Unit roots, random walks and the sources of business cycles: a survey*

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This paper presents and discusses some methodological aspects of econometric techniques that have recently been used to characterize the dynamic behavior of real GNP and its permanent and transitory components. Issues such as unit roots, stochastic trends, persistence of innovations, cyclical decompositions, structural models and spectral based tests are addressed.

1. Introduction; 2. Unit roots: why do we care? 3. The Beveridge-Nelson decomposition; 4. Structural decomposition of economic time series; 5. The measurement of persistence of shocks; 6. Permanent shocks: frequency and intensity; 7. The random walk null in the frequency domain; 8. The business cycle in a multivariate framework; 9. Concluding remarks.

1. Introduction

The issue of decomposing output fluctuations into permanent (trend) and transitory (cycles) components has received much attention in the recent literature. The traditional practice until the early 1980s was to fit a deterministic polynomial trend to the data and interpret the residuals as cycles. However, the pathbreaking work of Nelson and Plosser (1982) revealed that such a procedure was not appropriate since real GNP – as well as several other macroeconomic variables – appears to have an autoregressive unit root, and this implies that this series is integrated (or difference stationary) and hence has a stochastic trend.

Nonetheless, once a stochastic trend is allowed myriad possible decompositions can be obtained depending on the process attributed to the trend component. A wide field of research was then opened and many interesting

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(oftentimes conflicting) results were gotten using different econometric techniques. In this context, several questions arose. Are there business cycles? If so, how large is the cyclical component in GNP? How volatile is the stochastic trend? What is the role played by supply and demand disturbances in output fluctuations? How persistent are economic shocks? This paper's aim is to discuss the methodological aspects involved in some of the econometric techniques used to provide these questions with answers.

The questions posed above are important both theoretically and empirically. Macroeconomic theory has been primarily concerned with deviations of GNP from its trend, and the extent to which such deviations occur depends primarily on the nature of the trend component. In a more general context economic theory tries to place restrictions on the dynamic interrelations of several variables, and misspecification of the trend components will lead to incorrect inferences about the validity of different theories (Watson, 1986. p.50). On the empirical side, it has been shown by Nelson and Kang (1981,1984) and Durlauf and Phillips (1988) that fitting a deterministic polynomial trend to the data when these are generated by an integrated process introduces several spurious patterns in the detrended data. The questions above also have important policy implications since stabilization policies are designed to 'stabilize' short-run fluctuations, and again the extent to which such fluctuations occur is closely related to the dynamic behavior of the long-run component.

In particular, a puzzling question regarding unit roots in GNP is: why do we care? This question can be answered on different levels. To a policymaker the answer could be: "Because the policy implications are different". To a macroeconomist, it could be answered that "there are theoretical implications on several theories and models such as on the consumption theories" (*e.g.*, Deaton, 1991). Finally, an econometrician would be satisfied with the answer: "Because the asymptotics are different."

The remainder of the paper is organized as follows. The next section discusses some implications of unit roots in macroeconomic time series. Section 3 presents the Beveridge-Nelson decomposition of GNP into permanent and cyclical components. An alternative decomposition based on structural models is discussed in Section 4. Section 5 covers the controversial issue of measurement of persistence of innovations. The following section briefly summarizes some implications of large and infrequent shocks, as opposed to small and recurrent innovations. Section 7 turns to the frequency domain and presents some spectral based tests of the random walk null. Some strong results recently obtained using such tests are also covered. Some multivariate approaches to the characterization of output movements are discussed in Section 8. Finally, concluding remarks are given in the last section.

2. Unit roots: why do we care?

As discussed in the Introduction, the traditional view decomposes output fluctuations into a deterministic trend and cycles which follow a stationary stochastic process around the trend. Still according to this view, real factors such as technology, capital accumulation and population growth determine the growth in the trend component, whereas monetary disturbances are the source of business cycles.

However, since the influential work of Nelson and Plosser (1982), it has been argued that the secular component of GNP is best characterized by a stochastic rather than deterministic trend. These authors have shown that a time series has a stochastic trend if and only if it has a unit root in its autoregressive representation. Thus, testing for the presence of a unit root is equivalent to testing for the presence of a stochastic trend in the series.

The presence of a stochastic trend in GNP has far-reaching implications. The most important one relates to the persistence of economic shocks in the long-run. A deterministic trend implies that innovations are temporary, in the sense that their effects are felt only during the cycle. Put differently, innovations have only short-run effects. On the other hand, a stochastic trend implies that economic shocks are to some extent persistent in the sense that they have effects on the long-run level of output.

The key issue in the distinction between deterministic and stochastic trends is whether GNP is stationary, *i.e.*, whether its mean is time invariant and its variance is boundend and does not change over time (Cuthbertson, Hall and Taylor, 1992, p.129-30).

More formally, let y_t be the natural logarithm of real GNP and assume that y_t follows an AR(1) precess given by

$$y_t = \alpha y_{t-1} + \varepsilon_t,$$

where ε_t is second-order white noise.[1] Then, it follows that

$$E[y_t] = \sum_{j=0}^{\infty} \alpha^j E[\varepsilon_{t-j}] = 0,$$

$$var[y_t] = \frac{\sigma_\varepsilon^2}{1-\alpha^2}$$

and

[1] For a definition of second-order white noise processes, see Bloomfield (1991, p.153).

$$cov\,[y_t, y_{t-j}] = \alpha^{2j} E\,[y_t^2].$$

Hence, y_t is stationary if and only if $|\alpha| < 1$. Otherwise, *var* [.] and *cov* [.] would grow without bound.

Moving to a slightly more general, let y_t grow over time according to

$$y_t = \mu + \alpha_t + \varepsilon_t$$

This is in essence a deterministic trend model. Again, stationarity requires $|\alpha| < 1$. Note that the variance of y_t is bounded by the variance of ε_t. In economic terms this means that uncertainty is bounded even in the long-run.

Consider now the simplest integrated process: the random walk (with drift). This process is given by

$$y_t = \mu + y_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is second-order white noise. Note the presence of an autoregressive unit root on the coefficient of y_{t-1}. Equation (1) can be rewritten as

$$y_t = y_0 + \mu t + \sum_{j=1}^{t} \varepsilon_j, \tag{2}$$

where y_0 is the starting value of $\{y_t\}$. It is clear from (2) that innovations are full persistent, since a random walk is nothing more than an accumulation of disturbances. Note also that the variance of y_t grows without bound; this is because y_t is not stationary, although its first difference Δy_t is.

