The Problem of Economic Development*

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Summary: 1. Introduction; 2. Some development facts; 3. The neoclassical growth model; 4. The augmented model of Mankiw, Romer & Weil; 5. A theory of differences in production functions; 6. Conclusion.
Key words: economic development; technology adoption.

Why is the average citizen in Sub-Saharan Africa reportedly 1/30 as rich as the average citizen living in the United States or Switzerland? This paper assesses the progress that the economics profession has made in answering this question. In it, I argue that a theory which has the property that differences in policies translate into differences in steady state income levels rather than growth rates is more appropriate for understanding the problem of development. I do not, however, argue that the Solow model, even augmented with human capital accumulated via education, is a theory of development. Instead, I argue that a theory which seeks to explain differences in production functions by stressing firms' decisions to adoption better technologies provides the best hope for understanding why some countries are so poor relative to others.

Por que o cidadão médio da África subsaariana tem 1/30 da renda de um norte-americano médio? Este artigo avalia os progressos feitos pela economia para responder a esta questão. Para o autor, uma teoria que preconiza que diferenças em políticas se refletem em diferenças de níveis de renda estáveis em vez de taxas de crescimento é mais apropriada para a compreensão do problema do desenvolvimento. Entretanto, o autor não nega que o modelo de Solow, mesmo incluindo o capital humano acumulado via educação, seja uma teoria de desenvolvimento. Em vez disso, o artigo argumenta que uma teoria que busca explicar as diferenças entre funções produtivas enfatizando a decisão das empresas de adotar tecnologias mais avançadas dá mais esperanças de se entender por que alguns países são tão pobres em relação a outros.

1. Introduction

Why are some countries so poor relative to others? The average citizen in Sub-Saharan Africa, for instance, is reported to be one thirtieth as rich as

*Paper received in July and approved in Aug. 1997. Much of this paper draws on a research agenda that Edward C. Prescott and I began several years ago. Many of the ideas expressed in it are the outcomes of numerous discussions we have had over the years on this subject. I wish to acknowledge this contribution. Conversations with Richard Rogerson and Jim Schmitz have also influenced my views.

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the average person living in the United States or Switzerland. The purpose of this paper is to assess the progress the economics profession has made in understanding why the whole world is not rich. What I argue here is that while the *new endogenous growth theories* (i.e., theories which predict that differences in preferences and policies translate into permanent differences in balanced path growth rates) may be useful in explaining growth in knowledge, or technology, over time, they are not particularly useful in understanding international income differences. Such theories are, of course, capable of generating any level of income disparity. All they require is enough time to pass. However, they do so at the expense of other dimensions of the data. What I argue here instead is that a theory which predicts that differences in policies translate into differences is steady state income levels but not growth rates is more appropriate for understanding the problem of development. The question, then, is what factor or factors gives rise to such large differences in steady state output levels?

Mankiw (1995) has made a similar point about endogenous growth theory. His candidate factors are physical capital and human capital accumulated via education and on the job training. Mankiw, Romer and Weil (1992) and Mankiw (1995) claim that the neoclassical growth model augmented with human capital can explain most of the disparity in per capita outputs across countries. Unlike Mankiw (1995), I do not argue that production functions are the same across countries and that differences in physical capital and education explain most of the observed international output differences. In fact, I argue the opposite. My view is that a theory of differences in production functions, or technologies, is precisely what is needed. Towards understanding these differences, I favor a theory which emphasizes the adoption of better technologies by firms. This is to say that I believe technology adoption by firms, and not technology creation, is the relevant issue for poor countries. A large number of better technologies are available by which a poor country can increase its output. It need not invent new ideas or reinvent existing ones. This is another reason why I do not believe that many of the new endogenous growth models are useful in the context of understanding economic development.

My goal in this paper is therefold. First, I wish to explain the reasons why I believe that the new endogenous growth theory is not particularly relevant for understanding international income differences. This is section 2 of the paper. Second, I wish to make clear why it is that an exogenous growth theory which emphasizes differences in physical capital stocks and/or differences in education fails as a theory of economic development. I do this in two sections: in section 3, I show that the neoclassical growth model fails as a theory of international income differences. In section 4, I examine Mankiw's claim that the neoclassical growth model augmented to include human capital accumulated via education is a theory of development. In this section, I draw on works by Benhabib & Spiegel (1994), Bils & Klenow (1995), Hall & Jones (1996), and Klenow & Rodriguez-Clare (1997), which cast serious doubt on Mankiw's claim. My third goal is to describe a model in which all countries grow at the same rate in steady state but where differences in production functions arise endogenously on account of firms' decisions to adopt better technologies. This is section 5 of the paper. The theory of technology adoption and development that I describe is one put forth by Ed Prescott and myself in 1994.

2. Some Development Facts

This section documents some of the key development facts over the 1960-88 period and a longer historical time period. Much of this section is taken from work joint with Ed Prescott. The source of the data for the 1960-88 period is the Summers and Heston 1995 Penn world tables (PWT5.6). The set of countries that make up the analysis includes those in the PWT5.6 with populations in excess of 1 million in 1972 and with observations of real output per worker for each of the years in the 1960-88 period. The period chosen in the longest for which observations in each year for most countries in the world are available. There are 101 countries in the PWT5.6 that satisfy these criteria. The source of the data for the longer historical time period is primarily Maddison (1991).[1]

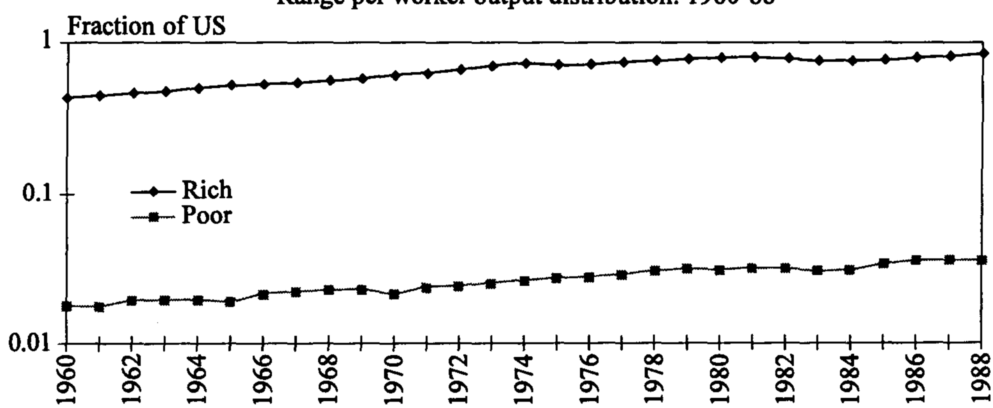
The development of the Summers and Heston data set has been without doubt among the most important ones in the last 15 years. It has allowed much more reliable comparisons of international income differences over a large set of countries and over a long period because it uses the same prices to value each country's goods. Prior to Summers and Heston, the only data

[1] *In Parente & Prescott (1993) nominal output per equivalent adult over the 1960-85 period was used.*

covering a large number of countries in the world for an extended period of time used exchange rates to convert each country's GDP in its own currency to a common currency unit (usually the US dollar). The problem with this approach is that it does not guarantee that the same price is used to weigh each country's quantity of a good, unless the law of one price holds on a good for good basis. As is well known, the law of one price does not hold across many individual goods, in particular nontradeables. The effect of this is to make the poor seem poorer than they really are because the relative price of non-tradeables to tradeables is typically lower in such countries. The work of Maddison is similarly noteworthy for avoiding this bias. Maddison does not cover such a large set of countries, but for the ones he does cover, he provides data going back as far as 200 years.

