Long-Run Limits on the Brazilian Government Debt*

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Summary: 1. Introduction; 2. Theory and evidence for the USA; 3. Empirical evidence for Brazil; 4. Conclusions.
Key words: intertemporal budget balance; internal debt; seigniorage.

This paper addresses the solvency condition of the Brazilian government. Two tests are performed to verify if the intertemporal borrowing constraint holds in present value terms. The first one evaluates the generating process of the stock of the debt, and the second analyzes the long run relationship between revenue and government spending. The empirical evidence is consistent with intertemporal budget balance. However, until February 1990, just before the government froze 80% of the financial assets, the debt shows an unstable path. Besides, the results indicate that seigniorage is an extremely important source of revenue for the government. If seigniorage is not considered, the government budget is not balanced in present value terms, the opposite happening if seigniorage is considered.

Este artigo analisa a condição de solvência do governo brasileiro. Dois testes são feitos para verificar se a restrição de endividamento intertemporal do governo é satisfeita em termos de valor presente. O primeiro teste analisa o processo gerador da dívida interna, e o segundo analisa a relação entre gastos e receitas do governo. A evidência empírica sugere a existência de equilíbrio orçamentário intertemporal. Contudo, até fevereiro de 1990, imediatamente antes do congelamento de 80% dos ativos financeiros, a dívida interna descrevia uma trajetória instável. Além do mais, os resultados indicam que a senhoriagem é uma fonte extremamente importante de receita para o governo. Se a senhoriagem é desconsiderada como receita, o orçamento do governo não é equilibrado em termos de valor presente, ocorrendo o oposto se a senhoriagem é incluída como receita.

1. Introduction

In *An open letter to the Brazilian finance minister*, Sargent (1993b:252) observes: "The key indication of the deficiency of tax collections relative to government expenditures is that you are borrowing more and more domestically. Arithmetic implies that your current policies can be continued only so

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long as you are able to borrow increasingly large amounts in real terms in the foreign and domestic markets. You will not be able to so for long. There are limits to how much any country can borrow, even a country with resources as great as yours. The limit is imposed by lenders' views of your capacity to run future government surpluses sufficiently large to service the debt. You have already reached the limit on your international borrowing. Limits on the volume of your domestic borrowing, indicated by the high real rates of interest, of 15% a year that you now pay, loom before you".[1]

The original letter was published in January 30, 1986. Since then, the unpleasant adjustments (higher taxes and/or lower government expenditures) considered inevitable by Sargent occurred with different degrees of success. The government even defaulted on its domestic debt. By March 1990, with the Collor Plan, all financial assets were blocked for 18 months. After that, they were released in 12 installments but the change in the rules regarding the payment of monetary correction corresponded to an implicit default.

The purpose of this paper is to assess the consistency of the time path of the Brazilian federal government expenditures, revenues, and debt with the assumption of intertemporal budget balance. The period to be analyzed goes from January 1980 to July 1993. Two types of tests will be performed. The first one checks the sustainability of fiscal policy by evaluating the generating process of the stock of the debt. The second is based on the long run relationship between revenue and government spending. This alternative is particularly interesting because it allows one to investigate directly the role played by the "inflation tax" in guaranteeing or not the intertemporal budget balance.

Section 2 discusses different tests for intertemporal budget balance existent in the literature. The results obtained for the USA are quite diverse. Wilcox (1989), and Hakkio & Rush (1991) conclude that USA data is inconsistent with the hypothesis of intertemporal budget balance, while Hamilton & Flavin (1986), Trehan & Walsh (1988, 1991), and Haug (1991) reach the opposite conclusion. Section 3 performs tests for the stationarity of the undiscounted debt as well as cointegration tests for revenue and government spending. Section 4 concludes.

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[1] *For a theoretical discussion of the government budget constraint and of its consequences for the behavior of monetary and fiscal authorities, see Sargent & Wallace (1981) and Sargent (1993a).*

2. Theory and Evidence for the USA

The government's budget constraint can be written as:

$$B_t = (1 + r)B_{t-1} - S_t \tag{1}$$

where B_t is the real market value of the debt held by the public, r is the ex post real interest rate, and S_t is the real noninterest surplus. The surplus is defined as $R_t - G_t$, where R_t is the real government revenue, and G_t constitutes real government purchases of goods and services excluding interest payments on debt. Money seigniorage can be included as revenue, in the sense that it also represents a source of retiring outstanding government debt.

Step by step forward substitution results in the intertemporal budget constraint:

$$B_t = \lim_{s \to \infty} (1 + r)^{-s} B_{t+s} + \sum_{s=1}^{\infty} (1 + r)^{-s}(R_{t+s} - G_{t+s}) \tag{2}$$

Taking expectations of (2) conditional on information available at time t, under the hypothesis of present value budget balance, the debt outstanding in the current period must be equal to the present value of all future surpluses:

$$B_t = \sum_{s=1}^{\infty} (1 + r)^{-s} E_t(R_{t+s} - G_{t+s}) \tag{3a}$$

From (2) this is mathematically equivalent to the restriction that the discounted value of the expected future stock of debt converges to 0 as time goes to infinite:

$$E_t \lim_{s \to \infty} (1 + r)^{-s} B_{t+s} = 0 \tag{3b}$$

As discussed more extensively by Barro (1987) and McCallum (1984), the expected limit of the discounted debt must equal 0 in order to rule out the possibility of the government running a Ponzi scheme with its debt. If this is not the case, the government can bubble finance its expenditures, with the old maturing debt being financed by new debt issues.

