Fiscal and quasi-fiscal deficits, nominal and real: measurement and policy issues*

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Summary: 1. Introduction; 2. The deficit of the non-financial public sector; 3. The quasi-fiscal deficit of the central bank; 4. The consolidated public sector deficit; 5. Some stylized examples; 6. Conclusions and further thoughts.

In many developing countries the central bank assumes an active role in the mobilization and allocation of domestic and foreign exchange resources. In these countries, central bank operations may result in significant imbalances between revenues and costs (quasi-fiscal deficits). We provide a framework for understanding quasi-fiscal deficits by modelling the interactions between the government and central bank accounts, in nominal and real terms. This framework is used to analyze the problem of foreign exchange losses in the central bank, including the reasons for their accumulation and the conditions under which they may be monetized. We also discuss alternative methods for measuring central bank and consolidated public sector deficits. The Yugoslav and Hungarian cases are used as illustrations.

Em muitos países em desenvolvimento o banco central assume ativo papel na mobilização e alocação de recursos em moeda nacional e estrangeira. Nesses países, as operações do banco central podem provocar grandes desequilíbrios entre receitas e despesas (déficits quase-fiscais). Para melhor compreender os déficits quase-fiscais, estabelecemos modelos de interações entre as contas do governo e do banco central, em termos reais e nominais. Assim analisamos o problema da perda de divisas no banco central e também os motivos que levam a sua acumulação e as condições nas quais podem ser monetizadas. Também tratamos de métodos alternativos para medir o déficit do banco central e o déficit consolidado do setor público. Utilizamos como exemplo os casos da Iugoslávia e da Hungria.

1. Introduction

In many developing countries the central bank assumes a very active role in the mobilization of domestic and foreign exchange resources and their allocation to the public and private sectors. In these countries, the central bank operations may result in significant imbalances between revenues and costs, usually referred to as quasi-fiscal deficits. In some cases, the imbalances include the accumulation and realization of foreign exchange losses and systematic default on central bank credits to the private sector. As a consequence, the deficits generated by central bank operations may become as large or even larger than the deficits of the non-financial public sector. Failure to take these operations explicitly into account may give rise to apparent puzzles such as the simultaneous occurrence of low fiscal deficits and very high inflation rates.

Although the importance of quasi-fiscal deficits has been widely recognized (see Alves, 1988; Anand & Van Wijnbergen, 1989; Blejer & Cheasty, 1991; Cysne, 1990; Cysne, Lees & Botts, 1990; Robinson & Stella, 1988; Simonsen & Cysne, 1989; Teijeiro, 1989; and The World Bank, 1988), there have been few attempts to integrate formally the accounts of the

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non-financial public sector and the central bank. The impact of inflation on central bank operations and accounts has not been thoroughly examined in the literature. This is surprising, since the distinction between nominal and real deficits is especially relevant to the central bank, given the financial nature of its operations. Other critical issues, such as the accumulation of foreign exchange losses by the central bank, are not fully analyzed in the existing literature either. Instead, the analysis of this issue is usually restricted to drawing a distinction between unrealized and realized losses, or between accrued and cash losses. The dynamic implications of systematic foreign exchange losses are not duly explored. Finally, although it is widely recognized that the quasi-fiscal functions of the central bank should be ideally transferred to the budget, the causes of chronic quasi-fiscal deficits, and the policies that are required to correct them, have not been thoroughly examined.

We seek to contribute to the literature on fiscal and quasi-fiscal deficits in several ways. First, we examine in detail the interactions between the government and central bank accounts, in nominal and real terms. Second, we analyze the problem of foreign exchange losses in the central bank, including the reasons for their accumulation and the conditions under which these losses may be monetized. The analysis is illustrated with referrence to the Yugoslav and Hungarian cases. Third, we identify some important policy issues related to the problem of quasi-fiscal deficits. Finally, we discuss the practical problems that are encountered in the measurement of central bank and consolidated public sector deficits, and explore alternative methods to measure these deficits from below the line.

The outline of the paper is as follows. Section 2 reviews alternative definitions of the non-financial public sector deficit. Important points giving rise to different definitions are the problem of foreign exchange losses, the appropriateness of including net government lending to the private sector and distinction between nominal and real deficits. Section 3 introduces alternative definitions of quasi-fiscal deficit, and discusses the same issues in the context of these definitions. Foreign exchange losses and the distinction between nominal and real deficits are discussed in greater detail, since these issues are even more relevant in the case of the central bank. Section 4 merges the accounts of the non-financial and financial public sector into a consolidated budget constraint. Section 5 illustrates the analysis of fiscal and quasi-fiscal deficits through a detailed examination of some stylized cases. Finally, the last section summarizes the major issues and derives some conclusions. The Appendix provides an analysis of different methods to estimate the public sector deficit from below the line.

2. The deficit of the non-financial public sector

The non-financial public sector (government for short) is broadly defined so as to include the central and local governments, state enterprises and other non-financial public institutions. The central bank and state-owned commercial banks are excluded. In the discussion of alternative definitions of the government's deficit, it is useful to start with the following definition:

$$D_g = D + i_g C_g + i_b B + (i^* + \hat{E})\, B^* E - i_l L = \dot{C}_g + (\dot{B} - \dot{L}) + (\dot{B}^* E) \qquad (1)$$

Here D = nominal primary deficit (total non-interest expenditures minus revenues, including the dividends from the central bank and from the state-owned commercial banks), i_g = nominal interest rate on central bank's credits to the government, C_g = nominal stock of central bank's credits to the government (including direct credits to the government as well

as any form of government debt held by the central bank), i_b = nominal interest rate on the government's domestic debt held outside the central bank, B = nominal stock of the government's domestic debt held outside the central bank,[1] i^* = nominal foreign interest rate on the government's external debt, B^* = nominal foreign currency value of the stock of the government's external debt, E = nominal exchange rate, i_l = nominal interest rate on government loans to the private sector, and L = stock of government loans to the private sector. The dots over the variables indicate time derivatives, and the hats percentage changes.[2]

The government deficit, as defined in (1), consists of the primary deficit, the interest expenditures on domestic debt held inside and outside the central bank, the depreciation-adjusted interest expenditures on external debt, minus the interest revenues on credits to the private sector. The last term in (1) indicates the three possible sources of financing for the non-financial public sector deficit. These are the changes in central bank credits to the government, $\dot{C}_g$, the changes in net domestic debt outside the central bank, $(\dot{B} - \dot{L})$, and the changes in external debt, $(\dot{B}^* E)$. It should be noted that, although the central bank's credits to the government are included, the central bank's operations with the private domestic and foreign sectors are excluded from the definition.

There are three noteworthy points related to this definition of the non-financial public sector deficit. First, the nominal capital losses on the stock of external debt arising from nominal exchange rate devaluations are included. Second, the increases in government lending to the private sector are not included. Instead, government lending is treated as a financing item, and subtracted from the changes in gross domestic debt with the private sector. Third, investment expenditures are taken into account in the calculation of the primary deficit. We now discuss these two points in more detail.

As mentioned above, the nominal deficit as defined in (1) includes all capital losses on the stock of external debt due to exchange rate devaluations, $\hat{E}B^* E = \dot{E}B^*$, independently of whether these foreign exchange losses are all realized in the current period.[3] Foreign exchange losses are realized through interest payments and through net repayments of the stock. The amount immediately realized depends on the schedule of interest and net debt repayments, and is usually a small fraction of the total loss. From another point of view, one can see these losses as being automatically financed by foreign lenders, who "agree" to increase the domestic currency value of their loans (since they are primarily concerned with the foreign currency value of these loans). Therefore, the inclusion of foreign exchange losses provides important information about the potential burden imposed by exchange rate devaluations on the public finances. However, these losses do not generate any significant immediate pressure on the economy, and a deficit definition that excludes them is also useful:[4]

$$D'_g = D + i_g C_g + i_b B + i^* B^* E - i_l L = \dot{C}_g + (\dot{B} - \dot{L}) + \dot{B}^* E \qquad (2)$$

The second question is whether government lending to the private sector should be included in the deficit definition. The answer to this question involves conceptual and practical

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[1] Net of government deposits in the commercial banks. The interest rate i_b is defined accordingly.

[2] $\dot{X} = dX / dt$ and $\hat{X} = \dot{X} / X$.

[3] Note that $(i^* + \hat{E})B^* E = i^* B^* E + \dot{E}B^*$, where the first term is a cash cost and the second the capital loss.

[4] Any possible link between unrealized capital losses on public debt and private savings along Ricardian lines (see Barro, 1974 and 1979) is ruled out here.

considerations. There are, in fact, solid arguments for not including the net changes in government loans to the private sector in the non-financial public sector's deficit. By borrowing from the lending to the private sector, the government is simply acting as a financial intermediary, just like any other private financial institution. If these financial operations are carried out at market rates and the loans are effectively recovered, they may not produce any significant impact on aggregate demand, inflation or the current account, relative to a situation where the lending is conducted by the private sector.

The macroeconomic impact of the government's loans to the private sector is greater in the cases where these loans are not effectively recovered, or where they are made at rates below market rates. Government loans that are not repaid are actually grants or transfers, and should obviously be included in the definition of the deficit. The problem of adjusting the deficit definition for these factors is not so much conceptual as practical. It may be very difficult to screen the government's loan portfolio and separate loans from transfers. The concession of government loans at below market rates also involves a transfer of resources to the private sector that should arguably be included in the deficit. The problem here lies in the measurement of the implicit subsidy. The concept of real deficit to be introduced below provides one solution to this problem. The subsidy and deficit definitions could be extended to include all foregone revenues.[5]

The inclusion of changes in government loans to the private sector in the government's deficit is still frequently advocated, independently of the whether these loans are collected and correctly priced. The argument is based on the observation that these loans effectively increase the government's financing requirements. The argument is sometimes stretched by the observation that an increase in the government's financing requirements might ultimately result in an increase in government's borrowing from the central bank and in monetary expansion. Thus, according to this argument, the relevant deficit definition for fiscal policy evaluation is:

$$D_g'' = D + i_g C_g + i_b B + i^* B^* E + (\dot{L} - i_l L) = \dot{C}_g + \dot{B} + \dot{B}^* E \qquad (3)$$

Equation (3) defines the public sector borrowing requirements (PSBR), a measure of the government's deficit which is commonly employed for the assessment of the fiscal policy stance. The term $(\dot{L} - i_l L)$ indicates the financing requirements implied by the expansion of government loans to the private sector.

The argument that government lending to the private sector should be included in the deficit because it increases financing requirements is frequently presented without further elaboration. As mentioned above, although financing requirements are obviously increased, the macroeconomic impact of government borrowing and lending activities is by no means obvious. If the government finances its loans to the private sector through issues of market-priced securities, that does not result necessarily in a crowding out of private sector activities by the public sector. The immediate result is rather a reallocation of scarce financial resources among different private sector activities. The final result depends on whether the government's intervention involves a gain or a loss in efficiency.

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[5] The problem of measuring a credit subsidy lies in the choice of a benchmark interest rate. One option is to compare nominal rates with the rate of inflation (the criterion implicit in the concept of real deficit used in this paper). If the subsidy and deficit definitions are defined so as to include all foregone revenues, the benchmark is the market interest rate.

Although the record of the government's intervention in resource allocation has generally been poor, especially in the case of developing countries, such intervention may be in principle justified in cases of severe credit rationing due to imperfect information and perceptions of excessively high credit risk. For instance, government loans to education as well as loans to small enterprises may correct market inefficiencies and generate high returns to the economy as a whole. Government housing loans may also alleviate housing bottlenecks, although in the latter case resources may be indeed diverted from other sectors of industry, with adverse consequences for growth.