I begin by depicting the range of the distribution of per worker output in each of the years of the 1960-88 period. The range is defined as the average per worker output of the richest 5% of the countries in the distribution in each year and the average per worker output of the poorest 5% of the distribution in each year. The set of countries whose per worker outputs are used in the calculations change from one year to the next to the extent that countries move in and out of the top and bottom 5% of the distribution. For expositional purposes, the averages are expressed relative to the 1988 US level of per worker output.

Figure 1
Range per worker output distribution: 1960-88



There are several things to note. The first is that the distribution of output per worker has been shifting up over time. All but 15 countries (most of which experienced some type of war or civil unrest), were richer in 1988 than in 1960. The average country grew at an average annual growth rate of 2%, which is roughly the average annual rate of growth for the USA over this period. The second thing to note is that the measured disparity in per worker outputs is huge. However, it has changed very little in this postwar period. The five richest countries were on average 30 times richer than the five poorest countries in 1960. This is also the factor difference that existed in 1988. While the disparity in per worker output increased from 1960 to the early 1970s, reaching a peak factor difference of 37, it has fallen subsequently so that in 1988 it stood at its 1960 level.[2]

It is this last property, the fact that the range of the distribution has not changed much over the 1960-88 period, which is so difficult to reconcile with the new endogenous growth theories.[3] According to these theories, differences in preferences and policies lead to permanent differences in growth rates. Therefore, such theories predict that the range of the distribution should have widened over time.[4] Of course, three or four hundred years ago all countries had more or less the same level of per capita output. Some countries clearly began what Kuznets called modern economic growth before others. But over the postwar period, a period of modern economic growth for essentially every country in the world, both poor and rich grew at essentially the same rate.

Looking back at this longer historical pattern, it is interesting to note what has happened to the average per capita output differences between the West and the East. Figure 2 shows the long run performance of the average per capita output of the West relative to the East. Output is in 1985 US prices. The numbers that are plotted are the total output of the region (East or West) divided by that region's total populations. The estimates are not, therefore, an average of the per capita outputs of the countries in the region.[5] What figure 2 shows is that while the West grew rich first, the East has made

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[2] *Per capita and per equivalent adult measures actually show a slight widening, but it all occurs over the 1982-88 subperiod.*

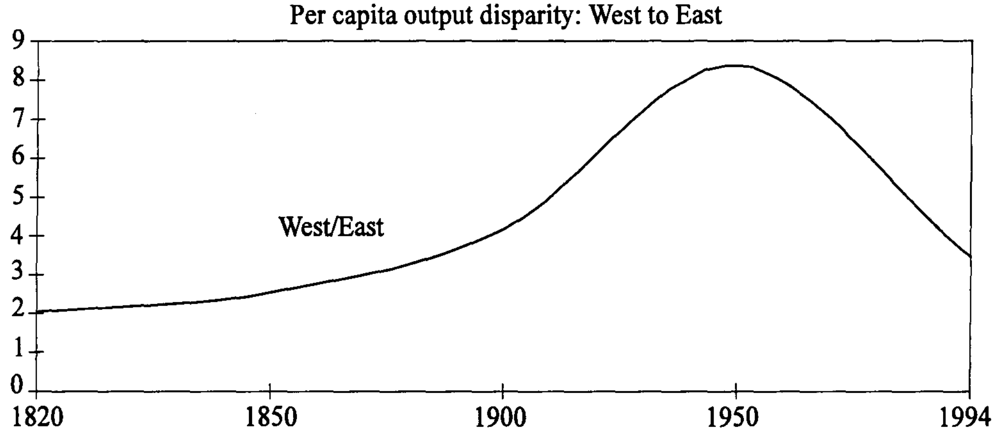
[3] *This is especially troubling for those theories which predict a poverty trap. The ten poorest countries in 1960 averaged an annual rate of growth of 1.7% over the 1960-88 period. All but two (Mali and Zaire) were richer in 1988 than they were in 1960.*

[4] *There are a few exceptions. See Rodriguez-Clare (1996) and Eaton & Kortum (1996).*

[5] *Countries included in the West are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Czechoslovakia, Denmark, Finland, France, Germany, Hungary, Italy, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK and USA. Countries included in the East are: Bangladesh, Burma, China, India, Indonesia, Pakistan, Philippines, South Korea, and Thailand. The data are taken from a variety of sources. The estimates, as well as the sources, are available by request.*

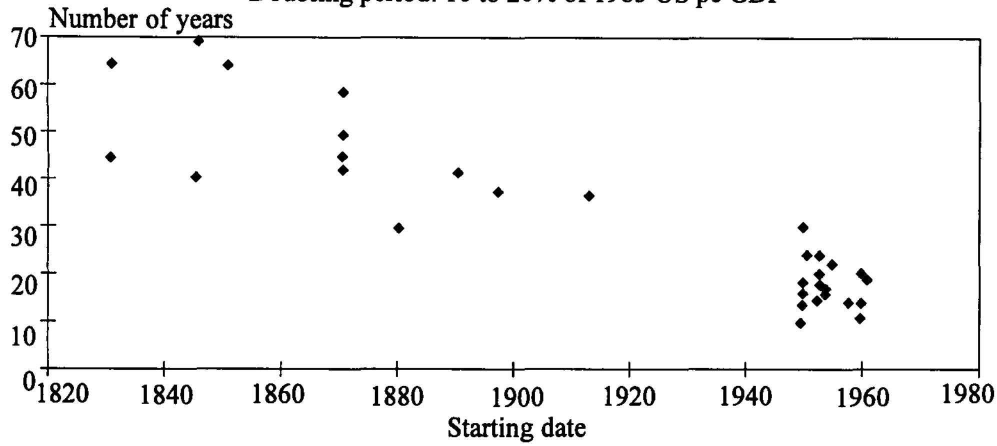
tremendous gains in the last 40 years. Two hundred years ago, the average citizen living in the West was roughly twice as rich as the average citizen living in the East. This difference increased to a factor 10 in 1950. Since 1950, the trend has begun to reverse itself so that today the average citizen living in the West is only four times as rich as the average citizen living in the East.

Figure 2
Per capita output disparity: West to East



The last property I wish to review appears in Parente & Prescott (1994). Figure 3 also presents a longer historical time period. It shows the time that individual countries took to go from one tenth of the 1985 US per capita output level to two tenths of that level. The figure plots the first year in which a country attained a per capita output level one tenth the 1985 US level against the number of years it took to double its per capita output. What the figure shows is that countries starting with this per capita output level after World War II were able to double their incomes in far less time than those countries that started with this level before World War II. I present this figure because it suggests that something or things have changed over time so as to allow countries in the postwar era to double their incomes in a decade. Prescott and I believe that the relevant change is the amount of technology available for countries to use. One hundred and fifty years ago a country which had one tenth the 1985 US per capita output level was probably very close to the technology frontier at that date. Today a country which has this level of income is probably very far from the current technology frontier.

