Optimal Indirect Taxes for Brazil: Combining Equity and Efficiency*

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Summary: 1. Introduction; 2. The model; 3. Household utility and demand functions; 4. The data; 5. Results; 6. Conclusions.
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This paper attempts to characterise the *optimal* structure of indirect taxes for Brazil, that is, the indirect tax structure that would allow the government to achieve certain redistributional objectives and raise enough revenue to finance its expenditures at the least possible cost in terms of efficiency. For this purpose, a computable optimal tax model is specified and solved under alternative assumptions about the extent of the government's concern with inequality, the constraints on its ability to tax, the preferences of households and the required level of revenue.

Este artigo busca caracterizar a estrutura *ótima* de impostos indiretos para o Brasil, isto é, a estrutura de impostos que permitiria ao governo alcançar certos objetivos redistributivos e arrecadar receita suficiente para financiar seus gastos ao menor custo possível em termos de eficiência. Para tanto, um modelo computável de impostos ótimos é especificado e solucionado sob diferentes hipóteses acerca da atitude do governo em relação à desigualdade, das restrições sobre sua capacidade para tributar, das preferências das famílias e do nível de receita requerido.

1. Introduction

The regressive nature of the Brazilian tax system has long been a source of concern for both economists and the public at large. The heavy reliance on indirect taxes as a source of revenue is widely believed to be a major cause of the inequalities of the system.[1] In an attempt to deal with this problem, the 1988 Constitution established that the main Brazilian indirect tax – the tax on the circulation of goods and transportation and communication services (ICMS) – could be selective according to the "essentiality of the product".

[1] *Empirical support for this view is provided by Eris et alii (1983), who show that the amount of indirect tax paid by households as a proportion of their disposable income decreases drastically as the level of income increases.*

The moves towards a selective ICMS have been modest, however, and do not seem always to advance the objective of improving equity. This may in part be due to the vagueness of the "essentiality criterion" and to a lack of a better understanding of the consequences of selectivity for economic efficiency and government revenue. Meanwhile, a proposal to create a new value-added tax on goods and services by merging the ICMS and the tax on industrial products (IPI) is at the center of current policy debates. If this new tax does not embody some degree of selectivity, it is expected to have a negative impact on the already unequal distribution of the tax burden and on government revenue.

To the best of my knowledge, there has been only one study which addresses the problem of indirect tax reform in Brazil by applying the tools of modern tax analysis. It is Souza (1992), which makes use of the theory of marginal reform to identify directions of changes in the indirect tax system that would improve social welfare. The marginal reform approach, however, applies only to "small" tax changes, while it seems that the achievement of distributional goals is likely to require more substantial changes in the tax structure.

The purpose of this paper is to analyse the appropriate structure of indirect taxes for Brazil in the light of the theory of optimal taxation.[2] More specifically, by using a computable optimal tax model, it attempts to characterise the indirect tax structure that would allow the Brazilian government to achieve certain redistributional objectives and raise enough revenue to finance its expenditures at the least possible cost in terms of efficiency. The model is solved under different assumptions about the extent of the government's concern with inequality, the constraints on its ability to tax, the preferences of households, and the required level of revenue. This approach has the advantage of being specific about the directions as well as the magnitudes of the desirable tax changes.

2. The Model

The assumptions regarding the production side of the economy are kept deliberately simple in order to concentrate on the twin concerns of consumer

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[2] *Comprehensive surveys of the theory of optimal taxation are available in Auerbach (1985) and Stiglitz (1987). A more introductory account of the theory, as well as examples of its practical applications, is found in Newbery & Stern (1987).*

welfare and revenue collection. Consequently, there are no profits, and producer prices are constant. This means that the effect of commodity taxes on consumer welfare works entirely through changes in consumer prices, ignoring all effects from changes in factor prices and profits.[3] The behaviour of households and the taxation problem faced by the government are detailed below.

2.1 Households

The model distinguishes between urban and rural population, a division that highlights two aspects of the dualism inherent in the Brazilian economy which may crucially influence tax design, namely:

(a) the severe constraints on the taxation of transactions within the rural sector;

(b) the disparities in the living standards of rural and urban residents.

The rural and urban populations are each divided into nine groups of households according to household expenditure. All households in a given expenditure group are assumed to be identical, so that each group's behaviour may be described by a "respresentative" household.