The conclusion seems to be that the inclusion of government loans to the private sector in the deficit definition without proper consideration of the nature and conditions of such lending activities may provide a misleading indication of the stance of fiscal policy. It is only after such examination is made that one can ascertain whether the fiscal deficit thus defined is crowding out productive investment, or pressing the current account, or still pressing domestic markets to the point where monetary accommodation becomes inevitable.

The government's real deficit can be analyzed along similar lines. Dividing all terms in equation (1) by the price level, P, multiplying and dividing the terms denominated in foreign currency by the foreign price level P^*, and splitting the nominal interest rates between the relevant real rate and the inflation premium, $i = r + \hat{P}$, one obtains:

$$d_g = d + r_g c_g + r_b b + (r^* + \hat{e}) b^* e - r_l l = \dot{c}_g + (\dot{b} - \dot{l}) + \dot{b}^* e \qquad (4)$$

Here r_g, r_b and r_l are, respectively, the real interest rates on government debt held inside and outside the central bank, and on government loans to the private sector, r^* is the real foreign interest rate, $e = EP^*/P$ is the real exchange rate. All other lower case variables are defined in real terms, $x = X / P$. Note also the extensive use of the identity $\dot{x} = \dot{X} / P - \hat{P}x$ in arriving at equation (4).

As equation (4) indicates, the real deficit is not just the nominal deficit divided by the price level. The critical difference is the exclusion of the inflation component from the government's interest payments and revenues. The argument for the utilization of a real definition of the deficit has been extensively discussed in the literature. When inflation increases and the real rate of interest remains constant, nominal interest payments also increase. However, these larger payments are just a compensation given to asset-holders for their nominal capital losses due to inflation, and therefore should not exert any additional pressure on aggregate demand. Instead, the asset-holders will be willing to invest these revenues in newly issued government securities, allowing the government to roll-over the existing stock of debt under the same conditions of price and maturity.[6]

Although the discussion in the literature has centered on the effect of inflation on interest payments on domestic debt, the same line of reasoning can be applied to the interest revenues on government loans to the private sector, and to the interest expenditures on the government's external debt. Finally, the same argument can also be applied to the problem of capital losses on the stock of external debt. While nominal exchange rate devaluations generate capital losses by increasing the domestic currency value of the stock of debt, domestic price inflation generates a capital gain on the stock. Thus, only real capital losses should be included in an inflation-adjusted definition of the deficit, as in equation (4).

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[6] See Eisner & Pieper (1984), and Eisner (1986, 1989a, and 1989b) for discussions of inflation-induced distortions in the measurement of the deficit and other macroeconomic variables, with focus in the US case. Cukierman & Mortensen (1983) provide a similar analysis for the OECD countries. Blejer, Tanzi & Teijeiro (1987) discuss the conditions under which the inflation-corrected deficit provides a better indication of the fiscal policy stance.

It should be noticed that the deficit, as given by (4), equals (minus) the variation in the government's real net worth. This means, of course, that all capital gains and losses due to inflation and exchange rate devaluations are properly taken into account. However, the question of whether unrealized real capital gains or losses on the stock of external debt should be included in the definition still remains. Likewise, the question of whether government lending to the private sector should be included in the definition also remains, whether the deficit is defined in nominal or real terms. Thus, defining equations (2) and (3) in real terms yields:

$$d'_g = d + r_g c_g + r_b b + r^* b^* e - \eta_l l = \dot{c}_g + (\dot{b} - \dot{l}) + \dot{b}^* e \tag{5}$$

$$d''_g = d + r_g c_g + r_b b + r^* b^* e + (\dot{l} - \eta_l l) = \dot{c}_g + \dot{b} + \dot{b}^* e \tag{6}$$

At this point one may become confused by the proliferation of alternative definitions of the deficit. However, a careful evaluation of fiscal policy may indeed require the use of more than one indicator. As mentioned above, measures of the deficit that include and exclude exchange rate-induced losses are both useful indicators of fiscal policy. In particular, the latter may provide very timely and useful information on future budgetary pressures arising from the realization of such losses and the need for fiscal adjustments.

Ultimately, none of the definitions presented above may fully capture the actual underlying behavior of economic agents. For instance, while it can be argued that the real deficit provides a more acurate indication of the fiscal policy stance than the nominal deficit under most possible circunstances, it is not itself free of problems. If there is some degree of money illusion, domestic asset-holders will tend to consume part of the inflation-related interest revenues and will be unwilling to refinance the roll over of the stock of government debt under the same conditions. In this case the real deficit will underestimate the extent of fiscal pressures on the economy.[7] Similarly, foreign holders of government debt may be unwilling to reinvest the foreign inflation component of interests. The reason for this may be money illusion or the availability of information that renders loans to the country an unattractive proposition.

Clearly, there is no easy solution to these and other problems. Instead, what the above discussion suggests is the need for good judgement in the selection and interpretation of the most relevant indicators. Knowledge about the specific country conditions and institutions must dictate the choice of the indicators and even the construction of taylor-made indicators.

3. The quasi-fiscal deficit of the central bank

Consider the simplified balance sheet of a representative central bank in figure 1.[8]

Figure 1
Balance sheet of the central bank

FA	$H = C_u + S$
C_g	FL
C_p	NW_{cb}

[7] See Blejer, Tanzi & Teijeiro (1987).

[8] The central bank's non-financial assets are ignored.

The central bank is assumed to hold foreign assets FA, and extend credits to the government C_g, and to the private sector C_p. Its liabilities are the money base H (equal to the sum of currency C_u, and bank reserves S), and foreign exchange liabilities, FL. Its net worth, NW, is the difference between its assets and liabilities, and its credits of the government are net from any government deposits. It is assumed that the private sector does not hold sight deposits at the central bank.[9]

The central bank's balance sheet may be written in terms of changes as:

$$\dot{NFA} + \dot{C}_g + \dot{C}_p = \dot{H} + \dot{NW}_{cb} \tag{7}$$

where $NFA = FA - FL$ denotes the net foreign assets of the central bank expressed in domestic currency. These may become large and negative if, for instance, the central bank borrows actively abroad, or manages schemes to attract the remittances of nationals working abroad, or still incurs in foreign exchange liabilities as a result of the implementation of foreign exchange insurance schemes. The reason net foreign assets turn out to be negative in these cases is that the central bank does not hold an equivalent stock of foreign assets. That, in turn, results from sales of previously acquired foreign exchange to the government or the private sector, and the channelling of the revenues from the sales towards credits denominated in domestic currency.

The variations in the central bank's net worth are identically equal to the difference between the central bank's revenues and expenditures, including the capital gains and losses due to exchange rate devaluations. Thus, these variations are obtained through a combination of its income statement and revaluation accounts:

$$\dot{NW}_{cb} = (i^* + \hat{E})NFA^*E + i_g C_g + i_p C_p - i_s S - \Delta \tag{8}$$

The first term on the right hand side of (8) is a depreciation-adjusted interest revenue or cost, depending on the sign of net foreign assets. Here we make the simplifying assumption that the central bank earns on its foreign assets the same international interest rate that it pays on its foreign liabilities. The second and third terms are the interest revenues collected on credits to the government and to the private sector, respectively. Next comes the interest expenses on bank reserves, and finally the dividend payments to the government, Δ.[10] Equation (8) does not cover all possible operations that might affect the central bank's profits. For instance, the central bank can run a profit (loss) if it sells foreign exchange at a rate higher (lower) than the buying rate. These and other operations are not included for the sake of simplicity.

Note that the central bank may experience a decline in its net worth while also generating cash profits. This situation may be caused by the provision of domestic credits at interest rates below the depreciation-adjusted cost of net foreign assets (assumed to be negative), or by the transfer of cash profits to the government. If the central bank transfers all its cash profits to the government, then it experiences a decline in its net worth equal to the foreign exchange losses in the period.

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[9] Otherwise, any such deposits should also be included in the definition of base money.

[10] It is assumed that the government it the central bank's sole shareholder.

If the central bank's nominal deficit is defined as the inverse of the variations in its net worth, then it is given by equation (8) (with a negative sign). In the special case where the government has no non-financial assets and does not invest, this definition is analogous to the definition of the government's deficit in (1):

$$D_{cb} = -\dot{NW}_{cb} = -[(i^* + \hat{E})NFA^*E + i_gC_g + i_pC_p - i_sS - D] = \dot{H} - N\dot{F}A - \dot{C}_g - \dot{C}_p \qquad (9)$$

As in the discussion of the government's deficit, it could be argued that the central bank's deficit should exclude the capital gains/losses on net foreign assets (as long as they remain unrealized) and, under certain conditions, include the changes in credits to the private sector. In these two cases, the nominal deficit of the central bank would be respectively defined by equations (10) and (11) — the counterpart of equations (2) and (3):

$$D'_{cb} = -[i^*NFA^*E + i_gC_g + i_pC_p - i_sS - D] = \dot{H} - N\dot{F}A^*E - \dot{C}_g - \dot{C}_p \qquad (10)$$

$$D''_{cb} = -[i^*NFA^*E + i_gC_g + i_pC_p - i_sS - D] + \dot{C}_p = \dot{H} - N\dot{F}A^*E - \dot{C}_g \qquad (11)$$

Of course, any of the equations (9)-(11) could be solved for the changes in base money, yielding the same result:

$$\dot{H} = -[i^*NFA^*E + i_gC_g + i_pC_p - i_sS - \Delta] + N\dot{F}A^*E + \dot{C}_g + \dot{C}_p \qquad (12)$$

Even though these are just manipulations of accounting identities, they do provide valuable insights on the sources of monetary expansion in any given country. Equation (12) states that base money expands whenever net foreign assets and domestic credits are expanded, or when the central bank runs an excess of interest costs plus dividends paid over interest revenues. This last term is usually disregarded in monetary policy analysis, but is at the core of the problem of monetary disequilibrium in some developing countries.

Notice that the inclusion of foreign exchange losses in the definition of the deficit does not make a difference, as far as the immediate impact on base money is concerned. This is actually the argument that is frequently used for its exclusion from the definition. An exchange rate depreciation generates an immediate increase in the domestic currency value of net foreign assets (assumed to be negative) and a matching increase in the stock of foreign exchange losses (a decrease in the central bank's net worth), with no immediate impact on base money creation.[11] However, the capital losses may be eventually monetized through two channels. First, through net repayments of foreign liabilities and, second, through larger interest payments on the stock of (negative) net foreign assets. These two effects are captured in equation (12).

It follows that the accumulation of foreign exchange losses does provide very important information for fiscal policy evaluation, even when these losses do not constitute a current source of monetary expansion. When foreign exchange losses are allowed to accumulate over time, the stock of central bank's earning assets shrinks relative to the stock of interest paying

[11] Notice that the capital loss terms, $\dot{E}NFA^*$, on both sides of equation (10) cancel out.

liabilities. This eventually generates cash losses that may have to be financed through monetary expansion, even if the principal of foreign liabilities is not being repaid. Of course, net repayments of foreign liabilities tend to worsen the financial condition of the central bank and may trigger further monetary expansion.