Figure 3
Doubling period: 10 to 20% of 1985 US pc GDP



3. The Neoclassical Growth Model

What accounts for the huge disparity in international income levels? To answer this question, I begin with the neoclassical growth model. I begin with it because the model's steady state properties roughly match the long run growth properties of the US economy. It is also consistent with the fact that the range of the per worker output distribution has neither widened nor shrunk over the postwar period and the fact that the length of time countries have taken to double their incomes has decreased over the last one hundred and fifty years.

3.1 The economy

The standard approach is to view each country as a closed system and assume that agents are the same across countries with respect to preferences. Agents are infinitely lived and have utility in each period defined solely over consumption of a single good. Instantaneous utility is assumed to be of the CES variety so that the discounted stream of utility over an agent's life is

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma} \tag{3.1}$$

where β is the subjective time discount factor and σ is the intertemporal rate of substitution.

The key feature of the model is the Cobb-Douglas function assumed for producing output,

$$Y_t = A_t K_t^{\theta_k} L_t^{1-\theta_k} \tag{3.2}$$

where A_t is the technology level, or Total Factor Productivity (TFP), K_t is the aggregate physical capital stock, and L_t is the aggregate labor supply. Since agents do not value leisure, the aggregate labor supply is just equal to the economy's population. For expositional purposes, I abstract from population growth.

Technology is assumed to grow exogenously at rate γ. This is

$$A_{t+1} = (1+\gamma)\, A_t. \tag{3.3a}$$

Given that technology is exogenous and disembodied, it is standard to treat both its level and growth rate as the same across countries. Physical capital is accumulated from forgone consumption. The resource constraint for this economy is thus

$$L_t c_t + X_t = Y_t \tag{3.4}$$

where X_t denotes aggregate investment at date t.

3.2 Relevant policy

As the key decision in the model is with respect to capital accumulation, policies which affect investment are the natural ones to examine in trying to explain international income differences. One can easily imagine a long list of factors that may affect private agents' investment decisions. Two policies that have been studied extensively are capital income taxation and distortions to the relative price of investment in terms of consumption.

These two types of policies are theoretically equivalent with respect to steady state output levels. Nevertheless, there are strong reasons why I prefer to model policies that distort the relative price of investment goods. While the two types of policies have the same implication for steady state output levels, they have different implications for relative prices of investment goods and saving rates across countries. Specifically, income tax policies do not imply differences in relative investment prices but do imply differences in saving rates. Policies which distort the relative price of investment goods imply that all countries save the same fraction of their output.

Empirically, the price of investment goods differs significantly and systematically across countries while capital income tax rates do not. Jones (1994) documents the differences in the relative price of equipment across countries and the strong positive correlation between these relative prices and countries' per capita outputs. He reports a range of relative prices on the order of 1 to 4 across countries, where the US is assigned a price of 1.[6] In contrast, empirical analyses of tax rates have found little systematic variation in rates of capital income taxation and income per capita (e.g., Burgess & Stern (1993), Easterly & Rebelo (1993)).

Empirically, savings, or investment rates, show no systematic variation across countries. According to the *IMF's Yearbook of Monetary Statistics*, the average investment rate in the industrialized countries over the 1966-90 period was 20%. This is the average investment rate for the developing countries as well. In figure 4, I have plotted the ratio of the average investment rate of the industrialized countries to the average investment rate of the developing countries, which as can be seen, is roughly one over the period. The equality in savings rates is not an artifact of the Newly Industrialized Countries being lumped together with Sub-Saharan Africa or Latin America in the set of developing countries. Indeed, the average investment rate for Africa was greater than the average for the industrialized countries over most of the postwar period.

This may come as a surprise, particularly since across section growth rate regressions made popular by Barro report a positive and significant coefficient on a country's investment rate. But these growth rate regression studies do not use the IMF savings/investment rates. Instead, they use the Summers and Heston investment rates which adjust prices for purchasing power parity. As Jones (1994) documents, the amount of capital that a country can buy with its savings does show a systematic variation across countries. These purchasing power parity adjusted investment rates do vary systematically with income, being roughly twice as high in the industrialized countries. This ratio is also shown in figure 4. However, in terms of the fraction of an economy's output which is not consumed privately or publically, there is no systematic difference across countries.

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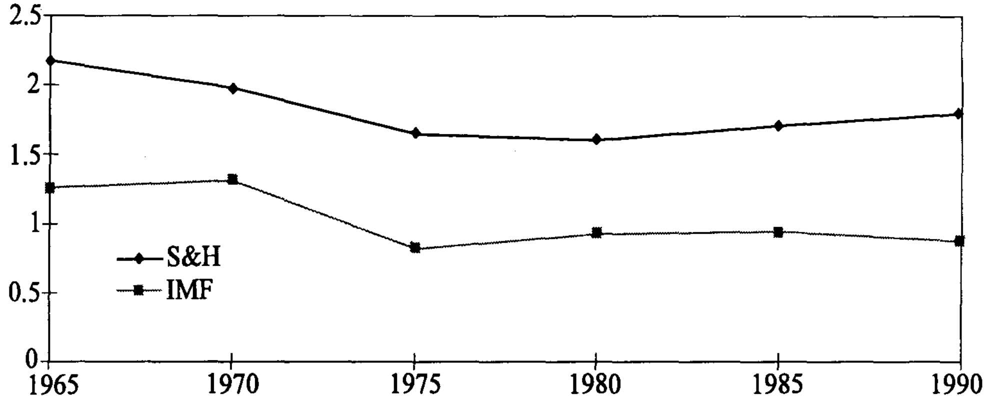
[6] *Restuccia & Urrutia (1995) report differences in the relative price of investment goods to consumption goods up to a factor 13 across countries from the Summers & Heston data.*

Figure 4
Investment rates ratio: industrialized to developing nations



I, therefore, conclude from these observations that it is more appropriate to consider policies which affect the price of investment goods relative to consumption goods. I model this by assuming that one unit of forgone consumption adds $1/\pi$ units of capital. The parameter π, thus, reflects the size of these distortionary policies. The law of motion for physical capital is thus described by

$$K_{t+1} = (1-\delta)\, K_t + \frac{X_t}{\pi}. \tag{3.5}$$

Following Parente & Prescott (1994), I refer to π as a barrier.

3.3 Steady state

It is well known that the growth rate of per capita output, consumption, investment, and capital along the balanced growth path are independent of preferences and policy parameters for this economy. Only the exogenous rate of technological change and the coefficient on labor in the production function enter. This growth rate is

$$1+g = (1+\gamma)^{1/(1-\theta_k)}. \tag{3.6}$$

Let lower case letters denote the corresponding per capita value of a variable. It is not difficult to solve for the steady level of per capita output and physical capital stock. They are

$$k_t^* = \Omega_1(1+g)^t \pi^{-1(1-\theta_k)} \tag{3.7}$$

and

$$y_t^* = \Omega_2(1+g)^t \pi^{-\theta_k/(1-\theta_k)} \tag{3.8}$$

where Ω_1 and Ω_2 are functions of the preference and technology parameters which do not vary across countries. Any difference in steady state output levels is the result of differences in barriers across countries.

3.4 As a theory of international income differences

The extent that differences in barriers generate large or small differences in steady state outputs levels depends only on the value of the coefficient in the production function, θ_k. None of the other parametric values matter for the level effect associated with barriers. As θ_k approaches one, the level effects associated with barriers become infinitely large. As θ_k approaches zero, differences in barriers have no consequences for steady state output levels.