The absence of wage and earnings data makes it imperative to assume that each household takes consumption expenditure as exogenously given. In addition, it is supposed that there are no savings, so that income and total consumption expenditure are interchangeable.[4] Further, it is assumed that, in addition to the income from their supply of labour, households may also receive lump-sum payments from the government, which are constrained to be the same for all households within a sector.

Formally, the consumer side of the economy is specified as follows. Urban households, indexed by ℓ, face a vector of urban consumer prices, q, and rural households, indexed by m, face a vector of rural consumer prices, s. The

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[3] *Another reason for making these assumptions is the lack of reliable data on profits and factor returns. In the empirical literature in this area, the traditional procedure used to exclude profits is to assume either constant return to scale (and competitive conditions in production) or 100% taxes on profits. Although the constancy of producer prices involves more restrictive conditions (Stern, 1987), the assumption of full shifting of commodity taxex into consumer prices is adopted in most incidence analyses, including Eris et alii (1983). For a discussion of the role of these assumptions in optimal tax models, see Stiglitz & Dasgupta (1971).*

[4] *Together these assumptions imply that the supply of labour is inelastic.*

budget constraint for each representative household in urban and rural areas then is respectively

$$\sum_{i=1}^{n} q_i x_i^{\ell} = y^{\ell} + I = Y^{\ell} \tag{1}$$

and

$$\sum_{i=1}^{n} s_i x_i^{m} = y^{m} + I' = Y^{m} \tag{2}$$

where:

i = index over the consumption goods;

$x_i^{\ell}(x_i^{m})$ = consumption of good i by household $\ell(m)$;

$y^{\ell}(y^{m})$ = fixed labour income of household $\ell(m)$;

$I(I')$ = lump-sum transfer received by each household in urban (rural) locations;

$Y^{\ell}(Y^{m})$ = total income received by household $\ell(m)$.

Each household ℓ and each household m is assumed to choose consumption goods so as to maximise their utilities subject to (1) and (2), respectively. This leads to the demand functions:

$$x_i^{\ell}(q, I) \text{ and } x_i^{m}(s, I')$$

And the indirect utility functions:

$$v^{\ell}(q, I) \text{ and } \nu^{m}(s, I')$$

2.2 Government

The government is assumed to be interested in using taxes both to raise a certain amount of revenue and to redistribute income. In view of the severe constraints faced by the Brazilian government on the implementation of a progressive and comprehensive system of income taxation,[5] the case where

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[5] *This is reflected by the fact that of Brazil's 60 million economically active individuals, it is estimated that just 6 million regularly report personal earnings, and, of that total, only 3 million effectively pay income tax (Exame, 1993).*

commodity taxes are the only policy instruments at the disposal of the government is emphasised. However, this situation will be compared with the more general case where, in addition to commodity taxes, the government can also grant lump-sum subsidies to households which may differ across rural and urban locations. It is interesting to note that allowing for poll subsidies to households (in addition to commodity taxation) is equivalent to allowing a linear income tax characterised by an exemption level and a constant marginal rate of tax both above and below this level.[6]

The government's social welfare function

It is assumed that the government's distributional objectives can be expressed through a social welfare function, W, based on the households' utility levels. In keeping with most studies, it is supposed that this function has the following specific form:

$$W = \frac{1}{(1-\varepsilon)} \left[\sum_{\ell=1}^{9} h^\ell \nu^\ell(q, I)^{1-\varepsilon} + \sum_{m=1}^{9} h^m \nu^m(s, I')^{1-\varepsilon} \right] \qquad (3)$$

when ε is not equal to 1, and

$$W = \sum_{\ell=1}^{9} h^\ell \log \, \nu^\ell(q, I) + \sum_{m=1}^{9} h^m \log \, \nu^m(s, I')$$

where ε is equal to 1. Where:

$h^\ell(h^m)$ = fraction of households in the group represented by household $\ell(m)$;

$\varepsilon = a$ parameter reflecting the government's aversion to inequality (Atkinson, 1970), with $\varepsilon \geq 0$.