The existence of central bank credits to the private sector creates another difficult conceptual issue. As in the case of government loans to the private sector, a judgment must be made on whether the changes in these loans should also be included in the definition of the deficit. The answer to this question requires a very careful examination of the central bank's portfolio, since the quality of central bank loans may vary widely across countries and over time. For instance, if the central bank borrows abroad and extends domestic credits to unprofitable enterprises, there is clearly a deficit associated with the operation, since the central bank is acquiring liabilities, but no meaningful assets as counterparts. Over time a serious situation may develop, as the central bank starts servicing its liabilities, but is unable to collect the interest and principal from its assets. In more general terms, if the central bank extends credits that cannot be recovered, that creates a serious problem of monetary management that usually results in excessive monetary expansion. In these cases excessive monetary creation may happen by omission, i.e., the new credits extended by the central bank are not offset by the repayment of outstanding credits.

On the other extreme, one finds ordinary liquidity rediscounts, which are extended exclusively for the purposes of monetary management and that are fully repaid by the borrowing banks. These rediscounts may be extended under conditions of monetary equilibrium at low rates of inflation. In this kind of situation it is clear that there is not a deficit associated with central bank lending to the private sector.

In most developed countries the central bank's financial operations are not a source of macroeconomic imbalances. The central bank exercises its traditional functions and earns profits, which are partially or fully transferred to the government's budget as a non-tax revenue. In contrast, in many developing countries the central bank intervenes extensively in the process of financial intermediation. Such intervention usually involves the allocation of large amounts of credits to favored sectors at subsidized terms, and may result in significant imbalances between revenues and costs, especially when the credits are backed by foreign liabilities.

For instance, the central banks of several developing countries borrow directly abroad, or run schemes to attract the savings of nationals working abroad, while extending credits in domestic currency at an interest rate that does not reflect the cost of foreign liabilities.[12] The introduction of foreign exchange insurance schemes by central banks in developing countries may create a similar problem, since the insurance premium is usually severely underpriced in these countries. Another common source of problems in developing countries is the concession of credits to banks or enterprises in fragile financial conditions. These credits may be extended in situations of emergency — the central bank may be induced to increase its lending in order to avert a financial crisis — or simply result from ordinary institutional practices. For instance, in some Eastern European countries all foreign borrowings were conducted by the central bank, which also extended credits to enterprises that were clearly unable to repay them.

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[12] This is actually the final result of three different operations. The first operation involves an equivalent increase in the central banks' foreign assets and liabilities. The second operation involves sales of foreign exchange by the central bank to importers. Finally, domestic credits are granted with the proceeds from the sales.

The lack of transparency involving central bank accounts in several developing countries may allow a deteriorating financial situation to remain undetected for a long period of time. In some cases this is caused by the deliberate window-dressing of central bank income statements. For instance, a large share of interest revenues may be accrued but not effectively collected, while interest expenses are effectively paid. Another example of creative accounting involves the inclusion of foreign exchange gains on foreign assets in the flow of interest revenues, together with the exclusion of foreign exchange losses on the foreign liabilities altogether from the income statement.[13] Of course, it is impossible to exclude the accumulation of foreign exchange losses from the balance sheet. However, these can be disguised under one of the several entries that comprise the net worth of the central bank.

In other cases the central bank's accounts may be just poorly interpreted, even when there is no attempt to conceal losses of any kind. Such misinterpretation may simply result from the failure to distinguish nominal from real interest flows in the central bank operations. For instance, consider the situation of a central bank that borrows abroad and provides subsidized loans in domestic currency. Suppose that this situation has resulted in a large accumulation of foreign exchange losses and has driven the stock of interesting-earning credits well below the stock of interest-paying foreign liabilities. It is clear that a central bank in this situation would be unable to balance its accounts under low rates of inflation and nominal interest rates. However, if inflation is high, the domestic nominal interest rates will be much higher than the foreign interest rates. Such difference in nominal rates may partly or fully offset the difference between domestic credits and foreign liabilities, giving the impression of an equilibrium when the central bank is in fact running a real deficit. Moreover, such a deficit may be an important source of monetary expansion.

This example suggests that the quasi-fiscal character of some of the central bank operations becomes more transparent when inflation is properly taken into account. Indeed, the distinction between nominal and real deficits is particularly relevant in the case of the central bank, since central bank operations are exclusively financial. To start the analysis of real quasi-fiscal deficits, consider first a definition identical to the variation of the central bank's real net worth. Such definition may be obtained by dividing equation (9) by the price level, splitting the nominal interest rates between the real rate and inflation premium and considering the identities $\dot{NW} / P = n^{\cdot}w + \hat{P}nw$ and $\dot{H} / P = \dot{h} + \hat{P}h$, where $\dot{H} / P$ are the total seignorage revenues, equal to the variations in real base money, $\dot{h}$, plus the inflation tax, $\hat{P}h$:

$$-n^{\cdot}w_{cb} = -[(r^{*} + \hat{e})nfa + r_p c_p + r_g c_g - \delta - i_s \dot{s} + \hat{P}h] = \dot{h} - n\dot{fa} - \dot{c}_g - \dot{c}_p \qquad (13)$$

Here the small case variables are defined as the real variables, as before ($\delta = \Delta/P$), and the r's are the relevant real interest rates. Note that such a definition of the real quasi-fiscal deficit includes the inflation tax, net of the interest paid on reserves, $\hat{P}h - i_s s$. That is, the real net worth of the central bank is increased by the inflation tax and decreased by the payment of interests on reserves. Note also that if no interest is paid on reserves ($i = 0$) the central bank's real net worth is increased by the full value of the inflation tax. If some interest is paid on reserves, the inflation tax is only fully collected on currency, while the amount which is effectively collected on reserves depends on the real interest rate. The net inflation tax revenues are then $\hat{P}c_u - r_s s = \hat{P}h - i_s s$.

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[13] Capital gains and losses should either be explicity shown in the income statement or put in a separate revaluation account.

Why would it make sense to define the central bank's deficit as (the negative of) its net worth variation, when (in the general case) the analogous definition for the government deficit was inappropriate? The central bank's non-financial assets are usually insignificant, when compared to its financial assets. They can therefore be valued with a reasonable degree of accuracy. It follows that the principal objection against using a net worth-based deficit definition does not apply in this case. However, the definition (13) of real central bank deficit, which equates the deficit to the (negative of) central bank's real net worth variation, leads to some counterintuitive results. One can envisage a steady state with a high rate of inflation where all variables in the central bank's balance sheet (including net worth) are constant in real terms. For instance, assume a central bank that has a large stock of foreign liabilities and that provides subsidized financing to the public and private sectors. In this case $r^* > 0$ and all r's < 0, i.e., the central bank runs a deficit in its real interest flows. However, the inclusion of the inflation tax as an ordinary source of revenue makes the real deficit, as defined in (13), equal to zero.

It follows that a definition that treats the inflation tax (net of interest on reserves) as a financing item, instead of an ordinary source of revenue, may convey more information about the effective financial situation of the central bank. Equation (14) defines the central bank's real deficit as (minus) its net worth variation net of any inflation tax revenues:

$$d_{cb} = -[\dot{n}w_{cb} - (\hat{P}h - i_s s)] = -[(r^* + \hat{e})nfa + r_p c_p + r_g c_g - \delta] = \frac{\dot{H}}{P} - i_s s - \dot{nfa} - \dot{c}_g - \dot{c}_p \tag{14}$$

Here $\dot{H}/P - i_s s$ is the net seignorage collected by the central bank, equal to gross seignorage less the intests paid on commercial bank reserves. As above, alternative definitions may exclude the capital losses/gains on net foreign assets and include the changes in credits to the private sector:

$$d'_{cb} == -(r^* nfa^* e + r_p c_p + r_g c_g - \delta) = \frac{\dot{H}}{P} - i_s s - \dot{nfa}^* e - \dot{c}_g - \dot{c}_p \tag{15}$$

$$d''_{cb} == -(r^* nfa^* e + r_p c_p + r_g c_g - \delta) + \dot{c}_p = \frac{\dot{H}}{P} - i_s s - \dot{nfa}^* e - \dot{c}_g \tag{16}$$

These definitions of the central bank's real deficit reveal clearly the transfers of real resources that may take place in an inflationary economy. They also indicate what type of corrections may be needed in order to stabilize an economy where the monetary imbalances are rooted in the central bank. If the seignorage collected by the central bank is being channelled towards the servicing of foreign liabilities and the concession of credit subsidies to the public and private sectors, monetary control may require a variety of measures, such as the increase in interest rates on central bank credits, the interruption of central bank's explicit transfers to the government's budget, and possibly the absorption of central bank foreign liabilities by the government. This last measure is required if the real stock of central bank credits has fallen to such low levels that an increase in real interest rates to reasonable levels does not generate enough real resources to cover the servicing of foreign liabilities.

For instance, the Yugoslav stabilization program of 1990 included the transfer to the federal budget of central bank credit subsidies to agriculture and exports, as well as the absorption of most of the central bank's foreign liabilities by the federation. Such absorption took place through the replacement of the large stock of foreign exchange losses in the central bank by an equivalent stock of federal government bonds indexed to the exchange rate and yielding an interest rate equal to Libor. Of course, all these measures required ultimately a fiscal adjustment by the federal government.[14]

The Yugoslav case suggests that proper examination of the central bank's foreign exchange losses may prevent the emergence of serious macroeconomic imbalances. If the inclusion of foreign exchange losses results in consistently large real deficits, that indicates that domestic credits are not correctly priced, and/or that the transfer of the central bank's cash profits to the government is excessive. The prevention of a serious financial situation requires not only a correction of interest rates but also the interruption of the transfer of central bank cash profits to the government. Failure to implement these corrective measures in time inevitably generates the need for an even larger fiscal adjustment in the future.

These corrective measures may not be easily implemented in many countries, since policy makers may interpret the existence of cash profits as an indication that the central bank enjoys a confortable financial situation, and that it would be a "waste" for the institution to retain these profits, as opposed to transferring them to the government. However, it is precisely the retention of cash profits that may prevent a drastic decline in the central bank's net worth and the emergence of a situation where the central bank starts running cash losses. Moreover, when inflation increases, the assessment of the situation may become even more complicated, because domestic nominal interest rates are likely to increase along with the rate of inflation, even when real interest rates remain negative.

Hungary and Turkey provide additional examples of real quasi-fiscal deficits caused by the combination of massive borrowings abroad by the central bank and the concession of low interest rate credits to the public and private sectors. In the late 80s the governments of both countries acknowledged the difficult financial situations faced by their respective central banks, and absorbed the stocks of foreign exchange losses. In the case of Hungary, the foreign exchange losses were replaced by a stock of credits to the government, while in Turkey the government replaced the central bank losses by a stock of long-term securities. However, in neither country the real quasi-fiscal deficit has been effectively eliminated. This is because the correction in the stocks has not been accompanied by a correction in the flows. In Hungary, the amount of interests effectively paid by the government on the converted stock seems to be negligible, while in Turkey the amount of interests paid is not only small, but is also financed by new central bank credits to the government.

The final outcome has been at best mixed in the Hungarian and Turkish cases. Although some of the institutional measures required to eliminate the real quasi-fiscal deficit were indeed implemented, the essential ingredient of the solution — a fiscal transfer from the government of the central bank — was not implemented. The high rates of inflation of both countries (30 and 70 percent in Hungary and Turkey, respectively) may be still masking the financial problems of their central banks. Another explanation for the lack of an effective adjustment lies simply in the difficulty to generate a sufficient fiscal adjustment at the govern-

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[14] See Coricelli & Rocha (1991) for an analysis of the Yugoslav and Polish stabilization programs of 1990 and Bole & Gaspari (1990), Mates (1991), Rocha (1991), and The World Bank (1989) for detailed studies of Yugoslavia's quasi-fiscal deficits and inflation.

ment level. In any case, the real quasi-fiscal deficit remains and seems to be affecting adversely the conduct of monetary policy in both countries.