Hamilton and Flavin (1986), examining the borrowing constraint in current terms, frame the alternative possibility that government deficits need not be met by future surpluses, or, equivalently, that the limit expressed in (3b)

is equal to a positive number, say A_0. The government budget constraint in this case can be written as:

$$B_t = A_0(1 + r)^t + \sum_{s=1}^{\infty}(1 + r)^{-s}S_{t+s} \qquad (4)$$

The present value borrowing constraint holds if A_0 is equal to 0.[2]

Hamilton and Flavin propose three tests to verify if A_0 equals 0 during the period of 1960 to 1984. The first test results from the observation that for any stationary process, the undiscounted debt is stationary for the sum of the expected discounted surpluses only if A_0 is equal to 0. They show that the undiscounted surplus and the undiscounted debt are both stationary, and therefore conclude that the government constraint holds.[3]

The second and third tests are performed by direct estimation of equation (4). Different assumptions about the information set underlying the formation of expectations of future surpluses are adopted. In the second test expectations of future surpluses are assumed to partially depend on past surpluses, so that the sum of expected surpluses in the right hand side of (4) is substituted by current and lagged values of the surplus. Lagged values of the debt are also included to eliminate the serial correlation of the error term. The regression equation is given by

$$B_t = A_0(1 + r)^t + c(L)B_{t-1} + b(L)S_t + \varepsilon_t \qquad (5)$$

where A_0 is a coefficient to be estimated, and the error term reflects "expected changes in real short-term interest rates, the term structure of long rates, and measurement error", and r is equal to the average ex post real rate over the sample period. No evidence of a violation of the borrowing constraint is found; A_0 is statistically insignificant and negative.

In Hamilton and Flavin's third test, expectations about the future surplus depended only on lagged values of the surplus. The equations of the debt as a function of expected future surpluses, and the surplus as function of its lagged values were estimated jointly by nonlinear least squares. Again, no role is found for the bubble term; A_0 is not significant and is negative.

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[2] *Equation (4) is mathematically equivalent to models of speculative bubbles, as the one initially formulated by Flood & Garber (1980).*

[3] *Stationarity of the undiscounted surplus is considered sufficient for stationarity of the sum of expected discounted surpluses if the real interest rate is positive.*

Wilcox (1989) proposes an alternative test. The present value borrowing constraint holds, and so fiscal policy is sustainable, if the forecast trajectory for the discounted debt converges to 0. The test consists of two steps. First it is necessary to check if the stock of the discounted debt is stationary. If nonstationarity is rejected, a test for a 0 mean in the representation of the debt process must then be performed. Intertemporal budget balance holds if and only if the discounted debt series is stationary with mean 0.

Wilcox's test is less restrictive than Hamilton and Flavin's test in two aspects. First, by discounting the government's debt back to a reference point in time, assumptions about the real rate process are avoided. Therefore the test allows for stochastic real rates as well, while Hamilton and Flavin's test depend heavily on the assumption of a constant expected real rate of interest. This is especially true with respect to their second and third tests. Only when the real rate is nonstochastic can the sum of expected future discounted surpluses be represented as a linear function of current and lagged surpluses and debt. Second, Wilcox's test can deal with different classes of violations of the borrowing constraint. Under the alternative hypothesis, that is, if the present value borrowing constraint does not hold, the value of the debt may differ from the sum of expected future surplus by, say, A_t, or equivalently:

$$A_t = \lim_{s \to \infty} E_t b_{t+s} \qquad (6)$$

where b_t is the discounted value of the debt.

From (6) we have:

$$A_{t+1} = A_t \, iff \lim_{s \to \infty} E_{t+1} b_{t+s} = \lim_{s \to \infty} E_t b_{t+s} \qquad (7)$$

Hamilton and Flavin concentrate on the case where A_t is constant. This happens if the debt trajectory converges to the same value each period, a condition which is satisfied when B_t is stationary. Wilcox refers to this as the nonstochastic case, given that A_t is constant. He claims, however, that equation (7) indicates that deviations of the debt from the expected sum of future surpluses can vary each period according to different possibilities. This corresponds to the stochastic case, and occurs whenever the discounted debt is nonstationary. Therefore, Wilcox's test is more general as it has power against stochastic violations of the borrowing constraint, while the second

and third tests proposed by Hamilton and Flavin assume that any violation of the borrowing constraint would be nonstochastic.[4]

Using the same data set used by Hamilton and Flavin, Wilcox finds evidence of parameter instability in fiscal policy. During the first half of the sample period (1962-74) the borrowing constraint seems to be satisfied, while for the post-1974 period the opposite happens.

Kremers (1988) shows evidence that the presence of first order autocorrelation invalidates the augmented Dickey and Fuller test which Hamilton & Flavin use to reject the nonstationarity of US government debt. This problem is solved with the addition of a second lagged dependent variable, but the results are reversed. The borrowing constraint is no longer satisfied.

