\frac{3\omega}{6-10\omega}y_{t-2}+\frac{\omega}{6-10\omega}y_{t-3}+\frac{5\omega}{6-10\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

$$t=8{,}20{,}32{,}...:$$

$$\begin{cases}\pi_t=\frac{13\alpha}{4-12\omega}y_{t-1}-\frac{1}{4-12\omega}\pi_{t-1}+\frac{\alpha}{4-12\omega}y_{t-2}+\frac{5-12\omega}{4-12\omega}\pi^*\\ y_t=\frac{4+\omega}{4-12\omega}y_{t-1}-\frac{\omega}{4-12\omega}\frac{1}{\alpha}\pi_{t-1}+\frac{\omega}{4-12\omega}y_{t-2}+\frac{\omega}{4-12\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

$$t=9{,}21{,}33{,}...:\begin{cases}\pi_t=\frac{8\alpha}{1-7\omega}y_{t-1}+\pi^*\\ y_t=\frac{1+\omega}{1-7\omega}y_{t-1}\end{cases}$$

$$t=10{,}22{,}34{,}...:$$

$$\begin{cases}\pi_t=\frac{15\alpha}{6-10\omega}y_{t-1}-\frac{1}{6-10\omega}\pi_{t-1}+\frac{\alpha}{6-10\omega}y_{t-2}+\frac{7-10\omega}{6-10\omega}\pi^*\\ y_t=\frac{6+5\omega}{6-10\omega}y_{t-1}-\frac{\omega}{6-10\omega}\frac{1}{\alpha}\pi_{t-1}+\frac{\omega}{6-10\omega}y_{t-2}+\frac{\omega}{6-10\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

$$t=11{,}23{,}35{,}...:$$

$$\begin{cases}\pi_t=\frac{5\alpha}{5-3\omega}y_{t-1}-\frac{3}{5-3\omega}\pi_{t-1}-\frac{1}{10-6\omega}\pi_{t-2}+\frac{5\alpha}{10-6\omega}y_{t-2}+\frac{\alpha}{10-6\omega}y_{t-3}+\frac{17-6\omega}{10-6\omega}\pi^*\\ y_t=\frac{5+2\omega}{5-3\omega}y_{t-1}-\frac{3\omega}{5-3\omega}\frac{1}{\alpha}\pi_{t-1}-\frac{\omega}{10-6\omega}\frac{1}{\alpha}\pi_{t-2}+\frac{5\omega}{10-6\omega}y_{t-2}+\frac{\omega}{10-6\omega}y_{t-3}+\frac{7\omega}{10-6\omega}\frac{1}{\alpha}\pi^*\end{cases}$$

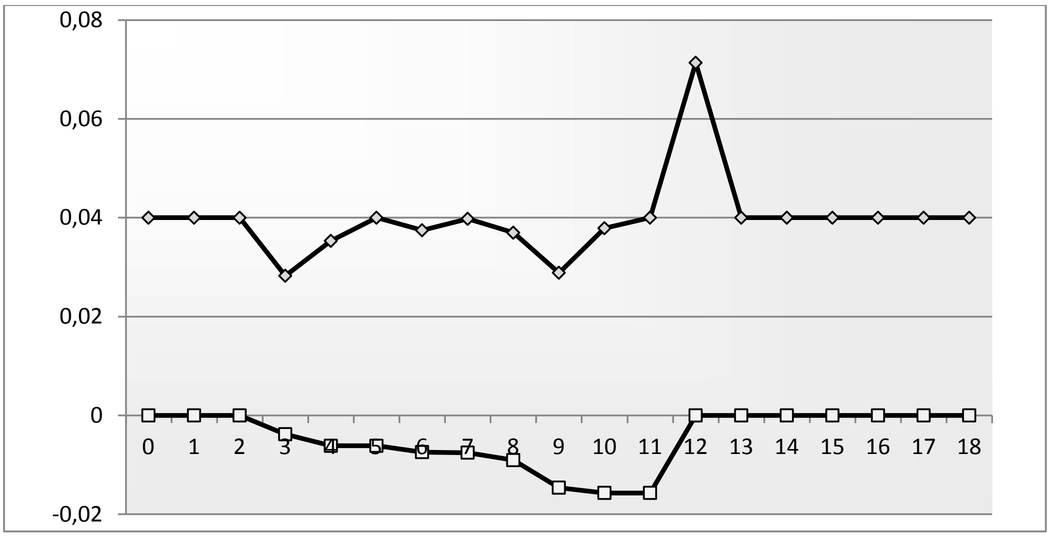
Equilibrium values (π_t,y_t) will remain at the steady-state level $(\pi^*,0)$ if this is the initial pair of values at $t=0$ and no perturbation affects the system. However, given that current values of inflation and output gap depend on the corresponding values in past periods, any disturbance occurring at a point in time with the exception of $t=0$ and $t=1$, will have a sluggish or gradual effect over the values of both variables. Furthermore, because equilibrium values are different from one period to the next, disturbances occurring at one point in time will have a distinct impact than disturbances occurring at a different period. Looking at the above equations, one understands that the consequence of considering firms with different information updating periodicities is that the current inflation rate and the current output gap will depend on different combinations of past values of these two variables.