4. The consolidated public sector deficit

The deficits of the non-financial public sector and of the central bank can be easily consolidated, whether they are defined in nominal or real terms. Consider, for instance the nominal deficits in (1) and (9). Adding these two equations, one obtains

$$\mathcal{B} = D_a + i_b B + (i^* + \hat{E})(B^* E - NFA^* E) - i_l L - i_p C_p - i_s S = \tag{17}$$

$$\dot{H} - \dot{C}_p + (\dot{B} - \dot{L}) + (B^* E - \dot{NFA}^* E)$$

Here D_a is the government's primary deficit, adjusted so as to exclude all dividends received from the central bank. This definition of the consolidated nominal deficit, $\mathcal{B}$, excludes government and central bank lending to the private sector, and includes the capital losses on the net external debt of the consolidated public sector — the government's external debt minus the net foreign assets of the central bank. The last term in (17) indicates the three possible sources of public sector financing. These are: (a) the changes in base money, (b) the changes in net domestic debt, and (c) the changes in net external debt. It is tedious to write other definitions, where the capital losses are excluded, and public sector lending to the private sector is included.

The real consolidated public sector deficit d can also be easily obtained by addition of (4) and (14):

$$d = d_a + r_b b + (r^* + \hat{e})(b^* e - nfa^* e) - \eta l - r_p c_p = \tag{18}$$

$$\frac{\dot{H}}{P} - i_s s - \dot{c}_p + (\dot{b} - \dot{l}) + (b^* e - \dot{nfa}^* e)$$

This deficit may be estimated either by excluding the inflation component from the nominal interest revenues and expenditures of the public sector, or by calculating the real changes in the net domestic and external debts and the net seignorage revenues on base money. Once again, the equations may be defined so as to exclude the real capital losses on the net external debt, or to include public sector lending to the private sector.

The consolidation of the government and the central bank cancels out all explicit and implicit transfers between the two entities. Of course, the consolidated deficit arises as a result of the government and central bank operations with the private domestic and foreign sectors. Thus, equations (17) and (18) are unaffected by the transfer or fiscal functions to the central bank and also indicate clearly the sources of public sector financing, irrespective of where the financing needs are located.

5. Some stylized examples

In order to clarify the analysis of real fiscal and quasi-fiscal deficits we provide three stylized examples in this section. The first of these is the case of government deficits financed exclusively from zero-interest central bank credits. In the other two examples we assume that the government's accounts are fully balanced and the pressures towards monetization are coming from different sources. Any one of these latter two cases would thus present a puzzle to the less informed observer. The imbalances are either in the private sector (enterprise or bank losses, subsidization of selected consumer goods) or in the central bank, which incurs valuation losses.

Case 1: primary deficits financed by interest rate subsidies

Consider the case of a central bank that does not have foreign assets or liabilities, does not extend credits to the private sector, does not pay interest on reserves, does not charge interest on its credits to the government, and does not pay dividends. To simplify, assume also that the government's net foreign and private domestic debts are both zero. In this case the real revenues collected from the holders of base money are fully transferred to the government. Part of this transfer is effected in the form of an interest rate subsidy, while the remainder, which equals the real fiscal deficit, takes the form of real credit expansion. Under these conditions, the nominal and real financing flows are as depicted in equations (19)-(23):

$$D_g = D = \dot{C}_g \tag{19}$$

$$d_g = d + r_g c_g = \dot{c}_g \tag{20}$$

$$\frac{\dot{C}_g}{P} = \dot{c}_g + \hat{P} c_g = \frac{\dot{H}}{P} = \dot{h} + \hat{P} h \tag{21}$$

$$d_{cb} = -r_g c_g = \frac{\dot{H}}{P} - \dot{c}_g \tag{22}$$

$$d = d_g + d_{cb} = \frac{\dot{H}}{P} \tag{23}$$

Equation (19) gives the government's nominal deficit, which is equal to its primary deficit (since $i_g = 0$). The government's real deficit is given by (20). Since $i_g = 0$, $r_g c_g = -\hat{P} c_g$. Equation (21) is a balance sheet identify for the central bank. Equation (22) gives the central bank's real deficit, which is equal to the subsidy given to the government. Finally, equation (23), which is the sum of (20) and (22), shows that the consolidated public sector deficit is the sum of the government and central bank deficits, and is entirely financed from seignorage revenues.

One can easily envisage a steady state with a constant rate of inflation, where the real stocks of base money and central bank credits are equal and constant, and the real net worth of the central bank is zero.[15] In this special case, the seignorage equals the inflation tax, and is fully used to extend an interest rate subsidy that matches the government's real primary deficit. Thus, the government's real deficit is zero. The central bank's real deficit, the consolidated public sector deficit, the government's real primary deficit, the seignorage, the inflation tax, and the interest rate subsidy from the central bank to the government are all equal.[16]

$$nw \equiv c_p \equiv 0$$

$$c_g \equiv h \equiv \text{constant}$$

$$d = \text{d} = d_{cb} = -r_g c_g = \hat{P} c_g = \frac{\dot{C}_g}{P} = \frac{\dot{H}}{P} = \hat{P} h$$

This example may give the impression that the methodology is just a convoluted way to arrive at obvious conclusions. Ultimately, the government's real primary deficit was financed by seignorage revenues, as it should be in a model where neither the government nor the central bank borrow from or lend to the foreign or domestic private sectors. However, the methodology does introduce a discipline in the assessment of central bank's accounts that proves very fruitful when the central bank operates extensively with the private and foreign sectors.

Case 2: private sector activities financed by interest rate subsidies

Suppose that the government's real deficit is zero, and the only activity of the central bank is the provision of zero interest rate credits to favored private sector activities. Assuming a steady state with constant real stocks for simplicity, the situation would be essentially as follows:

$$c_g \equiv nw_{cb} \equiv 0$$

$$c_p \equiv h \equiv \text{constant}$$

$$\text{d} = d_{cb} = -r_p c_p = \hat{P} h = \frac{\dot{H}}{P}$$

In this case, the real quasi-fiscal deficit is again equal to the consolidated public sector deficit. But now the central bank is transferring resources from the holders of base money to the private sector (to the recipients of credits at zero nominal interest rates). This situation is similar to the one where the government's deficit is due to the provision of ordinary subsidies

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[15] This steady state is consistent with a zero output growth rate.

[16] Notice that this is a case in which counting the inflation tax as a revenue of the central bank would yield zero deficits (both for the central bank and for the consolidated public sector). As noted above, this would be awkward, given that the inflation rate is positive and constant.

and transfers to segments of the private sector (e.g., food subsidies and payments to the elderly), and the deficit is financed from central bank credits. In both cases there is a deficit in the public sector, which originates from redistributional objectives, and which may result in excessive monetary creation and inflation.

In this example the rate of inflation is constant and positive, and the government does not borrow from the central bank or even from the private sector. Although this may appear as a puzzle, the solution consists in accounting for the real quasi-fiscal deficit, $-r_p c_p = \hat{P}h$. Indeed, the transfer of real resources associated to a credit subsidy is entirely equivalent to that produced by any other type of subsidy, ordinarily included in fiscal budgets.

Now consider a steady state where the nominal interest rate charged by the central bank on its loans to the private sector is positive, but still below the rate of inflation. In this case, the central bank has a positive nominal surplus equal to $i_p C_p$, but a real deficit equal to $-r_p C_p$. Note also that in this case the real net worth of the central bank is positive:[17]

$$c_p \equiv \text{constant} \qquad h \equiv \text{constant} \qquad nw = c_p - h \equiv \text{constant}$$

$$d = d_{cb} = -r_p c_p = \hat{P}h = \frac{\dot{H}}{P}$$

This example shows clearly that nominal surpluses are no guarantee of real surpluses, and that real, and not nominal deficits provide the correct measure of the pressures towards monetization. In this example the real deficit is associated with a credit subsidy. This type of subsidy is common in developing countries and difficult to eliminate in many cases.

Credit subsidies may be the instrument utilized by policy-markers to keep afloat loss-making enterprises and banks. Large enterprise losses can arise as a result of the removal of protection, large exchange rate devaluations, price controls, excessive personnel, or sheer inefficiency. Fear of the social costs of adjustment may induce policy-makers to delay adjustment, and keep afloat a large number of enterprises through credit subsidies from the central bank. In many cases enterprise losses spill over into commercial banks, through massive defaults on bank loans. In these cases, central bank credits may be directed towards the commercial banks.

When the central bank's deficit is directly or indirectly associated to the subsidization of loss-making enterprises, the ultimate source of inflation does not lie within the bounds of the public sector. In these cases it is unlikely that inflation will be stopped by measures that, dealing exclusively with the financial system, aim at eliminating the central bank's deficit. An increase in central bank real interest rates to positive levels does not necessarily correct the fundamental problems that require subsidization.[18] If the enterprises fail to adjust by cutting wages or investment, or by increasing efficiency, they will have to continue receiving resource flows. These flows usually take the form of distress borrowing, such as massive de-

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[17] It is easy to show that

$$nw = \frac{i_p c_p}{\hat{P}} > 0.$$

[18] If these higher rates are reflected in higher commercial bank lending rates, the financial problems faced by enterprises may actually be aggravated.

faults on bank loans or greater recourse to inter-enterprise credits. Of course, in the medium or long run a wave of bankruptcies or subsidization through widespread debt forgiveness are the only possible outcomes.

Under these circumstances, a successful stabilization may require closing down some inefficient enterprises. Measures such as corrections of enterprise prices and the financial restructuring of potentially viable enterprises and banks may also be needed. Financial restructuring invariably requires some fiscal support. Therefore, the stabilization of inflation and the elimination of credit subsidies may ultimately require a fiscal adjustment, even when the need for such fiscal adjustment is not obvious.

Case 3: foreign exchange losses at the central bank

Suppose now that the central bank borrows abroad and provides credits in domestic currency. Suppose further that the foreign exchange losses incurred by the central bank are not properly taken into account. Instead, the central bank's cash profits are integrally distributed to the government. To focus on the consequences of excessive dividend payments, it is assumed that the domestic and foreign real interest rates are zero, the nominal interest rate on reserves is zero, and the real exchange rate remains constant:

$$i_g = \hat{P} \qquad i^* = \hat{P}^* \qquad i_s = 0 \qquad \hat{E} = \hat{P} - \hat{P}^* \qquad \hat{e} = 0$$