Other tests present in the literature try to verify the existence of a cointegrating relation between expenditures, revenues, and the outstanding stock of debt.

Haug (1991) assumes the same alternative hypothesis as Hamilton & Flavin, but establishes that cointegration of B_{t-1} and S_t is the sufficient condition for ruling out bubble financing. Using quarterly US data from 1960:I to 1987:IV he concludes that the government budget is balanced in present value terms.

Trehan & Walsh (1991) show that, when the expected real rate of interest is constant, if $(1 - \lambda L)S_t$ is a mean 0 stochastic process with $0 \leq \lambda < R$, then intertemporal budget balance is attained if, and only if, there is a linear combination of S_t and B_{t-1} that is stationary. In particular, if $\lambda = 1$, that is, if S is difference stationary, a necessary and sufficient condition for budget balance is the stationarity of the inclusive of interest deficit.[5] When the expected real rate of interest is variable, the cointegration test is no longer valid. In this

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[4] *In footnote 5, Hamilton and Flavin (1986:816) seem to recognize this limitation. They observe: "An added complication for our application is that under some specifications of the alternative hypothesis A_0 could be regarded as a random variable".*

[5] *Trehan and Walsh (1988) derive the stationary test, and show that it is equivalent to the cointegration test if the real interest rate is assumed constant.*

case, intertemporal budget balance is satisfied if $(1 - L)B_t$ is a stationary process, given a strictly positive expected real rate of interest.[6]

Using Hamilton and Flavin's data set, no evidence of a stationary linear combination of the stock of debt and the net-of-interest deficit is found. More precisely, it is not possible to reject nonstationarity of the debt process, but it is possible to reject nonstationarity of the exclusive of interest deficit process. However, the results obtained reveal that the first difference of the stock of the debt is stationary. Trehan & Walsh then conclude that a constant expected real rate is a weak approximation to the data, and as the inclusive of interest deficit appears to be stationary sustainability is ensured.

They also tested for the possibility of sample specificity, redoing the exercise using annual data over the period 1890-1986. Again the evidence of the presence of a unit root in the debt process, and of its absence in the net of interest deficit process is inconsistent with intertemporal budget balance. However, the rejection of the null of a unit root for the first difference of the debt process accords with fiscal sustainability. The conclusion is restated. The assumption of a constant expected real interest rate does not correspond to the data generating process, and so the present value borrowing constraint is not violated.

Smith and Zin (1991) test the present value borrowing constraint for Canada for the period 1946-84 using monthly data. Two tests are performed. The first one requires that the government surplus, exclusive of interest payments and inclusive of base money creation, and the government debt be cointegrated. The second one requires that the gross surplus $(S_t - r_t B_t)$ be stationary, a condition that can be tested without further assumptions on the interest rate process. Several tests, data measures, and sample periods indicate that the Canadian federal government does not obey a present value borrowing constraint.

Hakkio & Rush (1991) establish, as a necessary condition for the government to obey its present value budget constraint, cointegration between government spending, inclusive of interest payments and government revenue. Cointegration guarantees that spending and revenue do not drift far apart be-

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[6] *The results for both fixed and variable interest rates are given in the form of propositions whose proofs can be found in the original paper. The invalidation of the cointegration test in the case where the expected real rate of interest is allowed to vary results from the impossibility of recovering a constant coefficient from a regression of B_{t-1} on S_t.*

cause their difference is stationary. Another hypothesis that is part of their empirical procedure is that the cointegrating factor of the equilibrium equation equals 1. Although a value smaller than 1 is consistent with a strict interpretation of intertemporal budget constraint, it implies increasing difficulties in marketing the debt. If the variables are measured relative to the GNP, if the cointegrating factor is smaller than 1, the real value of the debt relative to the GNP tends to infinity. This increases the incentive to default and may eventually imply increasing problems for the government to market its debt.[7]

A constant expected value of the interest rate is not required as long as the real interest rate is stationary. Several sample periods are used for the cointegration tests. The whole sample covers 1950:II to 1988:IV, while the two subperiods cover the periods 1964:I to 1988:IV, and 1976:III to 1988:IV respectively. The set of variables comprise revenue and spending themselves, and their normalization using real GNP and population.

If the entire sample period is considered, government expenditure and revenue measured in real terms and per capita appear to be cointegrated. For the period 1964:I to 1988:IV, most of the tests suggest that the data are not cointegrated. All the tests fail to reject lack of cointegration in the last subperiod. There seems to be, therefore, indications that the behavior of revenue and spending have changed in recent years, indicating a violation of the government's intertemporal budget constraint. The cointegrating factor appears always significantly less than 1, even when revenue and spending are measured relative to the GNP and population. This condition is inconsistent with the government's ability to market its debt in the long run.

As the results summarized before indicate, there is no consensus with respect to the sustainability of US fiscal policy. This happens despite of the fact that Hamilton & Flavin, Wilcox, Kremers, and Trehan & Walsh use the same data set. The results also cannot be reconciled using the assumption about the expected real interest rate. Hamilton & Flavin assume a constant real rate and find that the intertemporal budget constraint holds, while Kremers, using the same data set and test, reaches the opposite conclusion. Trehan &

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[7] *This point is also made by Barro (1979), McCallum (1984), and Kremers (1989).*

Walsh, on the other hand, conclude that a constant real rate is not a good approximation but that budget balance is observed. Given these inconsistencies, we will perform as many tests as possible for the Brazilian case to see if they produce a more consensual outcome.