5. MONETARY POLICY AND PERPETUAL MOTION

To get further insights on the model's implications, we consider a numerical example. The following parameter values are assumed: $\alpha = 0.1$, $\theta = 1$, $\widehat{i} = 0.01$, $\phi = 1.5$, $\overline{\pi} = 0.02$. With these values, the steady-state inflation level is $\pi^* = 0.04$. For any initial values (π_0,y_0), the economy will remain forever at the steady-state if no disturbance occurs. Next, we consider monetary policy actions that may trigger a deviation from the steady-state.

We take two types of monetary policy. First, we assume a one period change on policy parameter ϕ. This change reveals that it is not innocuous the timing of the application of the policy. Figure 1 shows the effect of a change in ϕ from 1.5 to 1.75 and back to 1.5 in the following time period. Panel *A* considers the change at $t = 3$ and panel *B* at $t = 5$. Note that at $t = 13$ the system will be back at its initial stage, since at this point in time full information is recovered (notice that the upper trajectory concerns the inflation rate, and the one in turn of zero respects to the output gap). The evidence we withdraw is that there are significant differences for policies applied at distinct periods. Inflation and output gap values suffer different impacts when disturbed in different time moments.

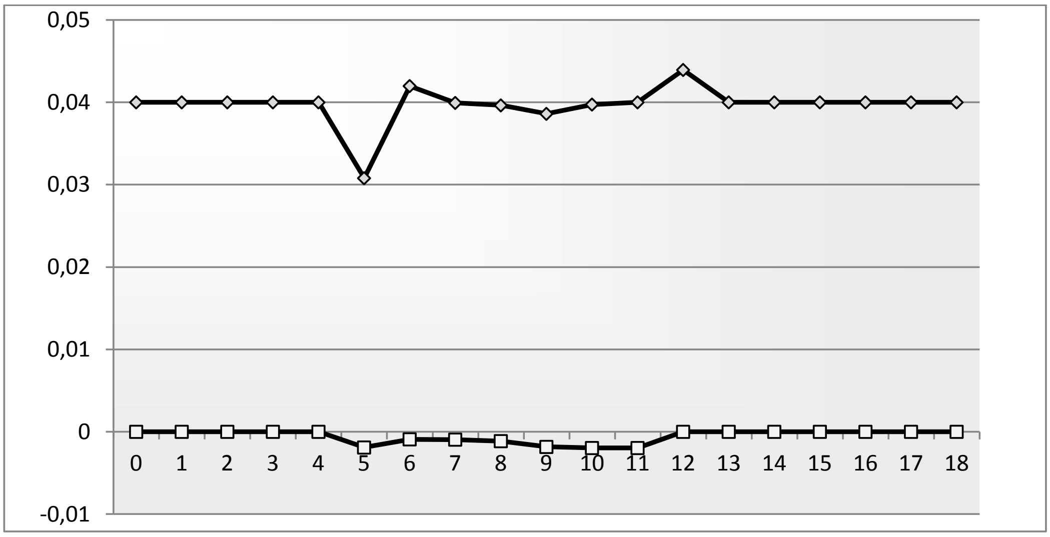
Figure 1 (Panel A) - Monetary policy disturbance at $t = 3$.



Secondly, we consider a more sophisticated type of policy. In this case, if the sum of the squares of the differences between the observed inflation rate and the target defined by the central bank is lower than a given threshold, υ, monetary policy is characterized by the selection of a policy parameter value ϕ_L; if such difference is a value above the threshold υ, the parameter defining monetary policy will be $\phi_H > \phi_L$. In this setting, a more aggressive monetary policy is followed when the accumulated distance between observed inflation and desired inflation is relatively larger. Letting β be a discount factor, monetary policy will be given by the following rule:

$$\begin{cases} \sum_{i=1}^{\tau} \beta^i (\pi_{t-i} - \overline{\pi})^2 < \upsilon \Rightarrow \phi = \phi_L \\ \sum_{i=1}^{\tau} \beta^i (\pi_{t-i} - \overline{\pi})^2 \geq \upsilon \Rightarrow \phi = \phi_H \end{cases}$$

Figure 1 (Panel B) - Monetary policy disturbance at $t = 5$.