The class of deterministic trend models is called "trend sationary" and the class of stochastic trend models is called "difference stationary". As shown above, the former implies a degree of persistence of shocks of zero, while the latter implies a positive degree of persistence of innovations.

Another important implication of the presence of a stochastic trend in GNP is that at least part of the short-run fluctuations are due to real factors. These factores are more important in this framework since they account for some of the short-run oscillations as well as the long-run fluctuations in GNP. Here a word of caution is in order. It is common practice to associate persistent shocks to movements in aggregate supply, whereas mean-reverting shocks are viewed as movements in aggregate demand, as, for instance, in Blanchard and Fischer (1989. p.14-5). However, this is a misleading and slippery view.[2] As noted by Plosser (1989. p.57) in the context of the

[2] I thank a referee for bringing this point to my attention.

so-called real business cycles models, real shocks do not easily translate into either demand or supply disturbances.

A final comment regards the test of the unit root hypothesis, *i.e.*, the test of the presence of a stochastic trend. The most used test in the literature is the (augmented) Dickey-Fuller (1979) test. This carried out by estimating the equation

$$y_t = \gamma_0 + p y_{t-1} + \sum_{j=i}^{k} \gamma_j \Delta y_{t-j} + u_t$$

by ordinary least squares (OLS) and testing the hypothesis that $t \overset{def}{=} (\rho - 1)/se(\rho)$ equals zero using the critical values in Fuller (1976, p.373). That is, the null hypothesis can be tested using the statistic

$$t_{\hat{\rho}} \overset{def}{=} \hat{s}^{-1} (\hat{\rho} - \rho) \sqrt{\sum (y_{t-1} - T^{-1} \sum y_{i-1})^2},$$

where $\hat{s}$ is the standard error of the regression. Failing to reject the null is equivalent to failing to reject the presence of a unit root or stochastic trend.

A problem with this approach is that it requires the disturbance term to be i.i.d. In order to extend this test to less restrictive settings, Phillips and Perron (1988) have proposed the following modification of the test statistic given above:

$$Z(t_{\hat{\rho}}) \overset{def}{=} (S_\varepsilon / S_{T\iota}) t_{\hat{\rho}} - \frac{1}{2} (S^2_{T\iota} - S^2_\varepsilon) \left(S_{T\iota} \sqrt{T^{-2} \sum y^2_{t-1}} \right)^{-1},$$

where, under the null hypothesis, S^2_ε is a consistent estimator of lim $T^{-1} \sum E[\varepsilon^2_t]$ and S^2_{Tl} is a consistent estimator of lim $T^{-1} E[(\sum \varepsilon_t)^2]$. The use of this teste statistic extends the unit root test to the general case of weakly dependent and heterogeneously distributed data. A nice feature of this test is that the critical values tabulated by Fuller (1976, p.373) are still valid.[3]

[3] See also Perron (1988) and Phillips (1987). For Bayesian tests of the unit root hypothesis, see Koop (1991,1992).

An even more serious problem with the Dickey-Fuller test is that it assumes that the order of autoregression (k) is known to the investigator, and yet this does not seem to be a realistic assumption in practice. Hall (1991) has derived the limiting distribution of this test K is selected using a data based method that is asymptotically independent of the test statistic. The main result his paper conveys is that the limiting distribution of the unit root test is not affected when one uses an information criterion to choose the order of the autoregression. This result deserves to be highlighted since it is very important for practical applications. See also Hall (1992).

It is also possible to design tests for the null hypothesis of stationarity against the alternative of a unit root; see, e.g., Kahn and Ogaki (1992) and Kwiatkoswki *et alii* (1992).

It should be remarked, however, that both tests of unit roots and tests of stationarity may have low power. That is, the powers of integration tests against plausible trend stationary alternatives can be very low, as can the powers of tests of the null hypothesis of trend stationarity against integrated alternatives. See DeJong *et alii* (1992) for further details.

3. The Beveridge-Nelson decomposition

A difficulty involved in trend/cycles decompositions once a stochastic trend is assumed is that one obtains a different decomposition for each stochastic process attributed to the trend component. A very useful decomposition is due to Beveridge and Nelson (1981). This decomposition is based on the assumption that the innovations in both components are perfectly correlated.

Let $z_t \overset{def}{=} y_t - y_{t-1}$ and assume thet z_t is covariance stationary. Then, its Wold representation (Wold, 1938) is

$$z_t = \mu + \sum_{j=0}^{\infty} \lambda_j \varepsilon_t - j, \quad \lambda_0 = 1, \tag{3}$$

where μ is the long-run growth rate[4] and $\in_t$ is second-order white noise. The permanent or trend component is given by

$$y_t^p = y_t + \left(\sum_{i=1}^{\infty} \lambda_i \right) \varepsilon_t + \left(\sum_{i=2}^{\infty} \lambda_i \right) \varepsilon_t - 1 + \dots \tag{4}$$

[4] This is because y_t is in logs.

The trend component in (4) follows a drifted random walk. To see this, take the first difference of y_t^p

$$(1-B)y_t^\rho = Z_t + \left(\sum_{i=1}^{\infty} \lambda_i\right) \varepsilon_t + \left(\sum_{i=2}^{\infty}\lambda_i\right)\varepsilon_{t-1} + \ldots - \lambda_1 \varepsilon_{t-1} \tag{5}$$

$$- \left(\sum_{i=2}^{\infty} \lambda_i\right)\varepsilon_{t-1} - \lambda_2 \varepsilon_{t-2} - \left(\sum_{i=3}^{\infty} \lambda_i\right)\varepsilon_{t-2} - \ldots$$

where B is the backward shift operator. Using (3), equation (5) can be simplified to

$$(1-B)y_t^\rho = \mu + \left(\sum_{i=0}^{\infty}\lambda_i\right) \varepsilon_t, \ \lambda_0 = 1$$

or

$$y_t^\rho = \mu + y_{t-1}^\rho + \tilde{\varepsilon}_t, \tag{6}$$

where $\tilde{\varepsilon}_t = \left(\sum_{i=0}^{\infty} \lambda_i\right) \varepsilon_t$. Equation (6) establishes that the trend in GNP follows a drifted random walk. Now let $\tilde{z}_t(k)$ be the conditional expectation of z_{t+k} at time t. We can then rewrite (4) as

$$y_t^\rho = y_t + \lim_{k\to\infty}\left\{\sum_{i=1}^{k} \tilde{z}_t(k) - k\mu\right\}.$$