3. Empirical Evidence for Brazil

As was discussed before, different tests on the government's budget constraint employ different assumptions about the expected real rate process. Therefore, some evidence with respect to the behavior of expected real rates seems a natural place to start. Since this issue constitutes itself an interesting topic of research, the procedure to be followed here is quite general.

We will look at the serial correlation structure of the ex post real rate.[8] The constancy of the real rate and market efficiency imply the null hypothesis that all autocorrelations are zero. Table 1 contains the first twelve autocorrelations of the ex post real rates for the period January 1980 to July 1993. The Box-Pierce Q-statistic, a measure of overall autocorrelation, exceeds the critical value of 21.03 indicating a rejection of the null hypothesis at the 5% level. Therefore, despite the lower power of this test pointed out by Nelson and Schwert (1977), the ex ante real rate seems to be variable.[9]

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[8] *This procedure is due to Fama (1975). He shows that if the hypothesis of a constant expected real rate, and market efficiency is true, the ex post rate must be a white noise.*

Formally, in an efficient market

$$f_m(\pi_{t+1}|I_t^m)=f(\pi_{t+1}|I_t) \qquad (a)$$

where I_t^m is the set of information used by the market at t, I_t is the set of information available at t, π_{t+1} is the inflation rate at t+1, f_m is the market assessed density function for π_{t+1}, and f is the true density function implied by I_t.

As in t only the nominal interest rate (i_t) is known, to assess probabilities to π_{t+1} implies to assess probabilities to the real expected interest rate (r_{t+1}). Therefore,

$$f_m(r_{t+1}|I_t^m,i_t)=f(r_{t+1}|I_t,i_t) \qquad (b)$$

Concentrating in the mean of the distribution, and assuming that the equilibrium expected real rate is constant through time:

$$E_m(r_{t+1}|I_t^m,i_t)=E(r_{t+1}|I_t,i_t)=E(r) \qquad (c)$$

One subset of I_t is the time series of past real interest rates. Then,

$$E(r_{t+1}|r_t,r_{t-1},\cdots)=E(r) \qquad (d)$$

If (d) holds, the autocorrelations of r_{t+1} for all lags are equal to 0. As this is a joint test, nonzero autocorrelations of r_{t+1} can result from market inefficiency or variable expected real rates. As usual in the literature, the efficiency of the market is not questioned.

[9] *They show that the autocorrelation function of the ex post real rate of interest can be significantly close to 0 for all lags, even if the ex ante real rate is variable, if the forecast errors for inflation are quite large.*

Given this result, we are left basically with three tests: Wilcox's test, that avoids assumptions about the expected real rate by examining the behavior of the discounted debt series; Trehan & Walsh's test, that is based on the assumption that a variable expected real rate of interest in fact characterizes the data generating process; and Hakkio & Rush's test, that allows for a stochastic real rate but only if it is stationary, a condition still to be verified. We decided to perform two tests. The first uses the debt series. In this case, Trehan & Walsh's test was chosen. The reason is that this test was also performed by Welch (1993) to verify the sustainability of Brazilian fiscal policy during the period March 1986 to February 1990, and therefore we have some basis for comparison. The other test is Hakkio and Rush's test. Despite all the qualitative and conceptual difficulties with measures of federal government revenues and expenditures in Brazil, we think that it is worth considering. There are not many studies working with this series, and a first approach that points out the limitations can be a good starting point for future research. Besides, it can give some additional information not revealed by the tests on the debt.

Table 1

Sample autocorrelations of ex post real rate

Jan. 1980 — July 1993

Lag	Autocorrelation	Lag	Autocorrelation
1	0.22	7	0.09
2	0.13	8	0.13
3	-0.04	9	0.03
4	0.01	10	0.08
5	-0.04	11	0.08
6	0.06	12	-0.13

Note: Standard error = 0.08; Box-Pierce Q = 22.05 (χ^2 with 12 degrees of freedom).

3.1 Testing the undiscounted debt[10,11]

We use monthly values of interest bearing privately held government debt, published by the Brazilian Central Bank (*Boletim do Banco Central do Brasil*, several issues), from January 1980 to July 1993, in constant prices of 1980. This series is plotted in figure 1. The Collor Plan, adopted in the middle of March 1990, is responsible for the sharp decrease in the stock of the government debt. It froze 80% of the money stock ($M4 = M1+$ all other financial assets). Figure 1 also shows the estimated trend line from a regression on a constant, a trend, and a dummy variable taking the value of 0 before and at March 1990, and a value of 1 from April 1990 onwards. The main feature of this graph is the one-time change in the intercept of the trend function. Despite this change, however, the trend is quite stable showing the same slope over the whole period.