Parameter τ corresponds to the number of time periods for which it is considered relevant to observe the difference between the inflation rate and the respective target. For the numerical illustration, we consider $\phi_L = 1.5$, $\phi_H = 1.75$, $\tau = 4$, $\upsilon = 0.0014$ and $\beta = 0.95$. This policy will imply the time series of inflation and of the output gap that are displayed in figure 2.

In figure 2, we represent a long-term setting, where the transient phase after selecting some initial pair (π_0,y_0) has already faded out. We observe that cycles of periodicity 36 are formed (each panel presents three complete cycles). Both the output gap and the inflation rate gravitate around their steady-state values without ever converging or diverging from such state. A simple policy rule combined with a scenario of heterogeneously inattentive agents (and just four different types of agents) is able to generate large periodicity cycles. The precise path followed by the variables will be subject to a kind of sensitive dependence on initial conditions. It is decisive the choice of a given date to initiate this kind of policy; starting the policy at each one of the twelve consecutive time periods that form the attentiveness cycle will lead to a different pattern in time.

Again, we highlight the idea that our analytical structure is relatively simple: a more sophisticated policy rule and a larger degree of heterogeneity may imply cycles of such large dimension that in practice we observe, given a reasonably extensive time length, a completely irregular path of evolution. Given the values of parameters, the type of policy, the degree of heterogeneity and the time period in which the policy is implemented, it is impossible to associate the motion of real and nominal variables to a simple unchangeable dynamic rule.

6. CONCLUSION

We have developed a simple aggregate model aimed at describing the most prominent characteristics of the evolution of the macroeconomic system. In doing so, we have identified and applied some of the features that are gaining a decisive role in the explanation of economic phenomena. The central of these features is agents' heterogeneity. Particularly relevant in our model is the fact that nonlinear

Figure 2 (Panel A) - Inflation rate time path (Monetary policy rule $\#2$).

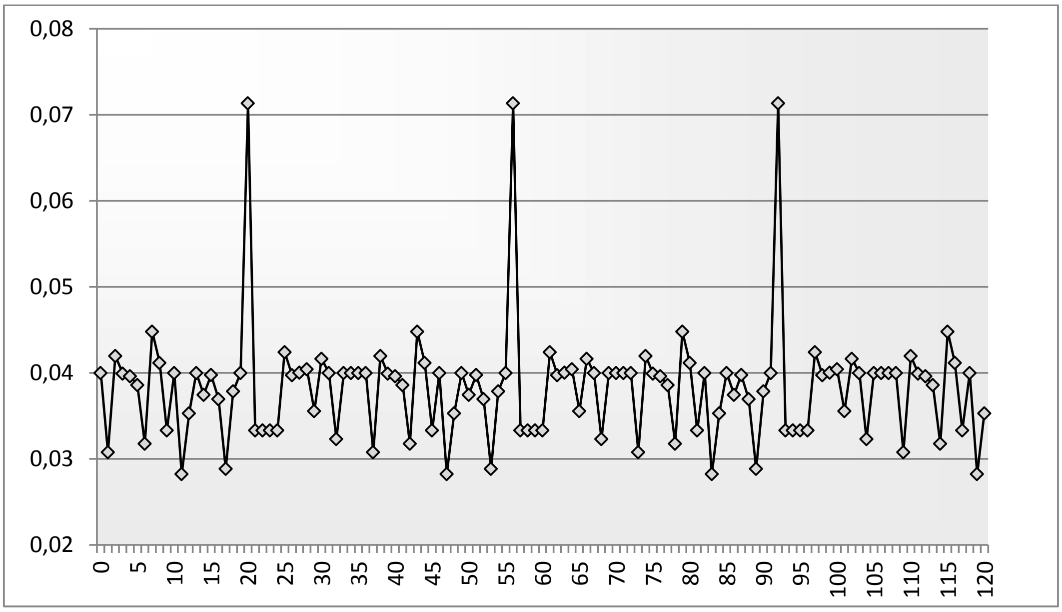
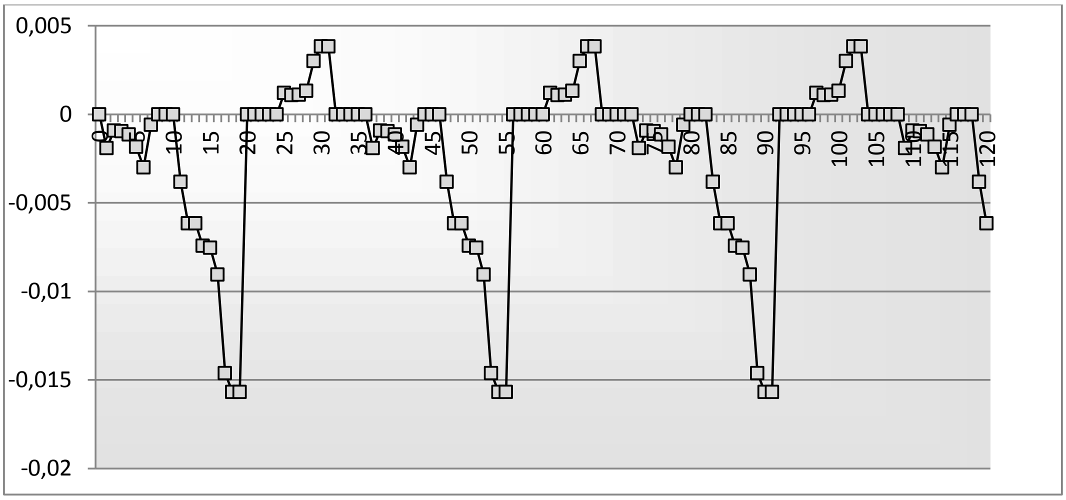