That is, the trend component is the sum of the current observation and all forecastable movements in the series apart the drift. It is the clear that the cyclical component (z_t^c) is given by

$$y_t^c = \lim_{k\to\infty}\left\{\sum_{i=1}^{k} \tilde{z}_t(i) - k\mu\right\}$$

or

$$y_t^c = \left(\sum_{i=1}^{\infty}\lambda_i\right)\varepsilon_t + \left(\sum_{i=2}^{\infty}\lambda_i\right)\varepsilon_{t-1} + \ldots \tag{7}$$

Thus, any integrated process can be decomposed into a trend which follows a drifted random walk [eq.(4)] and a cyclical component [eq.(7)] with the innovations to both components being perfectly correlated. It is important to mention that the assumption that shocks to both components are perfectly correlated is essential, given that not all integrated processes can be decomposed into a random walk component and an uncorrelated cyclical component, as shown by Watson (1986). One should note that there are no cycles when the series itself (z_t) follows a random walk with drift. Some other decompositions, as the one in Definition 2.1 of Quah's (1992) paper, do not allow the cyclical component to have variance zero.

A further issue in this context is how to compute the Beveridge-Nelson decomposition. Different strategies have been suggested by Cuddington and Winters (1987), Miller (1988) and Newbold (1990). We will focus on Newbold's approach, since it is "exact" in the sence that it does not require any truncation. His main result can be described as follows. Let z_t be represented by the ARMA (p,q) model

$$\phi(B)\,(z_t-\mu)=\theta(B)\varepsilon_t, \tag{8}$$

where ε_t is second-order white noise and the roots of the polynomials $\phi(B)$ and $\theta(B)$ lie outside the unit circle. Then,

$$y_t^c = \sum_{j=1}^{q}\tilde{z}_t^*(j) + \left(1-\sum_{i=1}^{p}\phi_i\right)^{-1}\sum_{j=1}^{p}\sum_{i=j}^{p}\phi_i\,\tilde{z}_t^*(q-j+1),$$

where $z_t^* = z_t - \mu$. The trend component is then obtained by summing y_t and y_t^c.

4. Structural decomposition of economic times series

The Beveridge-Nelson decomposition discussed in the last section is based on an ARIMA model building procedure, where one selects a model from a broader class based on the properties of the data. The structural decomposition approach, conversely, aims at decomposing the time series into unobservable components with direct economic meaning, such as trend, cycles and irregular.[5]

Following Harvey (1985), consider the following class of models:

$$y_t = \mu_t + \varphi_t + \varepsilon_t, \tag{9}$$

where μ_t is a trend, φ_t is a cycle and ε_t is an irregular component. This last component is second-order white noise and our goal is to find suitable characterizations for μ_t and l_t.

The cyclical component can enter the model additively as a separate component [as in eq.(9)] or can be specified into the trend component. The latter specification is known as "cyclical trend" and is given by

$$y_t = \mu_t + \varepsilon_t \tag{10}$$

and

$$\mu_t = \mu_{t-1} + \gamma_{-1} + \varphi_{t-1} + \nu_t, \tag{11}$$

where

$$y_t = y_{\gamma-1} + \nu_t \tag{12}$$

and ν_t and ν_t are uncorrelated second-order white noises.

The specification in (10)-(12) is still too general since no closed-form specification for the cyclical component is provided. A commonly used specification is given by

$$\begin{pmatrix} \varphi_t \\ \bar{\varphi}_t \end{pmatrix} = \rho \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} \varphi_{t-1} \\ \bar{\varphi}_{t-1} \end{pmatrix} \begin{pmatrix} \eta_t \\ \bar{\eta}_t \end{pmatrix}, \tag{13}$$

where η_t and $\bar{\eta}_t$ are uncorrelated second-order white noises whith variances o^2_η and $o^2_{\bar{\eta}}$, respectively. The parameter $\lambda (0 \leq \lambda \leq \pi)$ determines the frequency of the cycle and the parameter ρ is a damping factor on the cycle amplitude.

It follows from (13) that

$$\varphi_t = \frac{1 - \rho\cos\lambda . B)\eta_t + (\rho\sin\lambda . B)\bar{\eta}_t}{1 - 2\rho\cos\lambda . B + \rho^2 . B^2}, \tag{14}$$

[5] See Aoki (1990) and Harvey (1985, 1989).

The implication of (14 is that φ_t is constrained to follow an ARMA(2,1) process. When $\sigma_\eta^2 = 0$ and $0 < \lambda < 1$, equation (14) implies an ARMA(2,0) model for the cyclical component with the autoregressive operator having complex roots.

The parameter estimation of the model presented above can be carried out by writing the model in state space form and then using the Kalman filter (Harvey, 1989). For an analysis of diagnostic checking in structural models, see Harvey and Koopman (1992). For a definition of structural models with ARCH disturbances, see Harvey, Ruiz and Sentana (1992).

Although this is a very appealing technique given the economic meaning of the "strucural components", it faces several limitations. First, it restricts the class of ARIMA models and their respective paramater spaces on an entirely *a priori* basis (Newbold, 1991). Secondly, as shown by Newbold and Agiakloglou (1991), structural models can lead to implications on the unobservable components quite different from the ones obtained using the more conservative ARIMA model buiding procedure.

5. The measurement of persistence of shocks

It was shows in the last section that any time series with a unit root in its autoregressive representation can be decomposed into a random walk component (trend) and a stationary component (cycles). However, the random walk may have arbitrárily small variance.[6] Therefore, it is important to provide a measure of the size of the random walk component, which is done by measuring the degree of persistence of innovations in long-run GNP. Two measures will be discussed in this section.

The first measure is called the *cumulative impulse response function* and was proposed by Campbell and Mankiw (1987), who have built upon previous work by Box and Jenkins (1976). Let z_t be represented by (8). Then,

$$(1 - B)(z_t - \mu) = A(B)\varepsilon_t,$$

where $A(B) = [\phi(B)]^{-1}\theta(B)$. It follows from the stationarity of z_t that

$$A(1) \overset{\text{def}}{=} \sum_{i=0}^{\infty} A_i < \infty.$$

Since y_t is the logarithm of GNP and z_t is the first difference of y_t, then $A(1)$ is a measure of the long-run persistence of a one percent shock[7].