Figure 1
Brazilian government debt
Jan. 1980 — July 1993
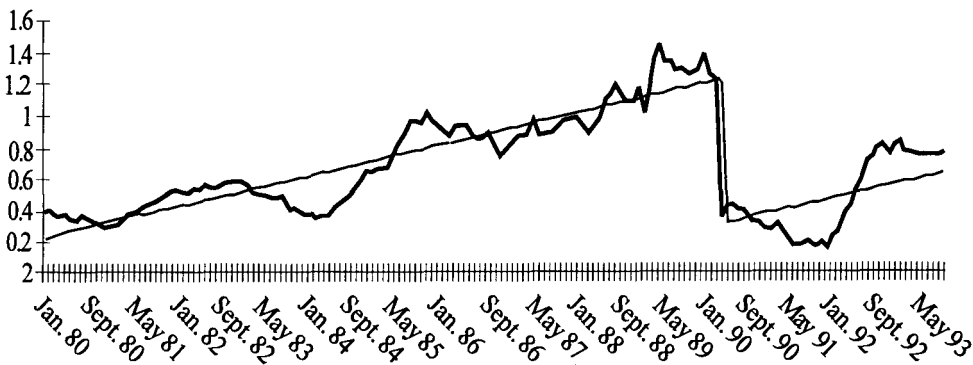
Note: The broken straight line is a fitted trend (by OLS) of the form $\widetilde{y}_t = \widetilde{\mu} + \widetilde{\gamma} DU_t + \widetilde{\beta} t$, where $DU_t = 0$ if $t \leq$ Mar. 1990, and $DU_t = 1$ if $t >$ Mar. 1990.

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[10] *In the Brazilian case, the government extensively used external debt to finance its deficits. However, the sustainability conditions for the external debt is guaranteed by superavits in the current account of the balance of payments and not by fiscal superavits or seigniorage. Tests for external debt are simply analogues to the tests developed for the internal debt (Trehan & Walsh, 1991). Although it would be interesting to describe the behavior of the external debt series, no tests are going to be performed here.*

[11] *The tests were repeated for quarterly data. The results are the same and are available from the author upon request.*

It seems, therefore, that we have a typical case of the crash model described by Perron (1989). The Collor Plan can be considered an exogenous shock in the sense that it is not a realization of the underlying data-generating mechanism of the debt series. We test then the null hypothesis that the debt series is characterized by the presence of a unit root, and possibly a nonzero drift, against the alternative hypothesis of a "trend stationary" system with a one-time change in the intercept of the trend function.

As the change in the trend function is instantaneous, the procedure is as follows.

Suppose that $\{y_t\}$ is the debt series. Let $\{\widetilde{y}_t\}$ be the residuals from a regression of $\{y_t\}$ on a constant, a time trend, and DU_t. Also, let $\widetilde{\alpha}$ be the least squares estimator of α in the following regression:

$$\widetilde{y}_t = \widetilde{\alpha}\widetilde{y}_{t-1} + \widetilde{e}_t \qquad t = 1, 2, \cdots, T \tag{8}$$

In the case where the errors are not uncorrelated, one possibility is to adopt the procedure suggested by Dickey & Fuller (1979, 1981) and use additional lags of the first differences of the debt as regressors in equation (8). The following equation is then estimated by OLS:

$$\widetilde{y}_t = \widetilde{\alpha}\widetilde{y}_{t-1} + \sum_{j=1}^{k} \widetilde{c}_j \Delta \widetilde{y}_{t-j} + \widetilde{\varepsilon}_t \tag{9}$$

where $\Delta \widetilde{y}_t = \widetilde{y}_t - \widetilde{y}_{t-1}$.

K indicates the number of regressors considered. The test, in both cases, is that $\alpha = 1$.

Although the choice of $K = 1$ is implied by the uncorrelated residuals, we decided to report results for $K = 1, \cdots, 5$. This procedure is adopted because, as observed by Perron, few lags may have a substantial effect on the size of the test. Therefore, it is wise to verify how sensitive the results are with the respect to the number of lagged regressors chosen.

Tables 2a and 2b present the estimated regressions, as well as the t statistics.

Table 2a

Tests for a unit root

$T_B = 03/90$	$\widetilde{\mu}$	$t_{\widetilde{\mu}}$	$\widetilde{\beta}$	$t_{\widetilde{\beta}}$	$\widetilde{\gamma}$	$t_{\widetilde{\gamma}}$
	0.22	8.06	0.008	21.28	-0.92	-2.17

Note: T_B is the period when the change in the intercept occurs.

Table 2b

Tests for a unit root

T	λ	K	$\widetilde{\alpha}$	$t_{\widetilde{\alpha}}$
161	0.74	1	0.788	-3.67^a
160	0.74	2	0.797	-3.33
159	0.74	3	0.815	-2.93
158	0.74	4	0.812	-2.87
157	0.74	5	0.812	-2.77

Notes: T is the whole sample size; λ is the ratio of the pre-break sample size to total sample size. ^{a} Denotes statistical significance at the 10% level. Critical values are from Perron (1989:1.376, table IV.B).

The unit root hypothesis can not be rejected for any of the values assumed for K, except for $K = 1$, but only at the 10% level.