The value which is typically assigned to θ_k is between 1/4 and 1/3. This is the range of estimated value for capital's share of income in the US economy. The reason θ_k is calibrated between 1/4 and 1/3 is that as long as factors are assumed to be paid their marginal products, θ_k in the model equals capital's share of income.

Taking the value of θ_k at the upper end of the large (i.e., $\theta_k = 1/3$) implies that a difference of a factor f in barriers across countries imply a factor difference in steady state per capita outputs equal to $f^{1/2}$. To generate a difference in steady state income levels of a factor 30, which is approximately the factor difference between the world's richest and poorest nations, the implied factor difference in the size of these barriers is 900. There is no evidence that such factor differences in the size of the distortionary policies exist. From Jones (1994), a plausible range for the difference in the size of the barriers is 4. A factor difference of 4 implies a factor difference in per capita output of only 2. The conclusion of this exercise is that some factor other than physical capital is needed to explain the huge observed differences in per capita output levels.

4. The Augmented Model of Mankiw, Romer & Weil

Mankiw, Romer & Weil (1992) (henceforth MRW) and more recently Mankiw (1995) claim that human capital is this other factor. MRW argue that the Solow model augmented with human capital can account for most of the observed disparity in per worker output across countries. For MRW, human capital is knowledge acquired by individuals through formal schooling and on the job training.

4.1 The economy

In the MRW model, production of output is given by

$$Y_t = A_t H_t^{\theta_h} K_t^{\theta_k} L_t^{1-\theta_k-\theta_h} \tag{4.1}$$

where H_t is economy wide human capital. For human capital, MRW assume that the production technology is the same as the technology for producing physical capital. They do not explicitely introduce barriers associated with investments in either type of capital. Human capital's law of motion is, thus,

$$H_{t+1} = (1-\delta)\, H_t + X_{ht}. \tag{4.2}$$

The basis for MRW's claim that differences in inputs across countries account for the majority of the observed international income differences is a regression of $\log(Y/L)$ on $\log(K/Y)$ and $\log(H/Y)$. The regression equation is derived by rewriting equation (4.1) as

$$\frac{Y_t}{L_t} = A_t^{\frac{1}{1-\theta_k-\theta_h}} \left(\frac{K_t}{Y_t}\right)^{\frac{\theta_k}{1-\theta_k-\theta_h}} \left(\frac{H_t}{Y_t}\right)^{\frac{\theta_h}{1-\theta_k-\theta_h}} \tag{4.3}$$

and then taking logs. The assumption in the regression is that the technology parameter is the same across countries. Using the log of 1985 per capita output from Summers & Heston with their constructed measures of physical capital and human capital to output ratios, they obtain estimates for the parameters $\theta_k = 0.31$ and $\theta_h = 0.28$ together with an $R^2 = 0.78$. From this exercise they conclude that the Solow model augmented to include human capital is a theory of international income differences.

4.2 Construction of capital stocks

What is crucial to the MRW findings is the capital stock estimates they use in their regression equation. MRW construct human and physical capital stocks measures using investment rate data and invoking steady state conditions. For physical capital, MRW take the average rate of physical capital investment over the 1960-85 period in Summres and Heston (PWT5.5) and use the steady state condition

$$\frac{K_t}{Y_t} = \frac{1}{(1+g)(1+\eta)-(1-\delta)} \frac{X_{kt}}{Y_t} \tag{4.4}$$

to derive each country's physical capital to output ratio, where η denotes a country's population growth rate. MRW do not abstract away from population growth, and this explains the presence of this term in equation (4.4). Equation (4.4) is derived from equation (3.5) using the steady state result that the aggregate capital stock tomorrow, $K_{t+1} = (1+\eta)(1+g)K_t$. In deriving these estimates, MRW use a depreciation rate of 3% and a steady state per capita ouput growth rate of 2%. The population growth rate, η, differs across countries according to Summers and Heston.

For human capital, MRW similarly estimate stocks from investment rates under the assumption that all countries are on their steady state paths. The equation which relates steady state investment rates to steady state human capital to output ratios is

$$\frac{H_t}{Y_t} = \frac{1}{(1+g)(1+\eta)-(1-\delta)} \frac{X_{ht}}{Y_t}. \tag{4.5}$$

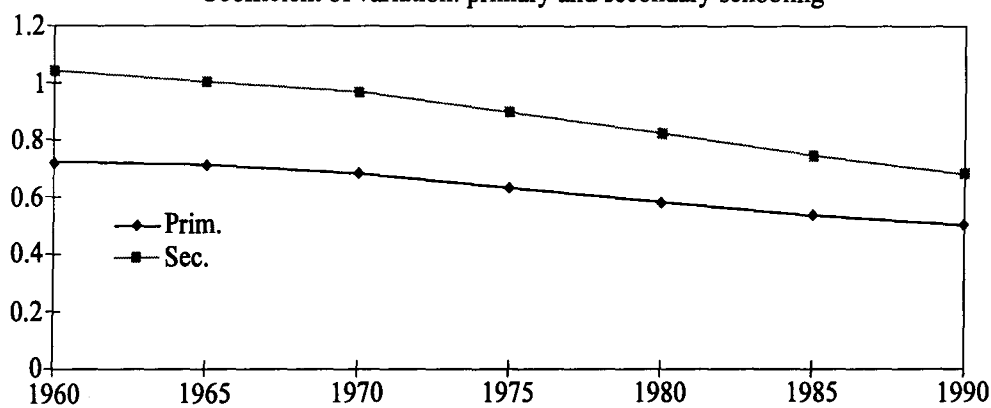
For X_{ht}/Y_t, MRW use the average ratio of secondary school students to the working age population over the 1965-85 period from the *Unesco Yearbook.* These are the capital stock numbers that MRW use in their regression analysis.

4.3 Criticisms of MRW capital stocks

It is precisely the MRW human capital stock measures, or perhaps, better said, the methods in which these capital stocks are constructed, which have been most heavily criticized. Two specific criticisms have been made. The first involves the use of secondary school students to the working age population reported by Unesco as the proxy for human capital investment. To be more precise, the criticism is that MRW ignore primary education. Ignoring primary education is important to their regression result because secondary enrollment rates vary by more than primary enrollment rates. Consequently, the human capital stock measures MRW construct show greater disparity than ones that use both primary and secondary enrollment rates.

To show this I have plotted in figure 5 the time paths of the coefficient of variation for average years of primary schooling in the population and average years of secondary schooling in the population over the 1960-90 period. Both averages are taken from Barro & Lee (1993). Throughout the 1960-90 period, the coefficient of variation for average years of secondary schooling in the population was about 1.5 times greater than the coefficient of variation for average years of primary schooling in the population. Additionally, it should be noted that both disparity measures show a decrease over time. No such decrease in the disparity of per worker output over the 1960-88 period is observed. In fact, the standard deviation of the logarithm of per worker output increased from 0.96 to 1.10 over the 1960-88 period.

Figure 5
Coefficient of variation: primary and secondary schooling



How does the model's explanatory power change when measures of human capital that reflect both primary and secondary schooling are used? Benhabib & Spiegel (1994) and Klenow & Rodriguez-Clare (1997) investigate this question. Benhabib & Spiegel (1994) use average years of schooling for the population from the Barro & Lee data set, as well as from a data set constructed by George Koyraciou, as their measures of the human capital stock. Klenow & Rodriguez (1997) use only the Barro & Lee numbers. Both sets of authors rerun the MRW regression using 1985 output. Benhabib & Spiegel construct physical capital stock measures from past investment rates in Summers & Heston (PWT5.5) while Klenow & Rodriguez-Clare use physical capital stocks provided in Summers & Heston (PWT5.6). When Klenow & Rodriguez-Clare rerun the MRW regression, they obtain an R^2 of 0.48. This is compared to the R^2 of 0.78 found by MRW. When primary schooling is included in the estimates of human capital, differences in inputs account for less than half of the disparity in international output differences.