Note that in this case $i_g = i^* + \hat{E}$, that is, there is no subsidy implicit in central bank credits. Thus, if the central bank interest revenues were internalized and reinvested, there would be no problem, since the foreign exchange losses would tend to be offset by the cash profits, resulting in a constant net worth. However, if the cash profits are transferred to the government, a serious situation may develop over time. To see this point, note that the changes in the central bank's net worth and in the base money are equal to:

$$\dot{NW}_{cb} = i_g c_g + (i^* + \hat{E})NFA - \Delta$$

$$\dot{H} = \dot{C}_g + E\dot{NFA}^* - (i_g c_g + i^* NFA) + \Delta$$

Here credits to the private sector are assumed to be zero for simplicity, and $\Delta = \max\{\alpha(i_g c_g + i^* NFA), 0\}$ $(\alpha > 0)$, where α indicates the proportion of cash profits that is distributed as dividends to the government. Notice the asymmetry in the determination of Δ: if cash profits are negative, there are no payments from the government to the central bank; instead, the central bank dividend is set at zero. Thus, there are two possible cases. First, if cash profits are nonnegative $(i_g C_g + i^* NFA \geq 0)$, the changes in base money are given by:

$$\dot{H} = (\alpha - 1)(i_g C_g + i^* NFA^* E) + \dot{C}_g + E\dot{NFA}^*$$

On the other hand, if there are cash losses $(i_g C_g + i^* NFA < 0)$, the expression giving changes in base money is:

$$\dot{H} = \dot{C}_g - i_g C_g + E\dot{NFA}^* - i^* NFA^* E$$

Suppose that (a) cash profits are initially positive ($i_g C_g(0) + i^* NFA(0) > 0$), (b) they are fully distributed ($\alpha = 1$), and (c) the country is unable to obtain further real foreign finance ($\dot{NFA}^* = 0$). Under these conditions,

$$\dot{H} = \dot{C}_g$$

and

$$\dot{h} + \hat{P}h = \dot{c}_g + \hat{P}c_g$$

If also h is assumed constant,

$$\dot{c}_g = \hat{P}(h - c_g)$$

and hence

$$c_g(t) = h + [c_g(0) - h]e^{-\hat{P}t}$$

So c_g converges asymptotically to h, and $nw - nfa$ converges asymptotically to zero. Assuming that initially $nw > 0 > nfa$, c_g exceedes h at time zero. This means nw and c_g are decreasing in time, and nw may become negative. In the long-run cash profits $i_g C_g + i^* NFA^* E$ will be positive, since nfa converges to zero. However, they can become temporarily negative.[19] As long as foreign liabilities are large, so that

$$\left|\frac{h}{nfa}\right| < \lambda \equiv \frac{\hat{P}^*}{\hat{P}}\hat{E} = \frac{i^*}{i_g}\hat{E}$$

cash profits will be decreasing.[20]

Of course, it is doubtful that such developments could take place. The simultaneous decrease of the government dividend revenues and of the real value of the government's liabilities towards the central bank can only be accomplished either in the presence of a primary

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[19] Notice that when this happens the regime of the system of differential equations changes. This is because the government does not compensate the central bank for negative cash profits. Assuming that the domestic inflation rate exceeds the foreign inflation rate, a necessary condition for the negativity of cash profits is that the assets $c_g + nfa$ of the central bank become negative. Indeed, it is easy to show that

$$i_g c_g + i^* nfa = -\hat{E}nfa + \hat{P}(c_g + nfa)$$

If $\hat{P}^* < \hat{P}$, then $\hat{E} > 0$, and the negativity of cash profits requires $c_g + nfa < 0$.

[20] To see this, notice that

$$i_g \dot{C}_g + i^* E\dot{NFA} = P\left[i_g(\dot{c}_g + \hat{P}c_g) + i^*\hat{E}nfa\right] = P\left[i_g h + i^*\hat{E}nfa\right].$$

Also, when the domestic inflaton rate is very high $\left(\hat{P}^* / \hat{P}\right)\hat{E} \approx \hat{P}^*$.

surplus, or through increasing borrowing from the private sector.[21] Thus the failure to take into account the valuation losses leads to a situation that is unsustainable, and creates a need for larger fiscal adjustments in the future.

Figures 2 and 3 illustrate the problems that may result from the failure to cover foreign exchange losses. Figure 2 illustrates the case A where the interest rate charged on domestic credits covers the depreciation-adjusted cost of foreign liabilities, but the central bank distributes all its cash profits to the government. Figure 3 shows the case B where no profits are transferred to the government, but the interest rate is significantly below the cost of foreign liabilities. In both cases the exchange rate is assumed to follow PPP. The initial conditions and all other parameters are assumed to be the same in the two cases, as shown in table 1. The exercise also assumes that base money is constant in real terms, and that the nominal stock of foreign liabilities is constant in dollars. It follows that the real stock declines at the rate of foreign inflation.

In both cases there is a strong initial decline in the real stocks of domestic credits and net worth. The decline in domestic credits relative to the stock of foreign liabilities leads to a rapid decline in the real cash profits of the central bank (the nominal cash profits divided by the price level), which become negative. Note that in both cases the stock of credits also becomes negative during several periods. These trends are eventually reversed due to the continuous decline in the real stock of foreign liabilities. Note also that the parameters values imply that real stocks ultimately converge.

Table 1
Simulation parameters

	$C(0)$	$H(0)$	$FL(0)$	$NW(0)$	$\hat{P}$	$\hat{P}^*$	$\hat{E}$	i_g	i^*	D
Case A	100	10	90	0	30%	3%	26%	35%	7%	100%
Case B	100	10	90	0	30%	3%	26%	20%	7%	0%

The exercise is admitedly mechanistic, and generates some implausible results, such as a decline of real credits to negative levels. However, it does illustrate the continous realization of foreign exchange losses through interest flows, and the resulting decline in central bank profits. The problems that may arise by overlooking the dynamic implications of foreign exchange losses are made evident. The decline in the stock of credits and cash profits usually leads the central bank to adopt expansionary monetary policies. This is inevitable if cash profits become negative and the central bank does not receive a fiscal support from the government. The monetary expansion may lead to an endogenous increase in inflation and domestic nominal interest rates, and a resulting increase in nominal interest revenues, masking a deteriorating financial situation within the central bank.

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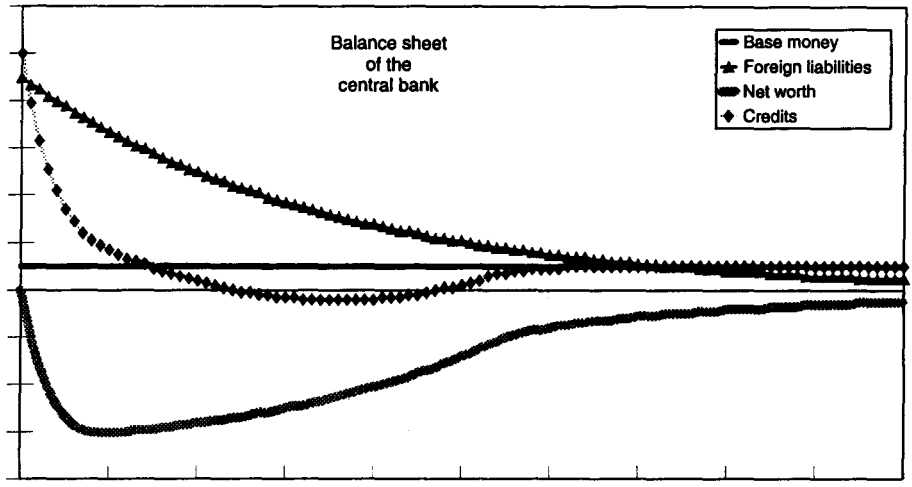
[21] Assuming that the government does not borrow abroad or lend to the private sector, $\dot{c}_g = d_g + \dot{b}$ and $d_g = d - \delta$. Since $\dot{c}_g < 0$, either there is a fiscal surplus ($d_g < 0$), or the government is increasing its debt towards the private sector ($\dot{b} > 0$). As the dividends paid by the central bank to the government dwindle, a fiscal surplus requires a primary surplus ($d > 0$).

Figure 2
Simulation: case A



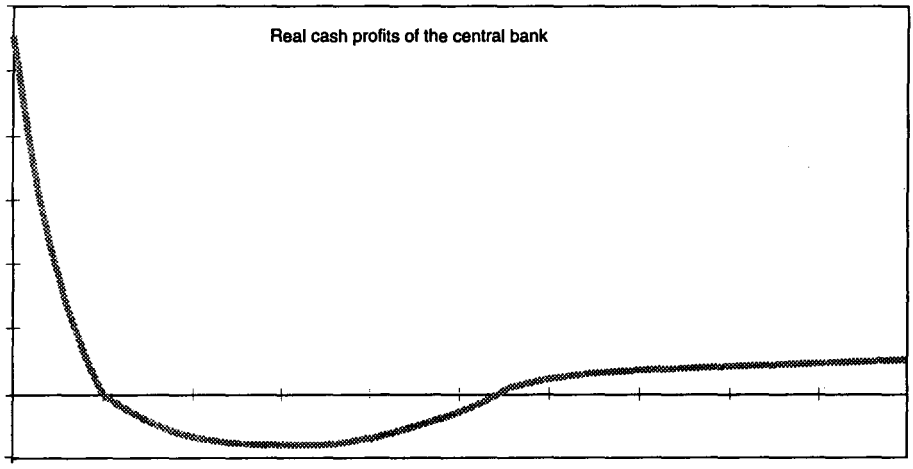
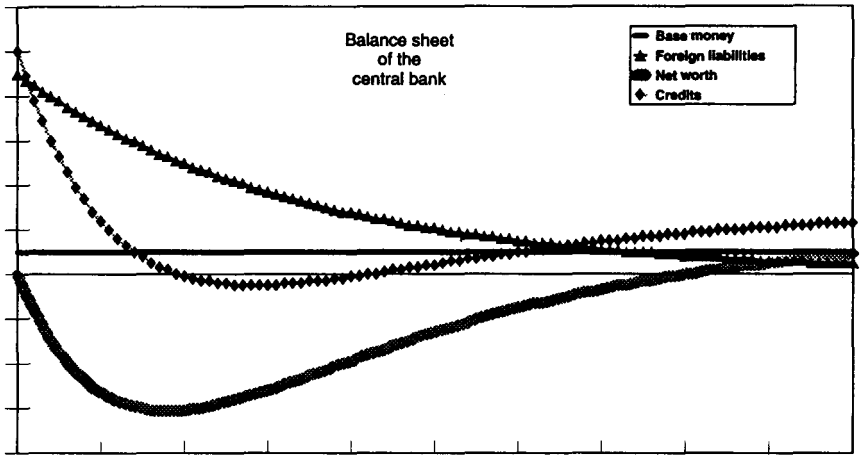
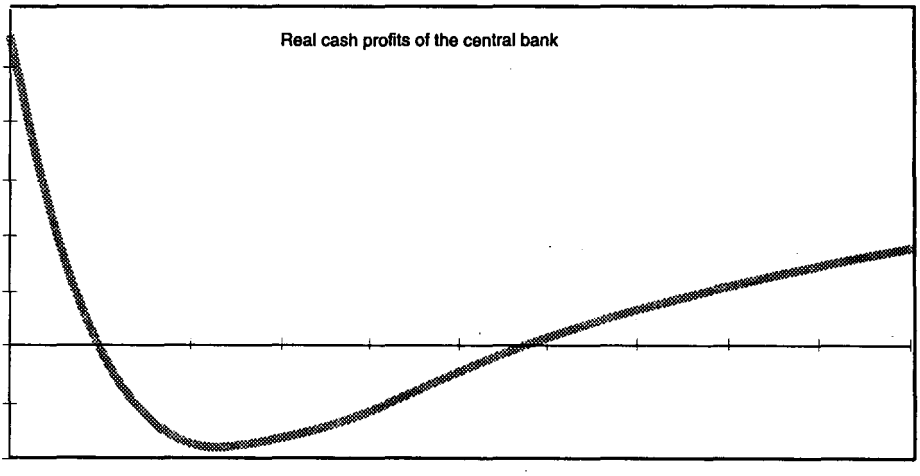


Figure 3
Simulation: case B





The situation of the Central Bank of Yugoslavia during the 80s illustrates very well this point. As shown in figure 4 and table 2, the policy of negative real interest rates on credits and the distribution of a share of the revenues to the Republican governments led to a sharp and continuous decline in the real stocks of credits and net worth. During the 80s, the ratio of credits to net foreign liabilities declined from 174 to only 12 percent. During the same period inflation increased continuously, leading to an increase in nominal interest rates and the Central Bank's nominal interest revenues. Since the nominal income statements did not show deficits, various governments were misled into overlooking the quasi-fiscal problem in successive attempts to stabilize the economy during the 80s. As mentioned before, the quasi-fiscal problem was only tackled in the stabilization program of 1990, when the stock of foreign exchange losses was replaced by a stock of interest-yielding bonds serviced by the government.