[6] In this case, unit root tests have arbitrarily low power in finite samples. See Cochrane (1991a).

[7] Recall that the difference of the logarithm of a series is (approximately) its rate of growth between consecutive periods.

This is a very intuitive measure and it has been widely used in the recent literature. Nonetheless, it faces several limitations. The most important of all is that different models may imply different degrees of long-run persistence, and yet there is no widely agreed upon criterion for selecting a model from a class of competing models. There are several criteria for model selection such as AIC (Akaike, 1974), AICC (Hurvich and Tsai, 1989), BIC (Schwarz, 1978; Rissanen, 1978) and HQ (Hannan and Quinn, 1979), and yet there is no consensus to which criterion should be used. Furthermore, as noted by Newbold, Agiakloglou and Miller (1992a), these criteria were designed to find a parsimonious representation for the series which means that p and q may not be sufficiently large to capture its long-run dynamics.[8] There is also the problem of sensitivity of the results to small parameter changes. A simple example will make this point clear. Consider two ARMA (1,1) models, the first with ϕ_1=0.9 and θ_1=0.8, and the second ϕ_1 = 0.8 and θ_1 = 0.9. For the first model, $A(1) = 2.0$ and for the second model $A(1) = 0.5$.

Another problem in computing the cumulative impulse response function relates to the estimation of the parameters in the selected ARIMA model. This estimation can be carried out using different methods. In an extensive simulation experiment, Ansley and Newbold (1980) have compared the performance of three competing methods, namely: exact maximum likelihood, exact least squares, and conditional least squares. Maximum likelihood estimates were obtained by maximizing the concentrated likelihood function using Ansley's (1979) algorithm. Exact least squares estimation was carried out by using one iteration and truncating an infinite sum. Finally, conditional least squares estimates were obtained by using the procedure suggested in Box and Jenkin (1976, p.211).

The three estimators described above are asymptotically equivalent in the sense that the difference between any of them vanishes in probability as the sample size tends to infinity. However, Ansley and Newbold's (1980) simulation results show that these estimators can give different estimates for mixed models in finite samples. In particular, this problem is more severe when the autoregressive operators are close to the stationarity boundary and/or there is near-cancellation of AR and MA parameters.

Based on their results, Ansley and Newbold (1980) recommend the use of the maximum likelihood estimator, since the possible losses are very small, whereas the possible gains may be large. According to them (p.181), "it seems likely that practical situations arise in which one of other of the least squares estimators performed so poorly that its use would be regarded as undesirable."

[8] For a thorough discussion of model selection criteria, see Choi (1992). For a Monte Carlo comparison of different criteria, see Mills and Prasad (1992).

It follows from the discussion above that it may be the case that two econometricians using the *same* data set, estimating the same ARIMA model, but using different econometric programs arrive at different estimates for A(1). This point was forcefully made by Newbold, Agiakloglou and Miller (1992b). These authors have used several diferent packages to analyze five series, including real GNP.[9] They have shown that different econometric programs can lead to different parameter estimates for both seasonal and non-seasonal series.

An alternative and nonparametric measure, known as the variance ratio, was proposed by Cochrane (1988). This measure is given by

$$V_k \overset{\text{def}}{=} \frac{1}{k}\,\frac{var\,(y_t - y_{t-k})}{var\,(y_t - y_{t-1})}. \tag{15}$$

It is easily shown that if y_t is trend-reverting, then $\lim k \to \infty\ V_k = 0$, and if y_t evolves as a random walk, then $V_k = 1$ for all lags.

The variance ratio in (15) can be estimated as follows. Define

$$\hat{\mu} \overset{\text{def}}{=} \frac{1}{T}\,(y_T - y_0),$$

$$\hat{\sigma}_1^2 \overset{\text{def}}{=} \frac{1}{T-1}\sum_{t=1}^{T}(y_t - y_{t-1} - \hat{\mu})^2$$

and

$$\hat{\sigma}_k^2 \overset{\text{def}}{=} \frac{T}{k\,(T-k)\,(T-k+1)}\sum_{t=k}^{T}(y_t - y_{t-k} - k\,\hat{\mu})^2.$$

Then, an estimator for the variance ratio is

$$\hat{V}_k = \frac{\hat{\sigma}_k^2}{\hat{\sigma}_1^2}.$$

[9] They used the following packages: SPSS, SAS, SHAZAM, STATGRAPHICS, BMDP, AUTOBOX, MINITAB, RATS, TSP, SYSTAT and S-PLUS. It should be remarked, however, that S-PLUS is not exactly a software; it is more a high level programming language.

Moreover, it was shown by Lo and MacKinlay (1988,1989) that

$$V_k \overset{\text{def}}{=} \sqrt{T}(\hat{V}_k - 1)\sqrt{\frac{3k}{2(2k-1)(k-1)}}$$

converges in distribution to a standard normal. This is a very useful result since it enables one to build confidence intervals for the estimated variance ratio.

For a criticism of the variance ratio as a tool for distinguishing between trend stationary and difference stationary processes, the reader is referred to Christiano and Einchenbaum (1989). In short, these authors argue that the standard errors for the estimated variance ratio are too large,[10] and that it is not possible to be sure about the direction of the bias of this measure.

The two measures presented above are however closely related. Let R^2 be the fraction of variance of $y_t - y_{t-1}$ that is predictable from the past behavior of the series. Then, the following relation holds:

$$A_k(1) = \sqrt{\frac{V_k}{1-R^2}}.$$

R^2 is usually approximated by the square of the first order coefficient of autoconelation.

The main puzzle in the measurement of persistence of innovations relates to the difference in the results obtained from ARIMA (*i.e.*, cumulative impulse response function) and structural models. Campbell and Mankiw found a degree of persistence of innovations greater than one for postwar quarterly U.S. real GNP. However, studies based on structural models (*e.g.*, Clark, 1987; Watson, 1986) have conversely found that such a degree is less than one.