Benhabib & Spiegel (1994) also repeat the MRW analysis for 1965. There, they find that a country's human capital is insignificant in explaining its 1965 output. Benhabib & Spiegel go on to regress the difference of log Y on the difference of log H, the difference of log K, and the difference of log population, L, between 1965 and 1985. Independently of the data set they use, they find the coefficient on human capital is negative and insignificant. There was little relation between countries' output per worker growth rates and education growth rates over the 1965-85 period. This is more evidence against education playing the key role in the development process. Of course, one could always argue that the Barro & Lee estimates do not accurately measure education attainments or do not reflect quality differences in education.

The second major criticism of MRW's analysis is the assumption that human capital is produced according to the same technology as physical capital. Bils & Klenow (1995) and Hall & Jones (1996) assume alternative human capital production technologies. The essential difference between these authors' approaches and the MRW approach is that the incremental effect of an additional year of schooling on one's human capital is not independent of one's current human capital stock. More specifically, additions to an individual's human capital stock become more expensive as his capital stock increases. For instance, Bils & Klenow assume that in addition to one's own years of schooling, an individual's experience and his teacher's human capital determine his human capital stock. Compared to the Barro & Lee measures, the resulting his human capital stock measures of both Bils & Klenow and Hall & Jones show far less disparity, as effectively secondary and post secondary years of schooling receive smaller weights in the construction of these measures. This can be seen by comparing the coefficients of variation for the distribution of human capital from Barro & Lee human capital measures is 0.57. For the Bils & Klenow measures, the coefficient of variation is 0.26.

How do these alternative capital stock measures change the explanatory power of the augmented Solow model? Hall & Jones (1996) calculate countries' TFPs by using output and physical capital stock estimates from the Summers & Heston PWT5.6 and their human capital stocks estimates. Klenow & Rodriguez-Clare (1997) effectively do the same using the Bils & Klenow human capital stock estimates. Both sets of authors find a strong correlation between TFP residuals and per capita output levels. Klenow & Rodriguez-Clare report a correlation between the log of TFP and the log of per capita output of 0.90 in 1985. The findings in these two papers suggest that a theory of differences in production functions, or technologies, is precisely what is needed to understand the problem of economic development.

As to MRW's claim of the opposite, I might add that in their construction of physical capital stocks there is an implicit assumption that production functions differ across countries. The assumption is not with respect to the production function of output, however, but rather with respect to the production function of physical capital. Recall, MRW use physical capital investment rates from Summers & Heston. The Summers & Heston numbers, of course, correct for price differences of investment goods across countries. By using the Summers & Heston investment rates, MRW admit that one unit of forgone consumption results in different increases in physical capital stocks in different countries. In terms of MRW's model, the use of investment rates that make no price adjustments, such as those that appear in the *IMF's Yearbook of Monetary Statistics*, actually seem more appropriate. However, as I have already pointed out, these show no systematic variation between rich and poor countries over the postwar period. Now, I am not trying to suggest here that one should choose the IMF investment rates over the Summers & Heston investment rates if one is attempting to construct capital stock measures. I merely wish to point out is that these investment price differences may be important to understanding international income differences. These price differences are certainly consistent with the existence of different production functions across countries. A theory of such differences is presented in the next section.

5. A Theory of Differences in Production Functions

Ed Prescott and I in 1994 put forth a model in which production functions differed across countries. These differences in production functions arise because of firms' decisions to adopt better technologies. Critical to our theory is the assumption that the adoption of a better technology requires an investment by the firm making the adoption. This is a different view than Romer (1990), which takes technology as non-rival and partially excludible.

Differences in barriers result in firms in different countries using different technologies, and these differences imply large differences in per capita outputs.

Rather than present our exact 1994 model, let me present a simplified version. All essential features remain. The model is first simplified by assuming away the government sector. The economy thus consists of a household and business sector. The model is further simplified by assuming that household utility is defined only over consumption of a final good and leisure. The discounted stream of utility of a household is thus,

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \phi \log(1 - h_t)\right] \tag{5.1}$$

where h_t denotes hours worked. In Parent & Prescott (1994), utility was also defined over services generated from household durables. Government spending is nearly 20% of US output reported in the national accounts, and the stock of household durables is nearly as large as the stock of physical capital in the business sector. The exclusion of government policy and of the stock of household durables does have some implications with respect to the model's calibration. In particular, their exclusions require that certain adjustments be made to the national income and product accounts so as to match the model with the data. However, with these adjustments, their exclusions have very little importance with respect to the quantitative predictions of the model for international income differences.

The labor/leisure decision cannot be abstracted from. The inclusion of leisure in the utility function is a notable difference between our model and other growth models. We cannot and do not abstract from this decision because it has important implications for the speed of convergence to the steady state. As I shall show, the transitional dynamics of the model are used to tie down the value of a parameter which has important implications for differences in steady state outputs. The disparity in output implied by the model for any given disparity in barriers would be far different were we to abstract from this decisions. This is the major reason we do not abstract from this decision. Additionally, countries such as Japan, West Germany, and France, that seemed to have been converging to higher steady states, did experience decreases in the average length of the manufacturing workweek. This decrease in the workweek is consistent with the transitional dynamics of an exogenous growth model in which utility is also defined over leisure.

The other important difference between the model I present here and our 1994 paper is that here I assume that barriers also apply to investment in business physical capital. In Parent & Prescott (1994), barriers applied only to investments associated with technology adoption. The law of motion for per capita physical capital is thus,

$$k_{t+1} = (1 - \delta_k)\, k_t + x_{kt}/\pi_k \tag{5.2}$$

where π_k is the size of the barrier associated with investments in physical capital. Households are assumed to own this capital, make investments in it, and rent it to the business sector.

Production units in the business sector are plants. The output of a plant depends on the technology it uses, the number of workers it hires, the physical capital it rents from the household sector, and the number of hours the plant is operated. There is a minimum number $\bar{N}$ of workers that is required to operate a plant. Above this number, additional workers make no contribution. The production function for a plant that is operated for h_t hours and that employs at date t at least $\bar{N}$ workers is

$$Y_t = h_t A_t \bar{N} K_t^{\theta_k} \qquad 0 < \theta_k < 1. \tag{5.3}$$

The functional form of the production technology for this economy warrants some discussion. Basically, the functional form allows us to maintain the perfectly competitive paradigm. The commodity space has many commodities. Workweeks of different lengths are different commodities, and plants with different technologies have different types of technology capital. Thus, there is a continuum of different types of both labor and technology capital. Given certain restrictions on parametric values, there is an optimal size firm. As the size of the economy increases, the number and not the size of plants increases. A doubling of every input results in a doubling of the number of plants in the economy and hence a doubling of aggregate output. The aggregate production possibilities set is subject to constant returns to scale.