When ε is zero, (3) corresponds to the classical utilitarian welfare function, which places equal weights on the utility changes of all households. As ε increases, higher weights are attached to changes in the utilities of the less well-off households. For example, a value of 1 for ε implies that if the utility of household ℓ is twice that of household m, then a marginal increase in the

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[6] *Since there are no savings, a uniform tax on all commodities is equivalent to a proportional tax on income. Below the exemption level, this tax is lower than the poll subsidy received by a household, so that the scheme corresponds to a negative income tax.*

utility of household m is worth twice the marginal increase in the utility of ℓ; a value of 5 for ε indicates that a marginal increase in the utility of household m is worth 32 times a marginal increase in the utility of household ℓ. As ε approaches infinity, the social welfare function (3) approximates the Ralwsian "maximin" criterion, by considering the utility only of the worst-off household. Optimal taxes are computed for values of ε of 0.1, 0.5, 1, 2, and 5, in order to cover a broad range of distributional judgements.

The government's budget constraint

Since the government wants to raise revenue to cover its expenditures, R, on some given activities, and also to finance the lump-sum payments to households, it faces a budget constraint:

$$R + I \sum_{\ell=1}^{9} H^\ell + I' \sum_{m=1}^{9} H^m = \sum_{i=1}^{n} t_i \sum_{\ell=1}^{9} H^\ell x_i^\ell + \sum_{i=1}^{n} t_i' \sum_{m=1}^{9} H^m x_i^m \qquad (4)$$

where:

$H^\ell (H^m)$ = number of households in group $\ell(m)$;

$t_i(t_i')$ = value of the tax on good i in urban (rural) areas.

Under the assumptions of the model:

$$t_i = q_i - p_i \qquad (5)$$
$$t_{i'} = s_i\, p_i \qquad (6)$$

for $i = 1, \cdots, n$, where p_i is the (fixed) producer price of good i, which is to be normalised at unity.

Tax restrictions

The model recognises that the government may not be able to tax all goods at will. In particular, it allows for the fact that the conventional features of agriculture in Brazil have effectively prevented the government from taxing internal trade within that sector.[7] In addition, it admits that due to the

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[7] *The difficulties of taxing agriculture are associated with, among other things, the presence of a large number of small-scale farmers and own consumption of agricultural goods (because this does not involve market transactions).*

possibility of arbitrage between the urban and rural sectors, the government may be constrained to tax certain goods in both sectors at the same rate. Accordingly, two kinds of restrictions on the possible structure of commodity taxes are considered:

(a) goods produced and consumed within the rural area cannot be taxed or subsidised,

(b) some goods must be taxed at the same rate in rural and urban rates.

Following Heady & Mitra (1986), these restrictions are represented as:

$$Cs = Cp \tag{7}$$

$$Dq = Ds \tag{8}$$

respectively, where C and D are diagonal matrices with elements of 1 and 0, which select the prices for which the restriction must hold.

In order to examine the effects of constraints on the government's ability to make lump-sum transfers, the following additional conditions are imposed:

$$I = I' \tag{9}$$

$$I = 0 \tag{10}$$

Expression (9) reflects the case where the government is constrained to set the value of the poll subsidies uniformly across rural-urban locations, whereas the combination of (9) and (10) reduces the model to the case where no lump-sum transfers are possible.

The government's problem

The government's problem is then defined as one of choosing commodity tax rates (or, equivalently, consumer prices) and poll subsidies to maximise the social welfare function (3) subject to the budget constraint (4) and the tax restrictions (7), (8), (9) and (10).

The Lagrangian for this problem is:

$$L = \frac{1}{1-\varepsilon} \left[\sum_{\ell=1}^{9} h^\ell (\nu^\ell)^{1-\varepsilon} + \sum_{m=1}^{9} h^m (\nu^m)^{1-\varepsilon} \right] + \lambda \left[\sum_{i=1}^{n} (q_i - p_i) \sum_{\ell=1}^{9} H^\ell x_i^\ell + \right.$$

$$\left. + \sum_{i=1}^{n} (s_i - p_i) \sum_{m=1}^{9} H^m x_i^m - I \sum_{\ell=1}^{9} H^\ell - I' \sum_{m=1}^{9} H^m - R \right] + \phi^T [Cs - Cp] +$$

$$+ \mu^T [Dq - Ds] + \nu [I - I'] + \omega I \tag{11}$$

where (5) and (6) have been substituted for t_i and t'_i in (4), respectively; λ, ν and ω are scalar multipliers corresponding to (4), (9) and (10), respectively; ϕ and μ are vectors of multipliers corresponding to (7) and (8), respectively; and T denotes the transpose operation.