This puzzle has recently been solved by Lippi and Reichlin (1992), who have shown that it is a mathematical consequence of the definition of structural models that the degree of persistence of innovations is constrained to the interval [0,1]. Their result can be stated more precisely as follows. Let y_t be a difference stationary random variable and consider the structural model given in (9) where $var\,[\varphi_t] > 0$ and $\mu_t = \gamma + \mu_{t-1} + \nu t$ ($var\,[\nu_t] > 0$). Then, the assumption that $cov\,[z_t,\ \varphi_{t-k}] = 0\ \ \forall k \geq 0$ implies that $A(1) < 1$.[11]

[10] They considered the standard errors given by Priestley (1981).

[11] Note that there is a typographical error in the equation defining the trend component in the statement of the theorem on page 91 of Lippi and Reichlin's (1992) paper. The way this component is defined implies that it is a white noise, rather than a random walk.

Therefore, the use of structural models in the context of measuring persistence of innovations constrains *a priori* the degree of this persistence, and one should bear that in mind.

6. Permanent shocks: frequency and intensity

Recent work has suggested that the issue of persistence of innovations can be analyzed in a slightly different setting. According to this work, persistence arises from large and infrequent shocks, rather than from small and frequent shocks. Perron (1989) used dummy variables to isolate the effects of some large shocks (mainly the Great Depression), and showed that it is possible to reject the null of a unit root once the effects of such shocks are removed. His analysis then suggests that a deterministic trend with a few discontinuities caused by some large and infrequent shocks together with stationary cycles best characterize the oscillations and fluctuations in output. For other applications of this approach, see Duck (1992), Inwood and Stengos (1991) and Serletis (1992). However, this approach does not seem to be robust to the measure of real economic activity and the number of lagged differenced residuals (Davis and Kanago, 1992). A more serious limitation of this kind of analysis is that since dummy variables are chosen in an *ex post* way, the *ex ante* probability assessed to future discontinuities equals zero. A more general framework was proposed by Balke and Fomby (1991a).[12] Consider the following model:

$$Y_t = \mu + y_{t-1} + \Xi(B)\zeta_t,$$

where the roots of $\Xi(B)$ lie outside the unit circle and

$$\zeta_t = I_t \varepsilon_t.$$

Assume that ε_t is second-order white noise and I_t is a Bernoulli trial, *i.e.*, $Pr(I_t=1)=\delta$ and $Pr(I_t=0)=1-\delta$. In this setting, a large and persistent shock occurs with probability δ. We then have that $var[\zeta_t]=\delta\sigma_\varepsilon^2$.

Note that this model has the deterministic trend model ($\delta=0$), the random walk trend model ($\delta=1$), and the discontinuous deterministic trend model ($0<\delta<1$) as special cases.

It is wel known that the Dickey-Fuller test discussed in Section 2 is asymptotically consistent regardless of the variance of the shocks, provided that this variance is strictly positive. However, the simulations in Balke and Fomby (1991a) show that when $0<\delta<1$ the Dickey-Fuller test is biased

[12] See also Balke and Fomby (1991b,1991c).

towards the rejection of the null, *i.e.*, it rejects the null of a unit root too often even when the series does have an autoregressive unit root.

7. The random walk null in the frequency domain

As mentioned earlier, the issue of persistence of innovations in real GNP is important because it is an indication of the size of its random walk component. This is in turn relevant because it helps in identifying the role of real shocks in output fluctuations.

It is well known that the presence of an autoregressive unit root in the series is equivalent to the presence of a stochastic trend. It is possible, however, that there are no cycles and hence the series and the stochastic trend coincide. This hypothesis can be tested changing the null from "the series has a unit root" "the series is a drifted random walk". This approach was followed by Durlauf (1989,1991). This section will discuss some spectral based tests used to test such a null.

If y_t follows a random walk, the z_t is an i.i.d. disturbance with mean μ, the drift. Let f_z be the spectral density of z and $0 \leq \lambda \leq \pi$. Then, the normalized spectral distribution

$$M_z(\lambda) \overset{\text{def}}{=} \frac{2\int_0^{\lambda} f_z(\omega)\, d(\omega)}{var(z)},$$

should be a diagonal line. Now define the following random function

$$U_T(t) \overset{\text{def}}{=} \sqrt{2T}\int_0^{\pi t}\left(\frac{I_T(\omega)}{var(z)} - \frac{1}{2\pi}\right) d\omega, \tag{16}$$

for $0 \leq t \leq 1$, and where $I_T(\omega)$ is the periodogram of z_t. The random function defined in (16) computes the deviations of the (periodogram-based) spectral distribution function from a diagonal line.

Under certain regularity conditions, $U_T(t)$ converges weakly to a Brownian bridge, that is

$$U_T(t) \overset{w}{\rightarrow} \bar{B}(t),$$

where $\bar{B}(t)$ is Brownian bridge.[13] This result establishes that $U_T(t)$ has asymptotic distribution $N(0, t-t^2)$, and it is the basis for the tests described below.

It is now possible to define two test statistics based on the random function $U_T(t)$ given in (16). The *Cramér-Von Mises statistic is* given by

$$S_t^{cm} \overset{\text{def}}{=} \int_0^1 U_T(t)^2\, dt, \tag{17}$$

and the *Kolmogorov-Smirnov statistic* is defined as

$$S_T^{ks} \overset{\text{def}}{=} \sup_{0 \le t \le 1} | U_T(t) |. \tag{18}$$

Another useful spectral based test is *Fisher's test.* Its null is that z_t is a Gaussian white noise, and it enables one to test for the presence of hidden periodicities with unspecified frequency. The test statistic is defined as

$$S_T^f \overset{\text{def}}{=} \frac{\max_{0<j<q+1} I_T(\omega j)}{q^{-1} \sum_{j=1}^{q} I_T(\omega j)}$$

where $q = \left(\frac{T-1}{2}\right)$.[14] It was shown by Fisher (1929) that

$$Pr\left(S_T^f \ge a\right) = \sum_{i=1}^{j} (-1)^{i-1} \binom{q}{i} (1 - ia)^{q-1}$$

Durlauf (1991) used spectral tests based on the statistics given in (17) and (18) to test the random walk null for U.S. real GNP from 1870 to 1989.[15] He showed that it is possible to reject this null at the usual significance levels. However, it is not possible to reject the same null for the subsamples 1870-1929 and 1947-89. These results suggest that the atypical period from 1930 to 1946 may be causing the rejection of the drifted random walk hypothesis for the whole sample.

Durlauf's (1991) results have strong implications. First, in the atypical

[13] See the appendix for a definition of a Brownian bridge. For a a definition of weak convergence, see Dhrymes (1989, chapter 4) and Pollard (1984).