In order for a plant to adopt a better technology, that plant must make an investment. The key element of our theory is the technology which determines the amount of investment a plant in a particular country must make in order to adopt a better technology. There are two key features to this investment technology. The first feature is the barriers to technology adoption, π_A, that exist in each country. Here, I allow for the possibility that the barrier size differs between types of investments in a given country. The second is the amount of scientific and general knowledge in the world. As this world knowledge increases, the amount of investment needed by a firm to adopt a better technology falls. Our motivation for this second feature is figure 3, which documents the decrease in the length of time countries have taken to double their per capita output. Our view is that countries that started the development process later were able to double their per capita outputs in less time because the set of available technologies to adopt has increased over time.

World knowledge in the model is taken as exogenous. We are interested in the problem of development, not the problem of growth in the richest countries. Firms in poor countries do not engage and R&D activities. From their standpoint, the assumption of exogenous world knowledge is appropriate. Let W_t denote the stock of world knowledge at date t. Then

$$W_{t+1} = W_t(1 + \gamma). \tag{5.4}$$

Since by assumption world knowledge at each date is the same for all countries, a property of the model is that all countries grow at the same rate in steady state.

Formally, the amount of investment, X_{At}, needed to go from technology A to A', $A' > A$, is

$$X_{At} = \pi_A(\alpha + 1)\int_A^{A'} \left(\frac{s}{W_t}\right)^{\alpha} ds. \tag{5.5}$$

While the barriers associated with investments in technology adoption, π_A, are country-specific, the parameter α is not. Its value is crucial to the model's predictions for international income differences. The parameter α affects the increase in technologies associated with any given investment.

To understand this, it is useful to interpret the value of the sum of a firm's past investments, $\{X_{As}\}_{s=0}^{t}$, as that firm's date t technlogy capital stock. For this purpose, define technology capital at date t as $Z_t \equiv A_t^{\alpha+1}/W_{t-1}^{\alpha}$, and integrate equation (5.5). Substituting Z into this integrated equation yields the law of motion for technology capital,

$$Z_{t+1} = (1 + \gamma)^{-\alpha} Z_t + X_{At}/\pi_A. \tag{5.6}$$

Effectively, for the same sequence of past investments, and hence the same date t stock of technology capital, a larger value for α implies a smaller current technology level, A_t. Smaller values of α, therefore, make technology investments more productive. Thus, a given difference in the size of the barriers to technology adoption imply bigger differences in technologies used in countries when α is smaller.

There are three other properties of this technology worth pointing out. The first is that more advanced technologies require greater amounts of investment to adopt (i.e., $\partial X_A/\partial A' > 0$). The second is that as world knowledge increases the amount of investment that a firm must make to adopt a more advanced technology decreases (i.e., $\partial X_A/\partial W < 0$). Since the adoption of any technology requires a smaller investment with growth of world knowledge, the value of a firm's past investment falls. In other words, growth of world knowledge effectively depreciates the technology capital stock of a firm. This is the meaning of the term $(1+\gamma)^{-\alpha}$ in equation (5.6). For this reason, I define $(1-\delta_z) \equiv (1+\gamma)^{-\alpha}$. The last property is that while the amount of investment needed to adopt a better technology decreases in the firm's current technology (i.e., $\partial X_A/\partial A < 0$), the size of these decreases become smaller (i.e., $\partial^2 X_A/\partial A^2 < 0$). In this respect there is an advantage to being technologically behing, ceteris paribus. Another way to say this is that if two firms in a country were to make the same investment then the firm with the higher current technology would still have in the next period a higher technology level, but the technology gap between the two would decrease. In what follows, it will be useful to use X_{zt} to denote X_{At}.

5.1 Steady state

In steady state, per capita output, physical capital, physical capital investment, technology capital investment, and consumption all grow at the same rate. Let lower case letters denote the per capita value of a variable. As in the neoclassical growth model, this growth rate is entirely independent of policy or preference parameters. It depends only on the growth rate of world knowledge, γ, and technology parameters, α and θ_k. This rate of growth is equal to

$$1+g = (1+\gamma)^{(1-\theta_z)/(1-\theta_k-\theta_z)} \tag{5.7}$$

where $\theta_z \equiv (\alpha+1)^{-1}$. In the case that $\pi_k = \pi_A = \pi$, the steady state per capita output, y_t, for this economy is just

$$y_t^* = (1+g)^t \Lambda \pi^{-(\theta_k+\theta_z)/(1-\theta_k-\theta_z)}. \tag{5.8}$$

The derivations of the steady state growth rates and levels are left to the appendix. The term Λ is a function of parameters β, γ, δ_k, α, ϕ and θ_k. As we assume that countries differ only in the size of their barriers, Λ does not differ across countries. Therefore, the extent that differences in policies translate into differences in steady state output levels depends on the values of θ_z and θ_k. Therefore, whether this theory can or cannot account for the huge observed international income differences depends on the sum of θ_k and θ_z. Larger factor differences in barriers are associated with larger factor differences in steady state per capita outputs the larger is the sum, $\theta_k + \theta_z$.

There is an important point to consider in trying to determine values for θ_k and θ_z. This point is that investments in technology capital are not part of the National Income and Product Accounts (NIPA). There is no technology capital investment rates or technology capital's share of output in the NIPA data to tie down a value for θ_z. For this reason the calibration of the model is not trivial. In the next section, I explain in some detail the steps needed to calibrate values for θ_z and θ_k.

5.2 Calibration

The difficulty in calibrating this model lies in the fact that current accounting procedures treat investments in technology capital as ordinary business expenses. To explain how a value for θ_z is determined it is necessary to describe how values for all other parameters are tied down. The other parameters of the model for which values must be determined are γ, (or δ_z), θ_k, δ_k, β, π, and ϕ. Given a value for θ_z, values for all other model parameters can be pinned down from US steady state observations. This section describes how values for the model parameters are determined.

First, however, it is necessary to reorganize the NIPA data around the model. There are two major adjustments. The first is to associate output in the US national accounts, y^{us}, with output less investments in technology capital in the model, $y - x_z$. This is necessary as investments in technology capital are treated as an ordinary business expense according to current accounting practices. Consequently, output in the model differs from NIPA output by the amount of investment in technology adoption. The second major adjustment is the treatment of government expenditures. This is of course necessary here in this version of the model because I have abstracted from government policy. It is standard to allocate a fraction of government expenditures as part of business physical capital investment and the rest as consumption. This adjustment yields a ratio of investment in physical capital to NIPA output, (x_k/y^{us}) equal to 0.20.

Given a value of θ_z, values for the remaining parameters can be determined from the following US growth observations:

(i) an average annual per worker output growth rate (g) of 2% per year;
(ii) a real rate of interest (i) of 4.5% per year;
(iii) a capital to NIPA output ratio (k/y^{us}) of 2.5, and
(iv) a fraction of non-personal and non-sleep time spent worker (h) of 0.40.