The first-order conditions for q_i, s_i, I and I' are:

$$\frac{\delta L}{\delta q_i} = \sum_{\ell=1}^{9} h^\ell (\nu^\ell)^{-\varepsilon} \left(\frac{\delta \nu^\ell}{\delta q_i} \right) + \lambda \left[\sum_{\ell=1}^{9} H^\ell x_i^\ell + \sum_{j=1}^{n} (q_j - p_j) \sum_{\ell=1}^{9} H^\ell \left(\frac{\delta x_j^\ell}{\delta q_i} \right) \right] +$$

$$+ \mu_i d_i = 0, \qquad i = 1, \cdots, n \tag{12}$$

$$\frac{\delta L}{\delta s_i} = \sum_{m=1}^{9} h^m (\nu^m)^{-\varepsilon} \left(\frac{\delta \nu^m}{\delta s_i} \right) + \lambda \left[\sum_{m=1}^{9} H^m x_i^m + \right.$$

$$\left. + \sum_{j=1}^{n} (s_j - p_j) \sum_{m=1}^{9} H^m \left(\frac{\delta x_j^m}{\delta s_i} \right) \right] + \phi_i c_i - \mu_i d_i = 0, \quad i = 1, \cdots, n \tag{13}$$

$$\frac{\delta L}{\delta I} = \sum_{\ell=1}^{9} h^\ell (\nu^\ell)^{-\varepsilon} \left(\frac{\delta \nu^\ell}{\delta I} \right) + \lambda \left[\sum_{i=1}^{n} (q_i - p_i) \sum_{\ell=1}^{9} H^\ell \left(\frac{\delta x_i^\ell}{\delta I} \right) - \sum_{\ell=1}^{9} H^\ell \right] +$$

$$+ \nu + \omega = 0 \tag{14}$$

$$\frac{\delta L}{\delta I'} = \sum_{m=1}^{9} h^m (\nu^m)^{-\varepsilon} \left(\frac{\delta \nu^m}{\delta I'} \right) + \lambda \left[\sum_{i=1}^{n} (s_i - p_i) \sum_{m=1}^{9} H^m \left(\frac{\delta x_i^m}{\delta I'} \right) - \right.$$

$$\left. - \sum_{m=1}^{9} H^m \right] - \nu = 0 \tag{15}$$

where $c_i = 1$ when the rural tax rate on good i must be zero and $c_i = 0$ otherwise; $d_i = 1$ when good i must have the same consumer price in rural and urban areas, and $d_i = 0$ otherwise.

3. Household Utility and Demand Functions

The main set of results are derived using the linear expenditure system (LES) to specify the households' demand and utility functions. For purposes of comparison, some results for the Cobb-Douglas specification are also presented.

The demand equations corresponding to the LES can be written as:

$$q_i x_i = q_i \alpha_i + \beta_i \left(Y - \sum_{j=1}^{n} q_j \alpha_j \right), \qquad i = 1, \cdots, n \qquad (16)$$

where x_i is the quantity of the i-th good consumed, q_i its price, Y total expenditure on the n goods, and α_i and β_i are parameters that satisfy the constraints:

$$\sum_{i}^{n} \beta = I, \qquad \beta_i > 0, \qquad x_i - \alpha_i > 0$$

for all i.[8]

The indirect utility function, ν, for the LES is:

$$\nu = \frac{\left(Y - \sum_{i=1}^{n} q_i \alpha_i \right)}{\prod_i q_i^{\beta_i}} \qquad (17)$$

Equation (16) is often interpreted as stating that the consumer first purchases "subsistence" or "committed" quantities of each good, $\alpha_i (i = 1, \cdots, n)$, on which a portion $\Sigma q_i \alpha_i$ of total expenditure is spent. The remainder of the consumer's total expenditure, $Y - \Sigma q_i \alpha_i$, termed "supernumerary expenditure", is then spent among the n goods according to fixed proportions $\beta_i (i = 1, \cdots, n)$.[9]