[14] See Brockwell and Davis (1991, chapter 10) for more details.

[15] See also Durlauf (1989).

period from the Great Depression to World War II GNP had a different stochastic characterization than in the rest of the series. As shown before, deviations of the series from its random walk component can be viewed as business cycles. In this sense, the results obtained by this author imply that – except for this atypical period – there is no evidence of business cycles in the United States. That is, all fluctuations and oscillations in GNP have been caused by the dynamic accumulation of persistent shocks.

Furthermore, it has been argued that it is not possible to identify a unit root since there are several caveats involved in testing this hypothesis (*cf.*, Christiano and Einchenbaum, 1989; Cochrane, 1991a). However, identifying a random walk in the level of series is certainly a stronger result than identifying a unit autoregressive root, since the former case implies the latter. Therefore, Durlauf's results show that on certain occasions it is reasonably safe to argue that the nature of the nonstationarity in GNP is stochastic.

8. The business cycle in a multivariate framework

The discussion has focused so far on the univariate properties of real GNP. However, it is possible to develop multivariate approaches that allow one to characterize the dynamic pattern of output movements. A useful concept to start with is the concept of *cointegration*. Suppose that y_{1t} and y_{2t} are $I(1)$, *i.e.*, integrated for order one. Assume further that there exists a linear combination of these series

$$m_t = \alpha_0 + \alpha_{1y_{1t}} + \alpha 2y_{2t}$$

which is both $I(0)$ and has zero mean. Then, y_{1t} and y_{2t} are said to be cointegrated.[16]

The concept of cointegration can be useful in characterizing persistent movements in GNP. Suppose one is willing to assume that persistent innovations are due to technology shifts. Since new technologies migrate quite easily among countries, one would expect per capita output in developed countries to be cointegrated. The underlying idea is that per capita GNP in two developed countries cannot drift apart in the long-run as a result of tecnological innovations, and hence long-run convergence is expected. This point was made by Durlauf (1989); see also Campbell and Mankiw (1989). However, as pointed out by Christopher Sims in his comments on Durlauf's paper, since countries are differently endowed shocks in technology will have persistently different effects on different countries.

[16] See Engle and Granger (1987). A useful survey is Muscatelli and Hurn (1992).

Another important concept in the multivariate framework is the concept of common features. A feature is said to be common to a multivariate data set if a linear combination of the series no longer has the feature. It is clear that this is a generalization of cointegration since the "feature" is not restricted to be the order of integration. Tests for common features were developed by Engle and Kozicki (1991). Here, an important feature to be considered is dependence, and the concept of *codependence* can be introduced as an indicator of comovement among stationary time series, as in Vahid and Engle (1991). A strong form of codependence is the *serial correlation common feature,* as in Engle and Kozicki (1991), in which case a linear combination of some stationary series eliminates all correlation with the past and is unpredictable with respect to the (joint) past information set. In this context, Vahid and Engle (1991) showed that "a common serial correlation feature among the first differences of a set of cointegrated $I(1)$ variables implies that the remainders after removing their common trends from their levels, i.e., their cycles, are also common" (p. 2). The authors have also presented a test for common cycles and showed how to estimate the number of common cycles given the existence of common trends. Engle and Issler (1992) applied these techniques to per capital sectoral real U.S. GNP. The results in their paper show that the cyclical fluctuations across sectors are fairly similar whilst the movements in the stochastic trends are quite different. Such a result is in agreement with Burns and Mitchell's (1946) classic book, since these authors worked under the belief that many economic time series are driven by a common cycle. An interesting result in Engle and Issler's (1992) paper is that cycles and trends are negatively correlated. This implies that temporary shocks to some sectors may have persistent effects on other sectors which breaks down the usual short and long-run dichotomy. A somewhat different approach was pursued by Clark (1992). He has studied the role of regional fluctuations in the business cycle in the United States. His results show that even after controlling for industry mix effects there are significant region-specific fluctuations in output. Indeed, he has shown that approximately "40% of the variance of the cyclical innovation in the average's region employment growth rate is particular to the region. Moreover, there is evidence that, over time, these shocks tend to propagate across regions" (p.1).

A different multivariate framework was adopted by Cochrane (1991b). He used consumption to decompose output movements into trend and cycles. His analysis goes as follows. Since consumption is nearly a random walk and the consumption GNP ratio is stable over long time spans,[17] then "a high consumption/GNP ratio signals that GNP must rise in the future, to reestablish the ratio, and today's consumption is nearly today's forecast of the long-run 'trend' ind GNP" (p.2). The author's results show that innovations in

[17]. This because consumption and GNP are cointegrated.

GNP that do not change consumption are almost entirely transitory, whereas shocks that induce changes in consumption are persistent. In this framework, one can use consumption as a good approximation to the trend in GNP since it provides a measure of consummer's expectations of long-run GNP.

King *et alli* (1991) used the long-run restriction of the class of real business cycles models that productivity shocks are shocks to the common stochastic trend in output, investment and consumption to shed some light on the sources of the business cycle. These authors have shown that, when nominal variables are introduced, productivity shocks account for less that half of the business cycle. This result casts some doubt on the claim that productivity shocks are the primary source frequency oscillations in output.

9. Concluding remarks

The purpose of this paper was to present and discuss several techniques that have been used in the recent literature to characterize the dynamic behavior of real GNP. The analysis reveals that the results obtained may be largely sensitive to the choice of the technique, and that there are several caveats in modeling GNP. Despite all controversies, however, it has become a stylized fact that economic shocks to output are to some extent persistent in the long-run. Furthermore, the presence of a stochastic trend in GNP seems to suggest that real factors have accounted for at least part of the short-run fluctuations in addition to the long-run movements in output.

Appendix: Brownian motion

A *Brownian motion* (or Wiener process) is a stochastic process satisfying the following set of conditions: *(i)*$B_0 = 0$; *(ii)* $\{B_t : t \geq 0\}$ is a Gaussian process; and *(iii)* For any $t \geq s \geq 0$, $B_t - B_s$ has mean zero and varianct $t - s$.