Given these observations and a value for θ_z, values for δ_k, ϕ, θ_k, β, and γ are determined by solving the following system of equations:

$$\phi \frac{c}{1-h} = w'(h) \tag{5.9}$$

$$\beta^{-1}(1+g) = 1+i \tag{5.10}$$

$$w'(h) = \frac{y}{h} \tag{5.11}$$

$$(i+\delta_k)\frac{k}{y} = \theta_k \tag{5.12}$$

$$(i+\delta_z)\frac{z}{y} = \theta_z \tag{5.13}$$

$$\frac{k}{y^{us}} = \frac{x_k}{y^{us}}(g+\delta_k)^{-1} \tag{5.14}$$

$$\frac{z}{y} = \frac{x_z}{y}(g+\delta_z)^{-1} \tag{5.15}$$

$$1+g = (1+\gamma)^{(1-\theta_z)/(1-\theta_k-\theta_z)} \tag{5.16}$$

$$(1-\delta_z) = (1+\gamma)^{-(1-\theta_z)/\theta_z} \tag{5.17}$$

$$y^{us} = y - x_z. \tag{5.18}$$

Equations (5.9) and (5.10) follow from utility maximization and the result that in steady state consumption grows at rate g. Equations (5.11)–(5.13) follow from profit maximization. Equations (5.14) and (5.15) are the steady state laws of motion for the two capital stocks. Equation (5.16) is the steady state growth rate of the economy. The last two equations are definitional. The value of π in the United States is only important to the extent that it determines the units in which output and technology capital are measured. Without loss of generality the units are chosen so that π is 1 in the United States. This is the reason it does not appear in any of the above equations. Subsequently, π refers to the size of a country's barriers relative to the United States.

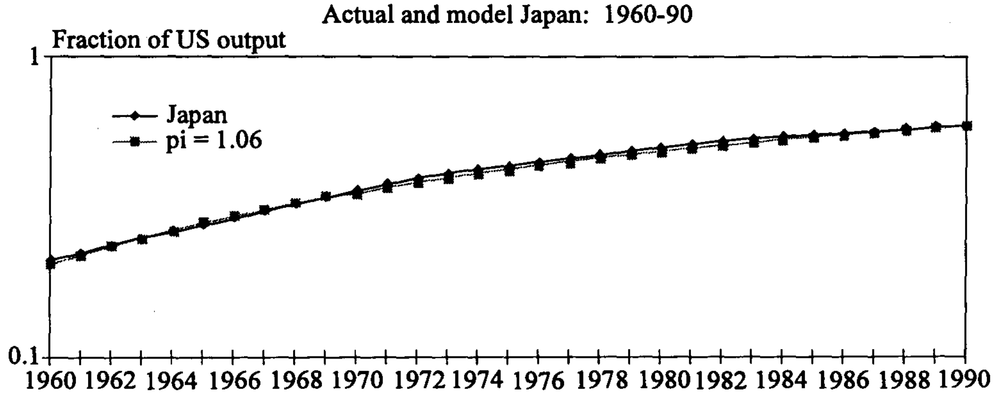
It is important to note that in the calibration it is not possible to use physical capital's share of NIPA income to tie down the value of θ_k. Since output equals income, the fact that there is unmeasured output means that there is unmeasured income. It is not clear what form this unreported income takes or how it is divided among workers and shareholders. Consequently, reported factor shares of income cannot be used in the calibration of this model. In the standard neoclassical growth model, there is no unmeasured income and so it is appropriate there to use physical capital's share of income in the calibration. This explains why we use the physical capital to NIPA output ratio observation for the United States instead in the calibration.

Since the steady state properties of the model are not sufficient to tie down a value for θ_z, we exploit the model's non-steady state properties. The value for θ_z has important implications for the model's transitional properties. For any given size barrier, smaller values for θ_z imply smaller steady state technology capital stocks, and hence faster convergence to the steady state. The idea is that if we take an observation of a country that appears to be transitioning to a new steady state, we can find a value for θ_z so that the model's transitional dynamics match the path of this economy. For our non-steady state observation we used the postwar development miracle of Japan. In 1960, the average worker in Japan was 21% as productive as the average US worker. In 1990, the average Japanese worker was 60% as productive as the average US worker.

Of course, along the transitional path, the speed of convergence depends on how far the economy starts from its steady state. The steady state depends importantly on the size of the barriers in the country. Thus, for any given initial output level, the speed of convergence over a particular period depends not only on the value of θ_z but also on the size of the barriers. For any given value for θ_z, it is possible to find a relative barrier size, π, as well as initial capital stocks so that the model matches Japan's beginning and ending per worker output levels. In effect, this is to say that there are many pairs (θ_z, π), for which the transitional path of the model matches Japan's 1960 and 1990 per worker output levels. If there were direct measurement of the size of the relative barriers in Japan and in the United States then a unique value for θ_z could be found. But there is no such measurement. Still, there is a way to eliminate most pairs of values as being implausible. This is possible because most pairs predict either too fast or too slow convergence over the first few years of the period, and the opposite for the last few years of the period.

The value of θ_z for which the model predicts neither too fast nor too slow convergence in the first or last few years of the 1960-90 period relative to the actual Japanese economy is $\theta_z = 0.50$. The associated relative size of barriers for our model Japan is 1.06. Figure 6 displays the path of per worker output relative to the United States for both Japan and the model over the 1960-90 period. The data for Japan has been smoothed using the Hodrick-Prescott filter. For this value of θ_z, the values of the parameters which are calibrated to the above US steady state growth observations are listed in table 1.[7]

Figure 6
Actual and model Japan: 1960-90



[7] *Prescott and I actually calibrated θ_z to 0.55 in our JPE paper. Part of the reason that the value of calibrated θ_z is different is that I have calibrated to the path of Japan's per worker output in the PWT5.6. Before, we calibrated to path of per capita output from the Summers & Heston PWT5.5. Additionally, Prescott and I allowed for the possibility that barriers in Japan were not constant over the postwar period.*

Table 1
Calibrated model parameters

$\phi = 2.57$	$\theta_k = 0.19$
$\beta = 0.98$	$\theta_z = 0.50$
$\gamma = 0.01$	$\alpha = 1.00$
$\delta_k = 0.06$	$\delta_z = 0.01$

5.3 International income differences

Given calibrated values for θ_z and θ_k, the model's implications for international income differences are easily determined. For calibrated values $\theta_z = 0.50$ and $\theta_k = 0.19$, relative barriers on both physical capital and technology capital investments, π, imply a factor difference in steady state per worker outputs between a country and the United States of a factor $\pi^{-2.25}$. This follows trivially from equation (5.8). Given that the parameter π in the United States is normalized to one, the calibrated barrier for Japan implies that the Japanese economy is converging to a per worker output 88% of the US level. For the model to match the disparity in per worker output that exists between the world's richest and poorest countries the implied disparity in barriers must be slightly less than 4.5. This is just slightly larger than the range of the disparity in equipment prices reported by Jones (1994). This model can account for the huge observed international income differences.

6. Conclusion

The last ten years has witnessed a tremendous outpouring of work in the area of economic development. My own view is that this outpouring has brought about an improvement in our understanding of the problem of economic development. I take the fact that fewer and fewer economists are now trying to explain the huge observed international income disparity by writing down endogenous growth models and that more and more economists are working with models that have the property that all countries grow at the same rate in steady state and that emphasize the adoption decision of better technologies as evidence of this enhanced understanding (e.g., Chari, Kehoe, & McGrattan (1997), Hall & Jones (1996), Jovanovic & Rob (1996)). This is not to say that endogenous growth theory has not made a major contribution. It has. It has helped us understand why there is growth in knowledge in the world and growth of per capita output in the industrialized countries. It is just not so relevant for understanding international income differences.