Figure 2 (Panel B) - Output gap time path (Monetary policy rule $\#2$).



results have emerged as the outcome of a single source of heterogeneity, with a small set of different groups and assuming a relatively simple policy rule.

The proposed macroeconomic framework has neglected learning mechanisms; for instance, the rule one has considered for the formation of expectations is straightforward and does not involve any evolution concept (agents have information and are able to formulate accurate forecasts or, in opposition, do not possess information and look at future dates as being the steady-state). However, the assumption of several types of agents with distinct information updating behavior was sufficient to build a setting in which a given phenomenon (in this case, aggregate supply conditions) cannot be described by a single dynamic rule. One has not derived a Phillips curve relation; instead, a different relation emerges at different time periods, given that at each date one finds distinct levels of aggregate attentiveness.

In the developed setting, we not only have heterogeneous firms, we also have firms that form their expectations having in consideration the overall ability of the economy to be attentive at each date. The attentiveness of others is relevant for the individual decision. Furthermore, the obtained results are specific to the kind of environment we have presented; if we change the structure of the problem (e.g., by adding agents, by removing agents, or by including different patterns of information updating), the time trajectories one would obtain for the inflation rate and for the output gap would be completely different from the ones we have arrived to. In other words, changing the dimension or the shape of the system would not preserve the structure of the problem.

Another relevant feature in the framework we have analyzed is that there is a well identified steady-state. However, given that for each period a different supply side equation exists, it became necessary to compute an equilibrium for each period and, thus, different pairs inflation rate – output gap characterize a supply-demand equilibrium at each moment. Therefore, although the equilibrium may coincide with a steady-state fixed point, this is changed once we consider a scenario where policy parameters do not remain constant independently of economic conditions; a changing monetary policy may trigger a long-term outcome where a boundedly instability outcome is formed (this outcome is characterized by large periodicity cycles in which inflation and the output gap fluctuate around the steady-state result but do not ever converge to or diverge from the steady-state).

In a representative agent world, applying some kind of policy would have the same impact on economic aggregates, independently of the time period in which the policy change occurs. In our setting, triggering a policy change implies diversified effects on the economy depending on the exact moment they are applied. A disturbance occurring at a period of full attentiveness impacts on economic aggregates differently from a disturbance that takes place on a period when a more or less significant degree of inattentiveness exists. Moreover, we were able to assume a two-state policy that perpetuates cycles in time. These cycles can have large periodicities, even when the heterogeneity in information updating is relatively low. When the heterogeneity level is high, the periodicity of a cycle can have such a large dimension that it becomes hard for someone to conceive such a significant time interval in which the same series of events repeats itself.

By going beyond the representative agent benchmark paradigm, one has offered a view of the economic system containing traces of complexity: heterogeneity has conducted us to a scenario of path dependence, structures exhibiting perpetual change, adaptive and evolving behavior of the system and of its components and relevant departures from equilibrium positions. Although simple and tractable from an analytical point of view, the advanced theoretical structure is rich enough to explain inertia and fluctuations in the time paths followed by meaningful economic variables.

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