If all of the α_i's equal zero, the model reduces to the Cobb-Douglas case, so that demands are given by

$$x_i = \frac{w_i Y}{q_i}, \qquad i = 1, \cdots, n$$

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[8] *These restrictions are required for consistency with utility maximisation subject to a budget constraint.*

[9] *This interpretation allows the indirect utility function (17) to be seen as taking "real expenditure" as an indicator of welfare, in that it expresses utility as a function of supernumerary expenditure deflated by a price index, the latter calculated as the weighted geometric mean of the prices, with the marginal shares β_i as weights.*

where w_i is the (average) budget share of good i, and the underlying indirect utility function is of the form:

$$\nu = \frac{Y}{\prod_i q_i^{w_i}}$$

4. The Data

The data on household expenditure are obtained from *Estudo nacional da despesa familiar* – Endef (IBGE, 1981). Thus, the expenditure levels that define household groups in the model are those used in the Endef classification.

The government's revenue requirement (R in equation (4)) is assumed to be equal to the net revenue raised from households in the year of the Endef survey, which was approximately 10% of the total sum of all household expenditures, as estimated from the 1975 matrix of intersectoral transactions (IBGE, 1987).

Consumption goods are classified in nine broad categories, namely: food, clothing, housing, durables, personal care, transport, recreation, beverages and tobacco, and miscellaneous. The β parameters of the LES, termed marginal budget shares, for each of those categories and for each household group, are calculated by using the average budget shares, Rossi & Neves (1987)'s estimates of the expenditure elasticities, and the property of the LES that the expenditure elasticity for a given commodity equals the ratio of the marginal budget share to the average budget share for that commodity.[10]

By their turn, the α parameters are derived, from equation (16), using the β estimates, consumer prices (estimated from the 1975 matrix of intersector transactions), and assuming that *per capita* total committed expenditure ($\Sigma q_i \alpha_i$) for every household equals 90% of the *per capita* total expenditure of the poorest household.[11]

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[10] *Some adjustments had to be made to Rossi & Neves's estimates, since their commodity categories do not exactly coincide with those defined here. Details are provided in Siqueira (1995).*

[11] *These correspond to those households in the lowest expenditure group in rural areas, which have the smallest total expenditure per head. Note that by fixing total committed expenditure in* per capita *terms, total committed expenditure per household is made proportional to household size – a phenomenon observed, for example, by Lluch, Powell & Williams (1977) and one which is consistent with the subsistence interpretation of the α's. The procedure, however, ignores possible economies of scale.*

5. Results

Among many different possible assumptions about the government's ability to tax, five cases have been selected to be presented here, which are described below. The sensitivity of the results to the form of households' preferences and to the level of revenue raised is tested next.

5.1 Case 1: there are no commodity tax restrictions

For the sake of comparison, the first case considers a situation where the government can tax all goods at will and can choose two different sets of taxes, one for rural and one for urban areas. The results are shown in tables 1a and 1b.

The most striking feature of these tables is the difference between rural and urban tax estimates. Taxation in the urban sector is much heavier than in the rural sector for all values of the inequality aversion parameter, reflecting the disparities in household expenditures between the urban sector and the poorer rural sector.

Table 1a

Optimal tax rates for urban areas with varying degrees
of inequality aversion: case 1 (%)

Commodity group	Degree of inequality aversion ε				
	0.1	0.5	1.0	2.0	5.0
Food	24.4	1.9	-19.6	-33.0	-51.5
Clothing	33.8	38.7	41.2	40.2	38.7
Housing	34.1	38.2	37.1	24.4	16.4
Durables	37.2	54.6	76.1	96.0	96.4
Personal care	34.0	39.1	40.7	36.4	33.6
Transport	35.0	43.5	48.4	48.3	47.6
Recreation	35.1	43.5	48.3	47.6	47.3
Beverages & tobacco	29.3	20.3	11.9	12.8	5.1
Miscellaneous	35.0	42.7	45.8	39.0	35.5

Table 1b

Optimal tax rates for rural areas with varying degrees
of inequality aversion: case 1 (%)