Figure 4
Central Bank of Yugoslavia
Real balance sheet (base = 1980)

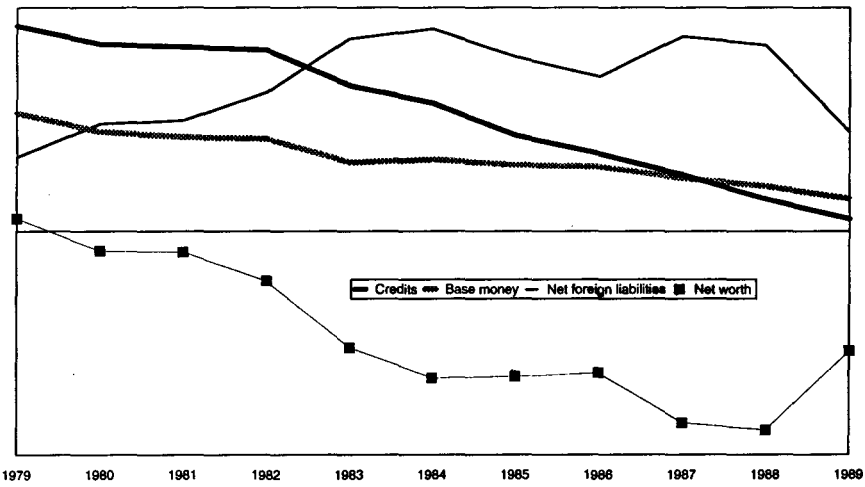


Table 2
Central Bank of Yugoslavia
Ratio of domestic assets to foreign liabilities and selected interest rates, 1980-89

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Net dom. assets/net for. liabilities	1.74	1.67	1.31	0.76	0.63	0.55	0.50	0.29	0.17	0.12
Average interest rate on domestic assets	3.5	3.5	3.5	6.5	20	38	48	61	100	1,000
Interest on foreign liabilities (DM)	8.1	8.9	10.3	6	6.4	5.6	4.8	4.8	3.8	7
Domestic inflation (Dec.)	37.5	35.7	32.7	60.0	53.0	75.0	92.0	169.0	240.0	2,685.0
Domestic inflation (Avg.)	30.9	40.0	31.5	40.2	54.7	72.3	89.8	120.8	194.0	1,240.0

The real balance sheet of the Central Bank of Hungary shows a similar pattern. As indicated in figure 5, there was also a decline in the real stocks of credits and net worth during the second half of the 80s, leading to a decline in central bank profits. The situation is less dramatic than in Yugoslavia, as indicated by a less pronounced decline in the stock of credits and net worth, and the fact that the stock of credits remained above the stock of foreign liabilities. However, the trends indicate a deteriorating situation due to the same causes, namely, underpriced credits and an unwarranted distribution of cash profits.

6. Conclusions and further thoughts

In this paper we review and discuss a number of issues related to the relevant definitions of fiscal and quasi-fiscal deficits, as well as the consolidation of the two. The major issues discussed are the distinction between nominal and real deficits, the treatment of foreign exchange losses, and the treatment of public sector loans to the private sector.

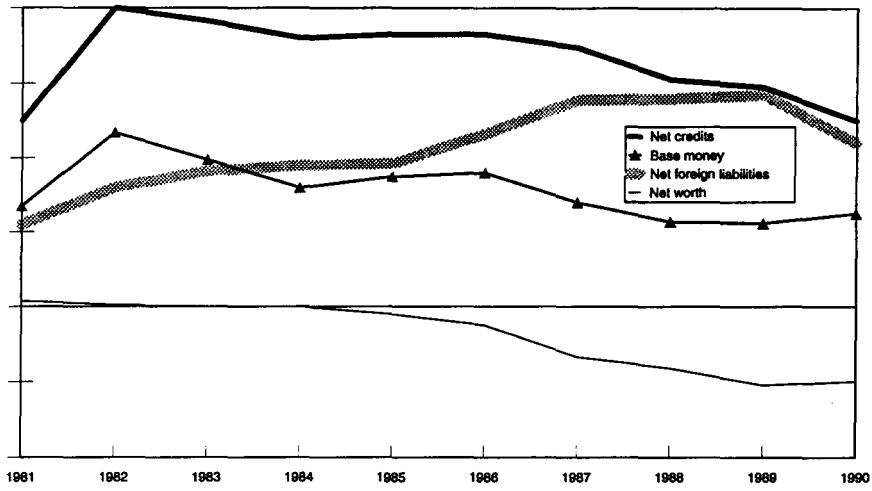
Despite their limitations, real measures of the deficit provide a less distorted indication of the actual fiscal policy stance than nominal measures. The distinction between nominal and real definitions is even more relevant in the case of the central bank's quasi-fiscal deficit, given the financial nature of the central bank's operations. Central banks rarely recognize explicitly their losses in their income statements. More often, these income statements show sizable nominal surpluses, even in the cases where there is actually a real deficit in the operations with the private domestic and foreign sectors.

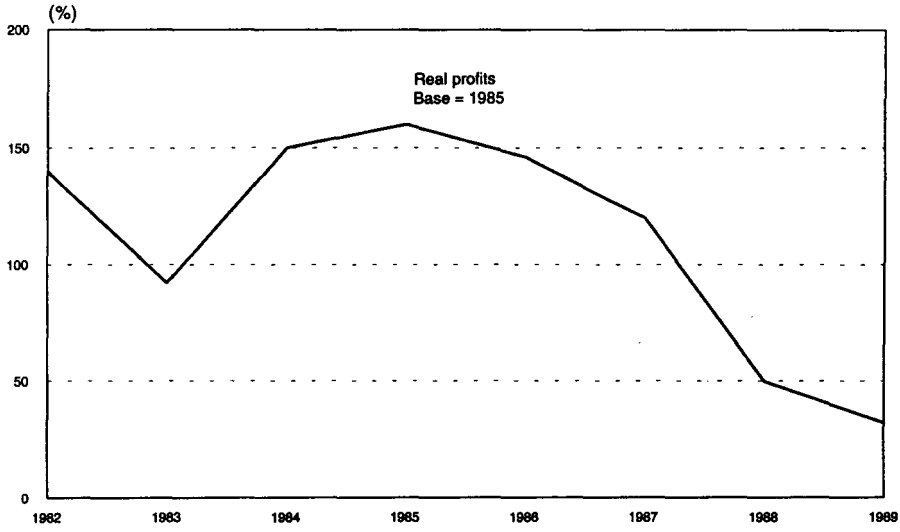
Real quasi-fiscal deficits usually reflect losses in other sectors of the economy and the need for a resource transfer. Therefore, a correction of these deficits may require more than a simple elimination of credit subsidies through an increase in real interest rates to positive levels. In fact, the elimination of losses at their source requires in many cases a fiscal adjustment, even when the need for such adjustment is not obvious.

The accumulation of foreign exchange losses imposes a burden on the consolidated public sector's finances. The fact that this burden is partly transferred to future periods frequently leads policy-makers to overlook its consequences. This is particularly true in the case of the central bank's own foreign exchange losses. Scenarios where these losses are allowed to accumulate to very large amounts, while interest rates on domestic credits are kept low and the central bank keeps transferring its declining cash profits to the non-financial public sector, are not uncommon. The mounting burden of net interest expenditures may then constitute a significant source of monetary expansion. This situation worsens considerably when the central bank is faced with a net repayment of its foreign liabilities.

We argue that proper consideration of foreign exchange losses provides very important information for the evaluation of the fiscal policy stance, even in situations where these losses seem to be largely unrealized. The distinction between realized and unrealized losses becomes meaningless over time, as losses are continuously realized through interest flows. A systematic accumulation of foreign exchange losses indicates the need to adjust the interest rates on domestic credits and/or to stop transferring central bank cash profits to the government. Failure to implement these measures in time can lead to inflationary episodes like the one observed in Yugoslavia during the 80s, where a large real quasi-fiscal deficit was the main factor leading to monetary expansion and accelerating inflation.

Figure 5
Central Bank of Hungary
Real balance sheet and profits





There are conceptual problems related to the treatment of central bank lending to the private sector that are similar to those related to government lending to the private sector. The existence of an element of subsidy in public sector credits does imply a transfer of resources to the recipients of those credits. This subsidy component should be included in the deficit definition. Uncollected or uncollectable credits should also be included, since they also imply a transfer of real resources to segments of the private sector. However, if the extension of loans by the central bank or the government to the private sector does not imply a negative variation in the public sector's net worth, the justification for the inclusion of these loans in the deficit is less clear.

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Appendix

1. Above and below the line deficit estimates

The consolidated public sector comprises non-financial and financial entities. The non-financial public sector includes the central and local governments, extra-budgetary entities (EBEs) and state enterprises (SEs). The financial public sector includes the central bank and other public sector banks and financial entities. Although it is obviously of interest to obtain measures of the deficit of the consolidated public sector, these measures are rarely computed. The deficit figures which are most commonly available and publicized are the deficits of central and local governments. In some cases, the figures include the deficits of EBEs and SEs. The operations of the financial public sector are rarely properly accounted for.

This lack of comprehensiveness of deficit figures is not surprising. The usual motivation to transfer fiscal or quasi-fiscal operations outside ordinary budgets is precisely to avoid the close monitoring of these operations by legislative bodies, domestic groups of interests, foreign creditors, and international organizations. In this way policy-makers hope to be less constrained in the implementation of fiscal policy. Therefore, in many cases the publicized figures turn out to be very poor indicators of the actual fiscal policy stance.

In principle, measures of the consolidated public sector deficit can be obtained by properly consolidating above the line calculations of the deficit of each segment of the public sector. That would amount to adding up the budgetary revenues and expenditures of central and local governments and the revenues and expenditures of other segments of the public sector, as recorded in their respective income statements. Of course, in following this procedure all transfers between segments of the public sector are automatically netted out, hence there is no double counting.

Simple as it may seem, the attempt to obtain an accurate measure of the consolidated deficit through this procedure may face serious statistical and methodological barriers. Sometimes the individual pieces of information are not available. The statements of specific segments of the public sector may be aggregated in such a way as to make very difficult the task of consolidation. Finally, the accounting methodologies followed by different segments of the public sector may differ significantly, rendering the above the line calculations of the consolidated deficit meaningless. Frequently, some segments of the public sector construct their accounts on a cash basis, while others mix accrued and cash revenues and payments, as well as capital gains and losses with ordinary payments.

Although EBE and SE accounts are known to constitute a problem in attempts to calculate the consolidated deficit, the most severe problems are usually encountered when the financial public sector is brought into the picture. This is particularly true in the case of central banks. As mentioned above, central banks around the world display a great imagination in the construction of their income statements. For example, interests on credits to the government may be accounted as a revenue, even though those interests are neither effectively paid nor included as expenditures in the budget. A similarly asymmetric procedure may be applied to credits to SEs. The flows do not cancel out, and above the line estimates of the consolidated deficit underestimate the true deficit. Another example of creative accounting is the inclusion of capital gains on foreign assets in interest revenues, while capital losses on the foreign liabilities are altogether excluded. The capital losses may be disguised in the balance sheet under one of the several "other accounts" created for this purpose, or recorded as special credits to the government. When these accounting procedures are utilized, a simple consolidation would also underestimate the true deficit.