Given that the debt series seems to be characterized by a unit root, it is interesting to verify if this behavior is also present in the samples before and after March 1990. The results of the augmented Dickey-Fuller (ADF) tests are presented in table 3, which also includes the result obtained for the whole sample. The number of lagged terms is chosen to ensure that the errors are uncorrelated.[12]

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[12] *Two forms of the Dickey-Fuller regression are going to be estimated throughout the paper:*

$$\Delta y_t = \beta_0 + \beta_1 t + (\alpha - 1) y_{t-1} + \sum_{j=1}^{K} \gamma_j \Delta y_{t-j} + u_t \qquad (a)$$

$$\Delta y_t = \beta_0 + (\alpha - 1) y_{t-1} + \sum_{j=1}^{K} \gamma_j y_{t-j} + u_t \qquad (b)$$

where u_t is a sequence of normal, independent random variables with mean 0 and variance σ^2. The ADF test is computed in the same way as the ordinary t statistic for $\alpha - 1 = 0$. Since, as is well known, this statistic does not have the student's t distribution, even asymptotically, it is sometimes named τ statistic. We will call the τ statistic based on (a) and (b), $\tau_{c,t}$ and τ_c respectively where c, t stands for "constant and trend" and c stands just for "constant".

Table 3

Tests for a unit root using split and full samples*

	Full sample	Pre-March 1990 sample	Post-March 1990 sample
τ statistic	-2.41	-3.14**	-2.44
Lags	12	8	2
n observations	150	113	37

*The asymptotic critical values for $\tau_{c,t}$ at the 10% and 5% levels are -3.13 and -3.41.
**The null hypothesis of non-stationary is rejected.

For the period January 1980 to February 1990 the results indicate that the unit root hypothesis can be rejected, but only at the 10% significance level. However, for the period that follows the break, the $\tau_{c,t}$ statistic implies a nonrejection of the unit root hypothesis.

Once we have strong evidence that the debt process really has a unit root, in order to implement Trehan and Walsh's test we have to verify the stationarity of the first difference of the debt series. As pointed out before, this is the condition for intertemporal budget balance.

With seven lags, the first difference of the stock of debt is stationary. The τ_c statistic is -3.45, implying a rejection of the unit root hypothesis at the 5% level.[13] Therefore, evidence for Brazil is consistent with intertemporal budget balance.

One possibility that still remains to be explored is that these results are sample-specific. This is specially important since Welch (1993) argues that the Brazilian economy never presented a government solvency problem, and an "untested diagnosis" resulted in the partial internal debt moratoria in March 1990. He implements Trehan and Walsh's test for the period March 1986 to February 1990, concluding that the debt is stationary in first differences, and so the government's budget is balanced in present value terms. If we consider, however, the period from January 1980 to February 1990, the results are quite different, as summarized in table 4.

[13] *The asymptotic critical values for τ_c at the 10% and 5% levels are -2.57 and -2.86 respectively.*

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Table 4

Tests for a unit root*

Jan. 1980 — Feb. 1990

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	τ statistic	Lags	n observations
$(1-L)y_t$	-2.37	8	112
$(1-L)^2 y_t$	-6.93**	8	111

*The asymptotic critical values for τ_c at the 10% and 5% levels are -2.57 and -2.86 respectively.
**The null hypothesis of non-stationarity is rejected.

During this period stationarity of the first difference of the debt process can also be rejected. In fact, the stock of the debt series is only stationary in second differences. Therefore, fiscal policy seemed unsustainable till March 1990. It seems, then, that by the end of the 80's the government has exhausted its opportunities to borrow domestically. Adjustments were necessary, although not necessarily the ones effectively implemented.[14]

3.2 Testing the long-run relationship between government revenue and spending[15]

In this section we perform the test suggested by Hakkio and Rush. As was shown before, this test implies cointegration of government spending, inclusive of interest and government revenue, for budget balance to hold.

Before we proceed, it is necessary to discuss carefully the data used. The data for government spending and revenue are approximated by the Treasury expenditure and fiscal revenue figures in *Boletim do Banco Central do Brasil*, several issues. They are in millions of cruzeiros, in constant prices of January 1980. The series of expenditures is, in fact, quite a poor proxy. It does not include purchases of goods, and disregards the social security system. However, it is the only series available, for the period under analysis, that is not annual.

Besides, the spending and revenues refer to the period they effectively occurred (*regime de caixa*). Therefore, the difference between the two is not

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[14] *All the results in this section are also valid if quarterly, instead of monthly, data is used.*

[15] *The tests were repeated for quarterly data. The results are the same and are available from the author upon request.*

compatible with official measures of deficit. These consider the period to which the revenue and expenditures are related to (*regime de competência*). For instance, the wages of the public employees related to April are considered to be spending of that month, although the greatest part of the employees are effectively only paid in the beginning of May.

Another important point is related to whether to include interest payments on government spending. Authors that use the deficit including interest (Trehan & Walsh, 1988; Hakkio & Rush, 1991) argue that this choice is based on McCallum's (1984) requirements for intertemporal budget balance. McCallum argues that a constant deficit inclusive of interest payments is consistent with optimizing behavior by bondholders — and would satisfy (3) —, while the same does not happen with a constant deficit exclusive of interest payments, in which case (3) would be violated. This question is, however, far from settled. The ideal would be that we had also a test based on spending exclusive of interest payments, to verify if different measures give the same or opposite indications of government policy. This not being the case, we proceed using government expenditure inclusive of interest payments, denoted from here on by GG.