Commodity group	Degree of inequality aversion ε				
	0.1	0.5	1.0	2.0	5.0
Food	-87.3	-107.9	-120.1	-121.9	-84.9
Clothing	-75.2	-58.8	-40.4	-13.2	18.1
Housing	-75.6	-63.3	-53.6	-43.2	-23.0
Durables	-70.6	-42.0	-18.0	5.9	31.2
Personal care	-73.9	-53.6	-32.6	-4.1	25.7
Transport	-67.0	-28.6	0.2	23.5	37.2
Recreation	-70.1	-41.9	-20.1	1.3	19.1
Beverages & tobacco	-82.3	-89.0	-90.6	-80.8	-34.9
Miscellaneous	-71.3	-45.9	-25.7	-5.6	14.2

Intersectoral rate differences are particularly acute at low levels of inequalities aversion, specifically $\varepsilon = 0.1$, where all urban goods are highly taxed and all rural goods are highly subsidised. This is because rate differentiation between sectors involves no distortion,[12] and thus becomes the principal means to carry out redistribution when one is primarily concerned with the efficiency cost of taxation. On the other hand, the rates within each sector are fairly uniform at $\varepsilon = 0.1$. This is not surprising, since that level of ε represents near neutrality with respect to distribution and it is known that if inequality was not a concern at all, tax rates within each sector should be uniform – for this is equivalent to a tax on the households' exogenous income and is thus non-distortionary.[13]

At higher levels of inequality aversion, however, achieving equity objectives requires a considerably higher degree of rate differentiation across commodities. In urban areas, the (positive) tax on food is replaced by a subsidy which increases in magnitude with increasing ε. The taxes on beverages & tobacco and housing are also significantly lower at higher values of ε. This reflects

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[12] *The model ignores intersectoral effects of taxation through, for instance, induced migration. Although Heady (1987, 1988) develops models that focus on optimal taxation in the presence of migration, the introduction of this additional element in the present model would bring severe empirical complications.*

[13] *This corresponds to the classical optimal taxation solution for a one-consumer economy where one factor is in completely inelastic supply. See, e.g., Dixit (1970) and Sandmo (1976).*

the fact that, according to Endef data, food and housing together take up more than 80% of the expenditure of the poorest household group, whereas the share of beverages & tobacco, though relatively small in the budget of all households, is highest for low- and middle-income groups. Thus, the burden of a tax on these three categories of consumption falls mainly on the less well-off. Meanwhile, the tax rate on durables, the principal luxury item for urban households, increases drastically as the degree of inequality aversion becomes stronger.[14] The other urban taxes vary relatively little as ε increases, remaining considerably high.

In order to allow reductions in the taxation of the urban necessities, the tax burden in the rural sector is increased for increased aversion to inequality. The rural subsidies for food and beverages & tobacco increase with ε at first but this increase is reversed at higher levels of inequality aversion. The subsidy for rural housing decreases with ε but remains negative over the entire range of the parameter, whereas the other rural subsidies are replaced by positive taxes at high values of ε.

5.2 Case 2: the rate of tax for each commodity is the same across rural-urban locations

Case 2 considers a more realistic setting where, in view of the difficulties of avoiding tax arbitrage between rural and urban locations, as well as due to concerns about the political acceptability of the tax system, the government chooses the same set of taxes for both rural and urban areas. The optimal tax rates for this case are reported in table 2.

The results from this table can be summarised as follows:

(a) at a low value of ε (specifically $\varepsilon = 0.1$), the tax rate on food is virtually zero, the rates on other goods except beverages & tobacco are approximately uniform, and the rate on beverages & tobacco is about half that on the other taxed items;

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[14] *The term "luxury" is used throughout this work to refer to those consumption items which are more important in the budget of high-income households, while "necessity" is used to refer to those goods which represent higher shares in the budget of low-income households.*

(b) at higher values of ε, food is subsidised, beverages & tobacco remains relatively lightly taxed,[15] and all the other goods – particularly durables, transport and recreation – are heavily taxed;

(c) the subsidy for food, as well as the taxes on the other goods except beverages & tobacco, increases with increases in ε, whereas the rate on beverages & tobacco decreases at first with ε and then increases.