If B_t is a Brownian motion, then the process defined as $\overline{B}_t = B_t - tB_1$, for $0 \leq t \leq 1$, is a *Brownian bridge*. It follows from the definition of Brownian motion that a Brownian bridge has distribution $N(0, t - t^2)$. It has a normal distribution because it is a linear combination of normally distributed variables. To see that it has mean zero, not that $E[\overline{B}_t] = E[B_t] - tE[B_1] = 0 - t0 = 0$. Finally, it has variance $t - t^2$ because $var[\overline{B}_t] = E[\overline{B}_t]^2 = E[B_t]^2 - 2tE[B_tB_1] + t^2E[B_1]^2 = t - 2t^2 + t^2 = t - t^2$.

For more details on Brownian motion, see Hida (1980).

Resumo

O presente artigo apresenta e discute vários aspectos metodológicos de técnicas econométricas que têm sido usadas recentemente para caracterizar o padrão dinâmico do PNB real e suas componentes permanente e transitória. Abordam-se assuntos como raízes unitárias, tendências estocásticas, persistência de inovações, decomposições cíclicas, modelos estruturais e testes baseados em análise espectral.

References

Akaike, H. A new look at he statistical model identification. *IEEE transactions on Automatic Control, AC-19*: 716-23, 1974.

Ansley, C.F. An algorithm for the exact likelihood of a mixed autoregressive-moving average process. *Biometrika, 66*: 59-65, 1979.

Ansley, C.F. & Newbold, P. Finite sample properties of estimators for autoregressive moving average models. *Journal of Econometrics, 13*: 159-83, 1980.

Aoki, M. *State space modeling of time series* 2nd ed. Berlin, Springer-Verlag, 1990.

Balke, N.S. & Fomby, T.B. Infrequent permanent shocks and the finite-sample performance of unit root tests. *Economics Letters, 36*: 269-73, 1991a.

Balke, N.S. & Fomby, T.B. Shifting trends, segmented trends, and infrequent permanent shocks. *Journal of Monetary Economics, 28*: 61-85, 1991b.

Balke, N.S. & Fomby, T.B. Large shocks, small shocks, and economic fluctuations: outliers in macroeconomic time series. Federal Reserve Bank of Dallas, 1991c. (Research Paper Nº 9.101.)

Beveridge, S. and & Nelson, C.R. A new approach to the decomposition of economic time series into permanent and transitory components with particular attention to measurement of the "business cycle." *Journal of Monetary Economics, 7*: 151-74, 1981.

Blanchard, O.J. & Fischer, S. *Lectures on macroeconomics.* Cambridge, The MIT Press, 1989.

Bloomfield, P. Time series methods. In: Hinkley, D.V.; Reid, N. & Snell E.J. (eds.). *Statistical theory and modelling: in honour of Sir David Cox, FRS.* London, Chapman and Hall, 1991.

Box, G.E.P & Jenkins, G.M. *Time series analysis: Forecasting and control.* San Francisco, Holden-Day, 1976.

Brockwell, P.J. & Davis, R.A. *Time series: theory and methods.* 2nd ed. New York, Springer-Verlag, 1991.

Burns, A.F., & Mitchell, W.C. *Measuring business cycles.* Cambridge, National Bureau of Economic Research, 1946.

Campbell, J.Y. & Mankiw, N.G. Are output fluctuations transitory? *Quarterly Journal of Economics, 102:* 857-80, 1987.

Campbell, J.Y. & Mankiw, N.G. International evidence of the persistence of economic fluctuations. *Journal of Monetary Economics, 23*: 319-33, 1989.

Choi, B.S. *ARMA model identification.* New York, Springer-Verlag, 1992.

Cristiano, L.J. & Einchenbaum, M. Unit roots in real GNP: do we know, and do we care? National Bureau of Economic Research, 1989. (Working Paper Nº 3.130.), 1989.

Clark, P. The cyclical component of US economic activity. *Quarterly Journal of Economics, 102:* 797-814, 1987.

Clark, T. Business cycle fluctuations in US regions and industries: the roles of national, region-specific, and industry-specific shocks. Federal Reserve Bank of Kansas City, 1992. (Research Working Paper Nº 92-05.)

Cochrane, J.H How big is the random walk in GNP? *Journal of Political Economy, 96:* 893-920, 1988.

Cochrane, J. H. A critique of the aplication of unit root tests. *Journal of Economic Dynamics and Control, 15:* 275-84, 1991a.

Cochrane, J.H. Permanent and transitory components of GNP and stock prices. Department of Economics, University of Chicago, 1991b. (mimeogr.)

Cuddington, J.T. & Winters, L.A. The Beveridge-Nelson decomposition of economic time series: a quick computational method. *Journal of Monetary Economics, 19:* 125-7, 1987.

Cuthbertson, K., Hall, S.G. & Taylor, M.P. *Applied econometric techniques.* Ann Arbor, The University of Michigan Press, 1992.

Davis, G.K. & Kanago, B. The robustness of the split-trend stationarity hypothesis for the U.S. *Southern Economic Journal, 59*: 9-14, 1992.

Deaton, A. Saving and liquidity constraints. *Econometrica, 59*: 1.221-48, 1991.

DeJong, D.N.; Nankervis, J.C.; Savin, N.E. & Whiteman, C.H. Integration versus trend stationarity in time series. *Econometrica, 60*: 423-33, 1992.

Dickey. D.A. & Fuller, W.A. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association, 74*: 427-31, 1979.

Dhrymes, P.J. *Topics in advanced econometrics: probability foundations.* New York, Springer-Verlang, 1989.

Duck, N.W. UK evidence on breaking trend functions. *Oxford Economic Papers, 44*: 426-39, 1992.

Durlauf, S.N. Output persistence, economic structure, and the choice of stabilization policy. *Brookings Papers on Economic Activity, 2*: 69-136, 1989.

Durlauf, S.N. Time series properties of aggregate output fluctuations. Department of Economics, Stanford University, 1991. (mimeogr.)

Durlauf, S.N. and Philips, P.C.B. Trends versus random walks in time series analysis. *Econometrica, 56*: 1.333-54. 1988

Engle, R.F. & Granger, C.W.J. Cointegration and error-correction: representation, estimation and testing. *Econometrica, 55*: 251-76, 1987.

Engle, R.F. & Kozicki, S. Testing for common features. Dapartment of Economics, University of California at San Diego, 1991. (mimeogr.)

Engle, R.F. & Issler, J.V. Estimating sectoral cycles using cointegration and common features. Department of Economics, University of California at San Diego, 1992. (mimeogr.)