The computation of consolidated deficits based on above the line figures could still be attempted on an *ad hoc* basis, by including some items in income statements while excluding others. However, this procedure would involve a number of arbitrary decisions, and might result in large measurement errors. One alternative route is the estimation of consolidated deficits from below the line, that is, from the computation of the changes in the assets and liabilities of the public sector.

Below the line estimates of the deficit are not free of problems either. Accurate estimates require a detailed compilation of the assets and liabilities of the various segments of the public sector. For instance, the computation of the stock of net domestic debt requires not only data on the stock of government securities held by the private sector, but also data on the stocks of credits to and deposits of the public sector in private domestic banks and other financial institutions. Credit and deposit figures may be aggregated in such a way as to make impossible a fine separation of the public and private sectors. Another typical difficulty is related to the impact of cross-currency fluctuations on the stocks of foreign assets and liabilities. Absence of detailed information on the currency composition of foreign assets and liabilities may render a straightforward computation of net changes meaningless. Still another difficulty is presented by the accumulation and decumulation of public sector arrears. Failure to include arrears in the stock of liabilities may also result in large measurement errors.

The comparative advantages and disadvantages of the above the line and below the line methods vary from country to country. However, even in those cases where the computation of the consolidated deficit from above the line is feasible and judged as reliable, below the line estimates may also prove useful for at least two reasons. First, these figures may be used for comparison and for checking the above the line figures. Second, below the line calculations provide useful detailed information on the sources of deficit finance.

In this appendix we discuss some measurement problems that are typically encountered in below the line computations of public sector deficits. These problems arise because below the line estimates involve the calculation of changes in stocks relative to the flow of output. If the numerator and the denominator are measured at different prices or affected by exchange rate movements, the calculated ratios will be distorted. The following sections analyze and compare alternative methods to measure the size of money finance, domestic debt finance and external debt finance relative to GNP.

2. Money finance

Gross seignorage revenues are identically equal to the sum of inflation tax revenues and the real variations in base money, as stated in equation (A.1). To obtain a measure of seignorage revenues as a share of GNP, equation (A.1) has to be divided by (instantaneous) real GNP, $y = Y/P$, where y = real GNP, Y = nominal GNP and P = price level. That yields equations (A.2) or (A.3), depending on whether the components of seignorage are defined as shares of real or nominal GNP.

$$\frac{\dot{H}}{P} = \hat{P}h + \dot{h} \tag{A.1}$$

$$\frac{\dot{H}}{Y} = \frac{\hat{P}H}{Y} + \frac{P\dot{h}}{Y} = \frac{\hat{P}H}{Y} + \frac{\dot{H} - \hat{P}H}{Y} \tag{A.2}$$

$$\frac{\dot{H}}{Y} = \frac{\left(\dot{H}/P\right)}{y} = \frac{\hat{P}h}{y} + \frac{\dot{h}}{y} \tag{A.3}$$

This section will explore alternative methods to compute the integrals of the instantaneous seignorage, as given by (A.2) or (A.3), from discrete data on monetary stocks, prices and output. The analysis will be illustrated with actual data from Turkey and Yugoslavia.

Method I: nominal discrete data

The most straightforward method of computing the ratio between seignorage revenues and GNP is to use the available data on end-of-year nominal money stocks, a price index and nominal GNP. Thus, equation (A.2) can be computed as (method Ia):

$$S_{Ia} = \frac{H_t - H_{t-1}}{\not{Y}_t} = \frac{\pi_t H_{t-1}}{\not{Y}_t} + \frac{H_t - (1+\pi_t)H_{t-1}}{\not{Y}_t} \tag{A.4}$$

Here H is the stock of base money at the end of t, $\pi_t = [P(t)/P(t-1)] - 1$ is the inflation rate between $t-1$ and t, and $\not{Y}_t = \int_0^1 Y(t-1+\tau)d\tau$ is nominal GNP at t. There is no obvious problem in computing seignorage revenues as a share of GNP through equation (A.4), although the breakdown between the inflation tax and the real variations in base money is likely to be distorted. This is because the flows in the numerator and the denominator are measured at different prices. For instance, the real variations component is measured at end-of-period prices, whereas the price level implicit in nominal GNP is an average price. That will tend to overestimate the gains and losses from this component. The reverse will happen with the inflation tax component.

Alternatively, the ratio between seignorage revenues and GNP can be computed as (method Ib):

$$S_{lb} = \frac{H_t - H_{t-1}}{\not{Y}_t} = \frac{\pi_t}{1+\pi_t}\frac{H_t}{\not{Y}_t} + \frac{H_t/(1+\pi_t) - H_{t-1}}{\not{Y}_t} \tag{A.5}$$

This method has a bias that is opposite to that of method Ia: the gains and losses of the real variations component are underestimated. Of course, the figures obtained from the two methods could be averaged to produce better estimates, since they have opposite biases.

Method II: integral of the numerator

The calculation of seignorage revenues from discrete statistical data can be refined through the computation of the integral of the numerators and denominators of (A.2) or (A.3). Thus, in the case of equation (A.2), total monetary revenues would be computed by:

$$\frac{\int_0^1 \dot{H}(t-1+\tau)d\tau}{\not{Y}_t} = \frac{\int_0^1 \hat{P}(t-1+\tau)H(t-1+\tau)d\tau}{\not{Y}_t} + \frac{\int_0^1 P(t-1+\tau)\dot{h}(t-1+\tau)d\tau}{\not{Y}_t} \tag{A.6}$$

Equation (A.6) can be computed with discrete data, assuming a certain growth path for prices and the nominal and real stocks of base money within the year. For instance, if these variables are assumed to grow exponentially during the year, the values of H, P and h at any point in time will be defined by:

$$H(t-1+\tau) = H(t-1)e^{\dot{H}_t\tau}$$

$$P(t-1+\tau) = P(t-1)e^{\dot{P}_t\tau}$$

$$h(t-1+\tau) = h(t-1)e^{\dot{h}_t\tau}$$

Here $\hat{H}_t, \hat{P}_t$, and $\hat{h}_t = \hat{H}_t - \hat{P}_t$ are the instantaneous growth rates of H, P and h. $\hat{H}_t$ can be computed for any year as $\hat{H}_t = \ln(H_t / H_{t-1})$. $\hat{P}$ and $\hat{h}$ are obtained through the same procedure. The time derivatives of H, P, and h can then be easily found. Under these assumptions, (A.6) becomes:

$$S_{lla} = \frac{H_t - H_{t-1}}{\not{Y}_t} = \frac{\hat{P}_t}{\hat{H}_t}\frac{H_t - H_{t-1}}{\not{Y}_t} + \frac{\hat{h}_t}{\hat{H}_t}\frac{H_t - H_{t-1}}{\not{Y}_t} \tag{A.7}$$

Method II yields the same result for total seignorage revenues as method I. However, note that the breakdown between inflation tax and real variations is different. Note also that the breakdown provided by equation (A.7) is quite intuitive. It amounts to splitting total sei-

gnorage revenues according to the shares of the growth rates of prices and real base money in the growth rate of nominal base money. Of course, the breakdown will depend on the specific assumptions about the intra-year growth path of H and P. Although the exponential rule is a good approximation in most practical cases, other growth rules might prove more appropriate under special conditions.

A similar method consists in calculating the integrals of the numerator and denominator of (A.3):

$$\frac{\int_0^1 \left[\dot{H}(t-1+\tau)/P(t-1+\tau)\right] d\tau}{\not{y}_t}$$

Using again the assumption of exponential growth rates for H and P yields:

$$S_{IIb} = \frac{\hat{P}_t}{\hat{h}_t}\frac{h_t - h_{t-1}}{\not{y}_t} + \frac{h_t - h_{t-1}}{\not{y}_t} \tag{A.8}$$

Here $\not{y}_t = \int_0^1 y(t-1+\tau)d\tau$ is real GNP during year t. In order to avoid serious biases in the calculation of (A.8) the various variables have to be deflated by the same price index.

Method III: integral of the ratio

Another alternative is to compute the integral of the ratio of seignorage to GNP directly, assuming a certain growth path for GNP. Again, that can be done by integrating either equation (A.2) or (A.3). For instance, the integral of (A.2), on the assumption of exponential growth rates for all variables is:

$$S_{IIIa} = \frac{\hat{P}_t}{\hat{H}_t - \hat{Y}_t}\left[\frac{H_t}{Y_t} - \frac{H_{t-1}}{Y_{t-1}}\right] + \frac{\hat{h}_t}{\hat{H}_t - \hat{Y}_t}\left[\frac{H_t}{Y_t} - \frac{H_{t-1}}{Y_{t-1}}\right] \tag{A.9}$$

This measurement technique is appealing, since it calculates the average of seignorage revenues as a share of GNP at every point in time within a year. However, care must be exercised in calculating the beginning-and end-of-period ratios of base money to GNP. For example, in (A.9) $H(t)/Y(t)$ is the ratio of the end-of-period stock of base money to *instantaneous* GNP at time t. In order to calculate the latter, one can assume that the intra-year real output growth rate is constant, and therefore equal to $\hat{y}_t = \ln\left(y_t / y_{t-1}\right)$, and solve the equation

$$\not{y}(t) = \int_0^1 y(t-1+\tau)d\tau = y(t-1)\int_0^1 e^{\hat{y}_t \tau} d\tau = y(t-1)\left[\frac{e^{\hat{y}_t} - 1}{\hat{y}_t}\right]$$

for y (t–1). Thus,

$$y(t-1) = \not{y}_t \left[\frac{\hat{y}_t}{e^{\hat{y}_t} - 1} \right] \tag{A.10}$$

Similarly, it is easy to check that

$$y(t) = \not{y}_t \left[\frac{-\hat{y}_t}{e^{-\hat{y}_t} - 1} \right] \tag{A.11}$$

One can then follow two alternative paths. First, for small $\hat{y}_t$, the terms inside brackets in (A.10) and (A.11) can both be approximated by one, so $y(t-1) \approx \not{y}(t) \approx y(t)$, and $\hat{y}_t$ can be approximated by zero. (A.9) then reduces to (A.8):

$$S_{IIIb} = S_{IIb}\ ^{22} \tag{A.12}$$

Alternatively, when $\hat{y}_t$, is large, it can be estimated as the geometric average of the adjacent real GDP growth rates:

$$\tilde{\hat{y}}_t \approx \sqrt{\frac{\not{y}_{t+1}}{\not{y}_{t-1}}} - 1$$

This estimate can be plugged in formulas (A.10) and (A.11) to obtain estimates $\tilde{y}_t$ and $\tilde{y}_{t-1}$, for y_t and y_{t-1}, respectively.[23]

Substituting these estimates in the "real" version of (A.9), one finally obtains:

$$S_{IIIc} = \frac{\hat{P}_t}{\hat{h}_t - \tilde{\hat{y}}_t} \left[\frac{h_t}{\tilde{y}_t} - \frac{h_{t-1}}{\tilde{y}_{t-1}} \right] + \frac{\hat{h}_t}{\hat{h}_t - \tilde{\hat{y}}_t} \left[\frac{h_t}{\tilde{y}_t} - \frac{h_{t-1}}{\tilde{y}_{t-1}} \right] \tag{A.1}$$

Table 3 provides a comparison of these different methods with actual data for Turkey and Yugoslavia. Note that Turkey experienced moderate to high inflation rates — ranging from 15 to 90 percent per annum, while inflation rates in Yugoslavia were much higher during the same period, ranging from 30 to 2,700 percent per annum.