As far as I see it there are several issues that remains. The first is the large predicted amounts of unmeasured output that is implied by a model of technology adoption. For calibrated Parente & Prescott economy, unmeasured output turns out to be 37% of NIPA output (i.e., $x_z/(y-x_z) = 0.37$). Some of this unmeasured investment takes the form of on the job training and learning by doing. Determining if these unmeasured investments are as large as the model predicts is an important challenge that remains for this theory.

Even if this unmeasured investment does not turn out to be this large, there are several ways to reconcile the model with the data. One possible way is to assume that the adoption of better technologies requires the purchase of new machines. Thus, investment in physical capital serves two purposes. Both Jovanovic & Rob (1996) and Rodriguez-Clare (1996) consider models where new technologies require the purchase of new machines. Neither model is capable of generating large level effects associated with barriers, however. Another line which I have pursued with Richard Rogerson & Randy Wright (1997) is to allow for home production as well as market production. Our preliminary finding is that with home production the sum of reproducible capital's share does not have to be as large for the model to match the observed disparity in per capita outputs. Thus, the model predicts far less unmeasured investment in the market sector. It, however, implies large unmeasured output in the home. It also predicts that the true difference in outputs across countries is smaller than the reported differences in the data. In the Parente & Prescott model, the measured differences are as large as the true differences.

The second issue relates to identifying the barriers which prevent the adoption of better technologies. Prescott and I (1997) consider how arrangements which bestow monopoly rights on coalitions of factor supplies affect technology adoption. Not only to we find that such rights can prevent the adoption of superior technologies, but we also find that they can result in the inefficient operation of existing technologies. We show that the existence of monopoly rights can translate into large differences in production functions and output. The work also suggests how difficult it is to get groups to relinquish such rights, thereby creating the opportunity for a country to become rich.

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Appendix

Steady State Derivation for Parente and Prescott Model

To simplify the analysis, I consider the decision to adopt a more advanced technology as one of accumulating technology capital, Z_t. I begin by substituting Z_t for A_t in the plant production technology with the optimal size firm, $\bar{N}$. With these substitutions, equation (5.3) becomes

$$Y_t = h_t(1+\gamma)^{(1-\theta_z)(t-1)}\, W_0^{1-\theta_z} Z_t^{\theta_z} K_t^{\theta_k} \bar{N}.$$

In the above equation, I have written $W_t = W_0(1+\gamma)^t$.

For expositional purposes, and without loss of generality, I assume in the derivation here that households own the technology capital and physical capital and rent them to plants. With these assumption, the plant maximization problem is static and is given as

$$\max_{(h_t,K_t,Z_t)} \left\{h_t(1+\gamma)^{(1-\theta_z)(t-1)}W_0^{1-\theta_z}Z_t^{\theta_z}K_t^{\theta_k}\bar{N} - r_{zt}Z_t - r_{kt}K_t - w_t(h_t)\bar{N}\right\},$$

where r_{zt} is the rental price of technology capital at date t, r_{kt} is the rental price of physical capital at date t, and $w_t(h_t)$ is the rental price of a worker which works a workweek of length h_t. The first order necessary conditions for profit maximization are

$$w_t'(h_t)\bar{N} = \frac{Y_t}{h_t}$$

$$r_{zt} = \theta_z\frac{Y_t}{Z_t}$$

$$r_{kt} = \theta_k\,\frac{Y_t}{K_t}.$$

If the population size is L, then in equilibrium there will be $L/\bar{N}$ such plants. Thus, the economy wide technology and physical capital stocks are $Z_tL/\bar{N}$ and $K_tL/\bar{N}$, and the per capita capital stocks are $Z_t/\bar{N}$ and $K_t/\bar{N}$. Denote per capita output, technology capital, and physical capital as y_t, z_t, and k_t respectively. Aggregating across plants to get total output and then dividing by the population yields the following per capita output production relation

$$y_t = h_t(1+\gamma)^{(1-\theta_z)(t-1)}W_0^{1-\theta_z}z_t^{\theta_z}k_t^{\theta_k}\bar{N}^{\theta_k+\theta_z}$$

By choosing the units in which output is measured, namely the value of W_0, we can express y_t as

$$y_t = h_t(1+\gamma)^{t\theta_z}z_t^{\theta_z}k_t^{\theta_k}. \tag{1}$$

In per capita terms, the first order necessary conditions for profit maximization are

$$w_t'(h_t) = \frac{y_t}{h_t}, \tag{2}$$

$$r_{zt} = \theta_z\,\frac{y_t}{z_t}, \tag{3}$$

$$r_{kt} = \theta_k\,\frac{y_t}{k_t}. \tag{4}$$

With the assumption that the household owns the physical and technology capital, the representative household's problem is to choose a sequence $\{c_t, h_t, k_{t+1}, z_{t+1}\}_{t=0}^{\infty}$ so as to

$$\max \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \phi \log(1 - h_t)\right]$$

subject to the following intertemporal budget constraint

$$\sum_{t=0}^{\infty} R_t^{-1} \left[w_t(h_t) + r_{zt} z_t + r_{kt} k_t + \pi_k (1 + \delta_k) k_t + \pi_A (1 + \delta_z) z_t - c_t - \right.$$
$$\left. - \pi_k k_{t+1} - \pi_A z_{t+1}\right],$$

where R_t^{-1} is the Arrow-Debreu date 0 price of the date t good, i.e.,

$$R_t^{-1} \equiv \prod_{s=0}^{t-1} (1 + i_s).$$

The first order necessary conditions for utility maximization are

$$\beta^{-1} \frac{c_{t+1}}{c_t} = 1 + i_t, \tag{5}$$

$$\phi \frac{c_t}{1 - h_t} = w_t'(h_t), \tag{6}$$

$$r_{kt+1} = \pi_k [i_t + \delta_k], \tag{7}$$

$$r_{zt+1} = \pi_A [i_t + \delta_z]. \tag{8}$$

Equations (1)–(8) together with

$$k_{t+1} = (1 - \delta_k) k_t + \frac{x_{kt}}{\pi_k}, \tag{9}$$

$$z_{t+1} = (1 - \delta_z) z_t + \frac{x_{zt}}{\pi_A}, \tag{10}$$

and the resource constraint for the economy

$$y_t = c_t + x_{kt} + x_{zt} \tag{11}$$

completely characterize the competitive equilibrium for this economy.

The steady state solution is determined by invoking the steady state condition that the length of the workweek is constant and that each variable grows at a rate which does not vary over time. From equation (5) it immediately follows that the real interest rate is constant at each date. Equations (7) and (8) imply that the rental price of technology capital, r_{zt}, and physical capital, r_{kt}, are constant in the steady state. From equations (3) and (4) it follows that y_t, z_t, and k_t all grow at the same rate g. Using this result and equation (1) yields the steady state growth rate given by equation (5.7).

The steady state per capita output level is determined by solving for i in equation (5) using the steady state result that $c_{t+1}/c_t = 1 + g$. From (7) and (8), values for r_k and r_z are determined. Using these values, equations (3) and (4) can be used to solve for z as a function of k, $z = z(k)$. Equations (9) and (10) together with $z(k)$ gives $x_z(k)$ and $x_k(k)$. Equation (3) with $z(k)$ gives $h(k)$. Given the functions $z(k)$, $h(k)$, $x_k(k)$, and $x_z(k)$, equations (2), (5), and (11) can then be used to solve for k_t^*. Having solved for k_t^*, it is straightforward to solve for y_t^*. The solution for the case where $\pi_k = \pi_A$ is given by equation (5.8).