As the Brazilian Central Bank cannot be considered an independent institution, it can be included as part of federal government. Therefore, Central Bank earnings are not treated as part of the federal government's receipts because they correspond only to a transfer payment from one branch of the government to an other. Changes in the monetary base, however, should be included as an additional source of revenue. We will, then, verify if inclusion or exclusion of seigniorage has a considerable effect on the results. Data on high powered money are also taken from *Boletim do Banco Central do Brasil*, several issues. RR will denote government revenue including seigniorage, and R will denote government revenue excluding seigniorage.

Given that Hakkio and Rush's test was derived under the assumption of a stationary real interest rate, the first step to be followed is to verify the behavior described by real interest rates. Figure 2 shows that the real rate series exhibits a tendency to return to a constant mean.

Figure 2
Real interest rates
Jan. 1980 — July 1993
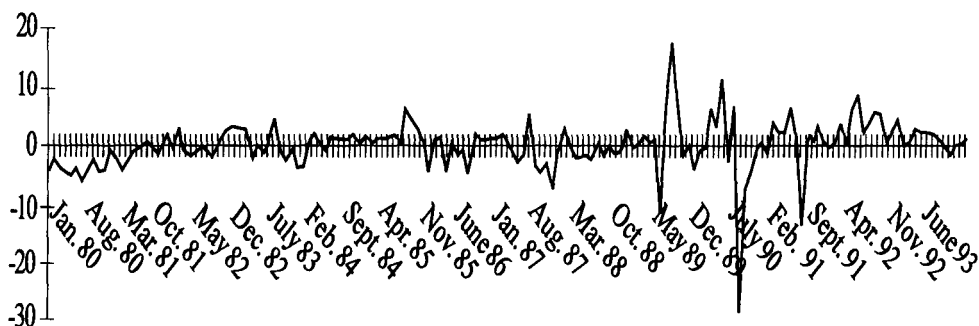
The formal results for the unit root test are contained in table 5. The τ_c statistic is smaller than the critical value, indicating that the real interest rate series is stationary.

We can then follow the rest of the procedure. Figure 3 plots real revenue, including interest payments, and real spending, including seigniorage, in millions of 1980 cruzeiros. The two series do not show an upward trend over the entire sample.
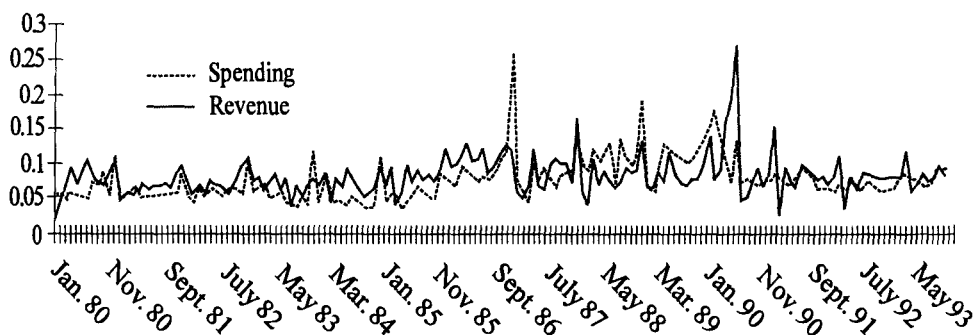
Table 5
Test for the stationarity of the real interest rates*
Jan. 1980 — July 1993

τ statistic	-3.93**
Lags	7
n observations	155

*The asymptotic critical values for τ_c at the 10% and 5%
levels are -2.57 and -2.86 respectively.
**The null hypothesis of non-stationarity is rejected.

Figure 3
Real spending (inclusive of interest payments) and
real revenue (inclusive of seigniorage)
Jan. 1980 — July 1993

The unit root tests are contained in table 6.

Table 6
Tests for a random walk*

Variables	Levels τ statistic	Lags	First difference τ statistic	Lags
Expenditure, including interest (GG)	-1.49	12	-4.64**	12
Revenue, excluding seigniorage (R)	-2.53	12	-5.31**	12
Revenue, including seigniorage (RR)	-2.45	12	-5.38**	12

*The asymptotic critical values for τ_c at the 10% and 5% levels are -2.57 and -2.86 respectively.
**The null hypothesis of non-stationarity is rejected.

We cannot reject the hypothesis of nonstationarity for real expenditures and revenues in levels. When measured as first differences, however, these variables are stationary. Then cointegration tests are adequate.[16]

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[16] *When we work with quarterly data, Hakkio & Rush's test cannot be implemented due to differences in the degree of stationarity of the revenues and expenditures series.*

The first cointegration test to be implemented is the one developed by Engle and Granger (1987) for the residuals of the cointegration regression. The results are given in table 7.

Table 7

Dickey-Fuller tests for cointegration*

Variables	τ statistic	Lags
R/GG	-2.87	12
RR/GG	-3.36**	12

*The asymptotic critical value at the 10% and 5% levels are -3.04 and -3.34 respectively.
**We can reject that revenue and spending are not cointegrated.