Fisher, R.A. Tests of significance in harmonic analysis. *Proc. R. Soc. London Ser. A, 125*: 54-9, 1929.

Fuller, W.A. *Introduction to statistical time series.* New York, Wiley, 1976.

Hall, A. Testing for a unit root in time series with pretest data based model selection. Departmente of Economics, North Carolina State University, 1991. (mimeogr.)

Hall, A. Testing for a unit root in time series using instrumental variable estimators with pretest data based model selection. *Journal of Econometrics, 54*: 223-50, 1992.

Hannan, E.J. & Quinn, B.J. The determination of the order of an autoregression. *Journal of the Royal Statistical Society B, 41*: 190-5, 1979.

Harvey, A.C. Trends and cycles in macroeconomic time series. *Journal of Business and Economic Statistics, 3*: 216-27, 1985.

Harvey, A.C. *Forecasting, structural time series models and the Kalman filter.* Cambridge, Cambridge University Press.

Harvey, A.C. & Koopman, S.J. Diagnostic checking of unobserved-components time series models. *Journal of Business and Economic Statistics, 10*: 377-89, 1992.

Harvey, A.C.; Ruiz, E.; & Sentana, E. Unobserved component time series models with ARCH disturbances. *Journal of Econometrics, 52*: 129-57, 1992.

Hida, T. *Brownian motion.* New York, Springer-Verlag, 1980.

Hurvich, C.M. & Tsai, C.L. Regression and time series model selection in small in small samples. *Biometrika, 76*: 297-307, 1989.

Inwood, K. & Stengos, T. Discontinuities in Canadian economic growth. *Explorations in Economic History, 28*: 274-80, 1991.

Kahn, J.A. & Ogaki, M. A consistent test for the null of stationarity against the alternative of a unit root. *Economics Letters, 39*: 7-11, 1992.

King, R.G.; Plosser, C.I.; Stock, J.H. & Watson, M.W. Stochastic trends and economic fluctuations. *American Economic Review, 81*: 819-40, 1991.

Koop, G. Intertemporal properties of real output: a bayesian analysis. *Journal of Business and Economic Statistics, 9*: 253-65, 1991.

Koop, G. "Objective" Bayesian unit root tests. *Journal of Applied Econometrics, 7*: 65-82, 1992.

Kwiatkoswki, D.; Phillips, P.C.B.; Schmidt, P. & Shin, Y. Testing the null hypohtesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root? *Journal of Econometrics, 54*: 159-78, 1992.

Lippi, M. & Reichlin, L. On persistence of shocks to economic variables: a common misconception. *Journal of Monetary Economics, 29* 87-93, 1992.

Lo, A.W. & MacKinlay, A.C. Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies*; *1*: 41-66, 1988.

Lo, A.W. & MacKinlay, A.C. The size and power of the variance ratio test in finite samples: a Monte Carlo investigation *Journal of Econometrics, 40*: 203-38, 1989.

Miller, S.M. The Beveridge-Nelson decomposition of economic time series: another economical computation method. *Journal of Monetary Economics, 21*: 141-2, 1988.

Mills, J.A. & Prasad, K. A comparison of model selection criteria. *Econometric Reviews, 11*: 201-33, 1992.

Muscatelli, V.A. & Hurn, S. Cointegration and dynamic time series models. *Journal of Economic Surveys, 6*: 1-43, 1992.

Nelson, C.R. & Kang, H. Spurious periodicity in inappropriately detrended time series. *Econometrica, 49*: 741-51, 1981.

Nelson, C.R. & Kang, H. Pitfalls in the use of time as an explanatory variable in regression. *Journal of Business and Economic Statistics, 2*: 73-82, 1984.

Nelson, C.R. & Plosser, C.I. Trends and random walks in macroeconomic time series: some evidence and implications. *Journal of Monetary Economics, 10*: 139-69, 1982.

Newbold, P. Precise and efficient computation of the Beveridge-Nelson decomposition of economic time series. *Journal of Monetary Economics, 26*: 453-7, 1990.

Newbold, P. Structural decomposition of time series with implications in Economics, Accounting, and finance research. *Review of Quantitative Finance and Accounting, 1*: 259-79, 1991.

Newbold, P. & Agiakloglou, C. Looking for evolving growth rates and cycles in British industrial production. *Journal of the Royal Statistical Society A, 154*: 341-8, 1991.

Newbold, P.; Agiakloglou, C. & Miller, J. Long term inference based on short-term forecasting models. Department of Economics, University of Illinois, 1992a. (mimeogr.)

Newold, P., Agiakloglou, C. & Miller, J. Adventures with ARIMA software. Department of Economics, University of Illinois, 1992b. (mimeogr.)

Perron, P. Trends and random walks in macroeconomic time series: further evidence from a new approach. *Journal of Economic Dynamics and Control, 12*: 297-332, 1988.

Perron, P. The Great Crash, the oil price shock, and the unit root hypothesis. *Econometrica, 57*: 1.361-401, 1989.

Phillips, P.C.B. Time series regression with a unit root. *Econometrica, 55*: 277-301, 1987.

Phillips, P.C.B. & Perron, P. Testing for a unit root in time series regression. *Biometrika, 75*: 333-45, 1988.

Plosser, C.I. Understanding real business cycles, *Journal of Economic Perspectives, 3*: 51-77, 1989.

Pollard, D. *Convergence of stochastic processes.* New York, Springer-Verlag, 1984.

Priestley, M.M. *Spectral analysis and time series.* New York, Academic Press, 1981.

Quah, D. The relative importance of permanent and transitory components: identification and some theoretical bounds. *Econometrica, 60*: 107-18, 1992.

Rissanen, J. Modeling by shortest data description. *Automatica, 14*: 465-71, 1978.

Schwarz, G. Estimation of the dimension of a model. *Annals of Statistics, 6*: 461-4, 1978.

Serletis, A. The random walk in Canadian output. *Canadian Journal of Economics, 25*: 392-406, 1992.

Vahid, F. & Engle, R.F. Common trends and Common cycles. Department of Economics, University of California at San Diego, 1991. (mimeogr.)

Watson, M.W. Univariate detrending methods with stochastic trends. *Journal of Monetary Economics, 18*: 49-75, 1986.

Wold, H. *A study in the analysis of stationary time series.* Stockholm, Almquist and Wicksell, 1938.