As shown in table 3, the differences among different measures of total seignorage revenues as a share of GNP are minor in both countries. As discussed above, methods Ia and Ib do not provide a satisfactory breakdown of total seignorage revenues between inflation tax and real variations, compared with methods IIa, IIb and III. As can be seen from table 3, the differences between the results yielded by these three last methods are small in both coun-

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[22] In this case real GDP, which appears as a denominator in (A.3), is constant. It follows that, for that equation, the ratio of the integrals equals the integral of the ratio.

[23] Notice that two (slightly) different estimates of y will be generated. One will be used in the estimation of the ratio seignorage revenues/GNP in year t (between times t–1 and t), and will be obtained by application of (A.11). The other will be used in the estimation of the same ratio in year t+1 (between times t and t+1), and will be obtained by the application of (A.10).

tries. Moreover, methods IIb and III yield the same results except in those years with large variations in real output. It is also interesting to note that the breakdown provided by methods IIa, IIb and III lies between methods Ia and Ib.

Table3
Turkey and Yugoslavia
Seignorage revenues as a share of GNP: 1980-89(%)

Turkey	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Ia. Discrete (start-period prices)										
1. Total	3.220	4.163	3.369	3.208	3.593	2.918	1.970	2.501	3.964	4.251
2. Inflation tax	5.946	1.888	2.133	3.221	3.717	3.236	2.220	3.414	3.806	3.471
3. Real variations	-2.726	2.275	1.235	-0.014	-0.124	-0.318	-0.250	-0.913	-0.158	0.780
Ib. Discrete (end-period prices)										
1. Total	3.220	4.163	3.369	3.208	3.593	2.918	1.970	2.501	3.964	4.251
2. Inflation tax	4.658	2.390	2.390	3.218	3.676	3.139	2.162	3.089	3.874	3.777
3. Real variations	-1.438	1.773	0.979	-0.010	-0.083	-0.220	-0.191	-0.589	0.090	0.475
IIa. Integral of numerator (nominal)										
1. Total	3.220	4.163	3.369	3.208	3.593	2.918	1.970	2.501	3.964	4.251
2. Inflation tax	5.208	2.140	2.264	3.219	3.695	3.184	2.190	3.237	3.843	3.634
3. Real variations	-1.988	2.024	1.104	-0.012	-0.102	-0.266	-0.219	-0.736	0.121	0.617
IIb. Integral of numerator (real)										
1. Total	3.776	4.267	3.349	3.186	3.688	2.961	1.965	2.427	4.057	4.350
2. Inflation tax	6.107	2.193	2.251	3.198	3.793	3.231	2.184	3.142	3.933	3.719
3. Real variations	-2.331	2.074	1.098	-0.012	-0.105	-0.270	-0.219	-0.715	0.124	0.631
III. Integral of ratio										
1. Total	3.778	4.263	3.349	3.186	3.689	2.962	1.966	2.428	4.057	4.350
2. Inflation tax	6.110	2.191	2.251	3.198	3.794	3.232	2.185	3.143	3.933	3.719
3. Real variations	-2.332	2.072	1.098	-0.012	-0.105	-0.270	-0.219	-0.715	0.124	0.631
Yugoslavia	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Ia. Discrete (start-period prices)										
1. Total	1.809	2.676	2.701	1.636	3.865	3.600	4.156	4.872	5.526	12.123
2. Inflation tax	4.274	3.322	2.953	5.073	3.442	4.372	4.407	6.769	7.087	17.034
3. Real variations	-2.464	-0.645	-0.252	-3.437	0.423	-0.773	-0.251	-1.897	-1.561	-4.912
Ib. Discrete (end-period prices)										
1. Total	1.809	2.676	2.701	1.636	3.865	3.600	4.156	4.872	5.526	12.123
2. Inflation tax	3.602	3.152	2.891	3.784	3.589	4.040	4.287	5.578	5.984	12.299
3. Real variations	-1.793	-0.475	-0.190	-2.147	0.277	-0.441	-0.131	-0.706	-0.458	-0.176
IIa. Integral of numerator (nominal)										
1. Total	1.809	2.676	2.701	1.636	3.865	3.600	4.156	4.872	5.526	12.123
2. Inflation tax	3.910	3.232	2.920	4.347	3.520	4.189	4.340	6.058	6.411	13.438
3. Real variations	-2.101	-0.555	-0.219	-2.711	0.345	-0.589	-0.184	-1.186	-0.885	-1.316
IIb. Integral of numerator (real)										
1. Total	1.842	2.781	2.750	1.611	3.719	3.639	4.320	4.886	5.280	11.791
2. Inflation tax	3.982	3.358	2.973	4.281	3.387	4.235	4.511	6.075	6.126	13.070
3. Real variations	-2.139	-0.577	-0.223	-2.669	0.332	-0.596	-0.192	-1.190	-0.845	-1.280
III. Integral of ratio										
1. Total	1.843	2.782	2.750	1.611	3.719	3.640	4.320	4.887	5.281	11.777
2. Inflation tax	3.983	3.359	2.973	4.280	3.387	4.236	4.512	6.077	6.127	13.055
3. Real variations	-2.140	-0.577	-0.223	-2.669	0.332	-0.596	-0.192	-1.190	-0.846	-1.278

Sources: Quarterly Bulletins of the Central Banks of Turkey and Yugoslavia.

3. Domestic debt finance

In the case of domestic debt finance, the problem is how to measure the value of changes in the stock of the public sector's net domestic debt relative to the flow of output. Again, the nominal changes in the stock of net domestic debt may be broken down between an inflation component and real variations of the stock, as in equation (A.13), which is formally identical to (A.1):

$$\frac{\dot{B}}{P} = \hat{P}b + \dot{b} \tag{A.13}$$

The measurement issues that arise in the calculation of domestic debt finance are thus broadly the same as those discussed in the case of money finance. The calculation of the nominal deficit from below the line involves the computation of the nominal changes in the stock of government debt relative to nominal GNP. In the case of the real deficit only the real variations of the stock will be computed, since the real deficit definition excludes the inflation component of interest payments on the stock. However, accuracy in measurement is more important in the calculation of debt finance, since only the real component is included in the computation of the real deficit. in the case of money finance, an accurate breakdown is desirable for informational purposes, but does not affect the total deficit calculation. This is because it is total seignorage revenues that matter in the calculation, regardless of whether the deficit is defined in nominal or real terms.

4. External debt finance

In the calculation of public sector deficits from below the line, the most severe measurement problems arise in the computation of the external finance component. The problem is how to separate actual financing flows from abroad from capital gains and losses resulting from movements in exchange rates. This problem is aggravated by the fact that the stock of net external debt is quoted in dollars, which subjects it to be influenced by the variations in the value of the dollar *vis-à-vis* other currencies.

Consider first the simple case where the foreign assets and liabilities of the public sector are exclusively denominated in US dollars. The problem of cross-currency fluctuations will be addressed further below. In this case, the external finance component is defined by:

$$\dot{Z} = \dot{E}\left(B^* - NFA^*\right) + E\left(\dot{B}^* - \dot{NFA}^*\right) \tag{A.14}$$

Here, by assumption, B^* and NFA^* are exclusively denominated in US dollars, and $Z = EZ^* = E\left(B^* - NFA^*\right)$. Equation (A.14) breaks down the changes in the stock of the

public sector's net external debt into financing flows and capital losses due to exchange rate depreciations.[24]

In theory, discrete devaluations are the easiest to deal with, since the stocks involved are constant when measured in foreign currency. That is, there are no "cross" terms to be dealt with.[25] Therefore, situations where there was a small number of devaluations during a given year can be easily dealt with. In years where devaluations were frequent one should still tackle any large devaluations individually. The year is then divided into subperiods comprised between two large devaluations. In each of these subperiods there may have been a large number of small devaluations, and either it is impractical to deal with them individually, or detailed data is not available. One can then aproximate the growth paths of the variables involved by assuming specific functional forms. In those cases where only the end points are known, exponential growth is the most sensible assumption, as it corresponds to a constant growth rate.

For instance, computation of the integrals of all terms in equation (A.15), assuming exponential growth rules for E and Z^*, yields:

$$Z_t - Z_{t-1} = \left(\frac{\hat{E}_t}{\hat{Z}_t}\right)(Z_t - Z_{t-1}) + \left(\frac{\hat{Z}^*_t}{\hat{Z}_t}\right)(Z_t - Z_{t-1}) \qquad \text{(A.15)}$$

Here $\hat{E}_t = \ln(E_t / E_{t-1})$, $\hat{Z}^*_t = \ln\left(Z^*_t / Z^*_{t-1}\right)$, and $\hat{Z}_t = \hat{E}_t + Z^*_t$ are the exponential growth rates of E, Z^* and Z within the year. In order to obtain a measure of the magnitude of the external finance component relative to GNP, it would suffice to divide equation (A.15) by nominal GNP in period t. This procedure is similar to the one of method II above. Of course, any one of the three methods proposed for the calculation of the ratio seignorage/GNP can be adapted for the calculation of the external finance component.

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[24] In the case of external finance, a straightforward aplication of discrete end-of-period data will never allow for a fine separation of the two terms on the right hand side of (A.13). The problem lies in the division of a cross product between capital losses and actual financing flows:

$$Z_t - Z_{t-1} = E_{t-1}\left(Z^*_t - Z^*_{t-1}\right) + \left(E_t - E_{t-1}\right)Z^*_{t-1} + \left(E_t - E_{t-1}\right)\left(Z^*_t - Z^*_{t-1}\right)$$

The first term on the right hand side of the equation captures actual financing flows, the second term captures the capital losses, and the third term is a cross product that captures both. One method frequently employed to obtain a division of the cross-product relies on the arithmetic averages. Indeed, the variations in the domestic currency value of the net external debt may be written as:

$$Z_t - Z_{t-1} = E'_t\left(Z^*_t - Z^*_{t-1}\right) + \left(E_t - E_{t-1}\right)Z'^*_{t-1}$$

Here E'_t and Z'^*_t are the average exchange rate and the US dollar value of the net external debt stock, respectively. However, it can be easily shown that this simple procedure will be optimal only in the unlikely case that the stock variables grow linearly ($E(t) = E(0) + at$ and $Z^*(t) = Z^*(0) + bt$).

[25] If $X = EX^*$ and E changes to $E + \Delta E$, then $X + \Delta X = (E + \Delta E)X^*$, or $\Delta X = \Delta E\, X^*$ if the devaluation was instantaneous, so that X^* could be considered as constant. Otherwise, $\Delta X = \Delta E\, X^* + E\, \Delta X^* + \Delta E\, \Delta X^*$. The "cross" term $\Delta E\, \Delta X^*$ frequently complicates matters in undesirable ways.

The calculation of the foreign finance component in real terms does not present any serious difficulty. Of course, in this case, all the variables in equation (A.14) would have to be defined in real terms:

$$\dot{z} = \dot{e}\left(b^* - nfa^*\right) + \left(\dot{b}^* - n\dot{f}a^*\right) \tag{A.16}$$

In actual calculations the nominal variables have to be deflated by the price indices judged as most appropriate. The use of domestic and foreign CPIs is one possible alternative. That permits the calculation of an expression exactly equivalent to (A.15).

Finally, the existence of assets and liabilities in several currencies may present a problem, due to fluctuations of the dollar *vis-à-vis* other foreign currencies. However, these cross-currency effects may be taken into account in two ways. First, the above calculations may be performed for each individual foreign currency. Alternatively, the total dollar value of foreign liabilities may be adjusted for cross-currency fluctuations.[26]

[26] See World Bank (1988) and Van Wijnbergen, Anand, Chibber & Rocha (1992) for an illustration of this second method for the case of Turkey.