The results for real revenue and real spending indicate a violation of the government's budget constraint if revenue excluding seigniorage is considered. The cointegration test does not reject the hypothesis of noncointegration. The results are reversed when the change in the monetary base is considered as part of the government's revenue. In this case, the cointegration test indicates that RR and GG are cointegrated.

We also apply the likelihood ratio test for cointegration due to Johansen (1988). The number of lags in the VAR model is set equal to 8. A lag length of eight periods is selected to capture the main short-run dynamics in a parsimonious way.[17] It also ensures that the disturbances are as close to being Gaussian as possible. Table 8 reports the results.

Table 8

Johansen cointegration test

Variables	Trace statistics		Eigenvalue statistics	
	$H_0 : r \leq 1$	$H_0 : r = 0$	$H_0 : r \leq 1$	$H_0 : r = 0$
R/GG	4.6771	18.7344	4.6771	14.0573
RR/GG	4.4537	23.3600	4.4537	18.9083

Note: The Johansen statistics test the hypothesis of at most one and zero cointegrating vectors, respectively. The 5% critical value for $H_0:r \leq 1$ is 9.2430 for the eigenvalue and trace statistics, and for $H_0:r = 0$, it is 15.6720 and 19.9640 for the eigenvalue and trace statistics respectively.

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[17] *Gonzalo (1990) studies the effects on the MLE of using incorrect lag lengths for the short-run dynamics. He shows that too long a lag results in just a small loss of efficiency, and also that too short a lag implies that the MLE is no longer the best method.*

The hypothesis of zero cointegrating vectors is not rejected when revenue excluding seigniorage is considered. When revenue from money creation is included, the hypothesis of at most one cointegrating vector is not rejected. Therefore, the results from the Dickey-Fuller tests for cointegration are confirmed. If seigniorage is excluded, the government budget is not balanced in present value terms, the opposite happening if seigniorage is included.

We then turn to the second issue raised by Hakkio and Rush. Lets assume that the "equilibrium regression" is given by:

$$Y_t = a + bX_t + u_t$$

where Y_t is real revenue exclusive or inclusive of seigniorage, X_t is real spending inclusive of interest payments, a is the constant term, b the cointegration factor, and u_t the equilibrium error.

We should then verify if the cointegrating factor equals 1. One way to do so is to examine the stationarity of the difference between spending and revenue. The "deficit" constrains the parameters of the cointegrating regression to be $a = 0$ and $b = 1$. The results are given in table 9.

Table 9

Tests for the stationarity of the "deficit"*

Variables	τ statistic	Lags	n observations
GG/R	-2.81**	5	157
GG/RR	-4.20**	4	158

*The asymptotic critical values for τ_c at the 10% and 5% levels are -2.57 and -2.86 respectively.
**The null hypothesis of non-stationarity is rejected.

As observed before, the differences between spending and revenue are not compatible with the official measures of the deficit. So the same care used to analyze the previous results is also necessary here.

The evidence is consistent with the results obtained for the cointegration tests. If revenue from money creation is not included, the "deficit" is nonstationary (in fact, it is stationary but only at the 10% level), while if revenue from money creation is included, the "deficit" is stationary.

It seems, therefore, that seigniorage adjusts exactly to fulfill the government's budget constraint in Brazil. Seigniorage supplies the incremental amount of revenue necessary to finance the government. As pointed out by Sargent (1993a:253) "the central bank is operating openly as an effective agent of your internal revenue service". Only when seigniorage is included revenue and spending are cointegrated.

The importance of seigniorage as a source of revenue for the government can be seen in table 10. It shows seigniorage as a percentage of real government revenues. In some years, revenue from money creation reaches more than 1/3 of total government revenues.

Table 10
Government fiscal revenue and seigniorage

Period	Seigniorage*/real government revenue (%)
1980	14.74
1981	15.13
1982	19.57
1983	12.33
1984	22.49
1985	19.96
1986	33.70
1987	19.90
1988	29.47
1989	42.90
1990	39.93
1991	21.06
1992	27.29

*Seigniorage is measured as the real value of the change in high powered money.

4. Conclusions

In this paper we have implemented two tests of intertemporal budget balance.

The first one, developed by Trehan & Walsh (1991), examines the process followed by the stock of the debt. The second, developed by Hakkio &

Rush (1991), requires government expenditures inclusive of interest be cointegrated with revenues. Both tests were consistent in showing evidence that the Brazilian government budget is balanced in present value terms.

This seems to happen, however, not because the government systematically follows a strict Ricardian regime for servicing the debt it issues. In this regime, a positive value of interest bearing debt signals a stream of present discounted surpluses, but in Brazil this is only true if we consider revenue from money creation.

Therefore, it is more realistic to assume that the Brazilian government is following a deficit-financing regime like the one described by Sargent and Wallace (1981). The persistent path of fiscal deficits requires increases in base money in order to maintain solvency. Government debt is issued but it is in part repaid by the issuing of additional base money. The problem of coordinating the actions of the monetary and fiscal authorities in Brazil seems to be "solved" by the fiscal authority moving first and the monetary authority levying whatever "inflation tax" necessary to balance the budget. As predicted by Sargent and Wallace high, and in Brazil's case, variable inflation has resulted.

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