Table 2

Optimal tax rates with varying degrees
of inequality aversion: case 2 (%)

Commodity group	Degree of inequality aversion ε				
	0.1	0.5	1.0	2.0	5.0
Food	-0.5	-31.5	-56.7	-74.7	-77.7
Clothing	13.3	20.4	26.5	34.5	36.7
Housing	14.8	24.7	29.5	31.1	31.7
Durables	17.8	39.4	56.9	65.2	63.1
Personal care	13.9	22.7	29.8	38.1	41.0
Transport	16.5	32.6	43.6	51.3	52.5
Recreation	16.4	31.5	41.3	48.4	49.8
Beverages & tobacco	8.1	-0.7	-5.1	4.9	13.9
Miscellaneous	15.6	28.1	35.0	38.6	38.6

The behaviour of the rate on beverages & tobacco, as the degree of inequality aversion increases, reflects the fact that the budget share for this category increases at first with increases in income, being highest for middle-income classes, and then decreases. This means that, as less weight is put on the welfare of middle-income households relatively to low-income households, it becomes less desirable to subsidise beverages & tobacco. Subsidy is then shifted to food, the share of which declines continuously with income, from 68% for the poorest household group to 13% for the richest group. On the whole, the results in table 2 suggest that the entire weight of the equity-improving aspect of the indirect tax system should be borne by food subsidies.

Note that although the magnitudes of the tax rates are quite sensitive to changes in the inequality aversion parameter, the tax structure at different levels of the parameter present a remarkable degree of consistency. To some

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[15]*It is actually subsidised at moderate levels of ε.*

extent, this is because with fixed incomes plus the limited substitution possibilities between goods imposed by the linear expenditure system, the optimal tax formula insures that luxuries will be taxed more heavily than necessities for positive values of ε.[16] At the same time, the disparities in the distribution and in the patterns of expenditure among households have certainly contributed to accentuate the progressivity and robustness of the estimated tax structures.

Another feature of table 2 that calls for attention is the relatively minute variation in the tax estimates as ε changes from 2.0 to 5.0. This indicates that the scope for redistribution through commodity taxes is near exhaustion when the rates are those calculated for $\varepsilon = 2.0$.

5.3 Case 3: food is untaxed

Table 3 reports the results for the case where, in addition to the impossibility to discriminate between rates on locational grounds, the government cannot tax or subsidise food. This additional restriction is meant to reflect the practical difficulties of taxing food within the rural sector.

The effect of food being untaxed can be seen by comparing tables 2 and 3. At a low level of inequality aversion, $\varepsilon = 0.1$, the optimal tax estimates are virtually the same in the two tables, since the rate of food, even in the absence of the restriction, is very close to zero.

On the other hand, at higher levels of inequality aversion, the inability to subsidise food permits a large reduction in the tax rates on all the other goods. It also implies a higher degree of rate differentiation across taxable items. In table 3, for $\varepsilon = 0.5$, for example, the subsidy on beverages & tobacco is much higher than it is in table 2, and the ratio between the highest and the lowest (positive) tax rates is 5:1, compared with 2:1 in table 2. This represents an attempt to transfer to beverages & tobacco, housing, clothing, and personal care some of the redistributive role associated with food subsidies in case 2, by switching taxation from these goods to those with higher degree of luxury.

Note, however, that only in the case of housing does the rate decrease monotonically with increasing inequality aversion. The rates on clothing and personal care decline initially as ε increases, reaching their lowest values at a

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[16] *A demonstration of this can be found in Deaton (1979).*

moderate level of inequality aversion ($\varepsilon = 1.0$), and then they increase with ε. Correspondingly, the subsidy on beverages & tobacco is highest for $\varepsilon = 1.0$ and decreases for higher values of the parameter.

This is because beverages & tobacco, personal care, and, in particular, clothing are more important in the consumption bundle of middle-income households. Consequently, as aversion to inequality reaches a level where one is primarily concerned with the welfare of the very poor, there is a shift in taxation towards those goods and away from housing, which figures heavily in the budget of low-income households.

Meanwhile, the tax rates on the categories that are chiefly consumed by high-income families (i.e., durables, transport, recreation, and miscellaneous) become very high with increased inequality aversion. Nevertheless, they are lower than the corresponding rates in table 2, indicating that the impossibility to subsidise food has reduced the redistributive power of the tax system.

Table 3

Optimal tax rates with varying degrees
of inequality aversion: case 3 (%)

Commodity group	Degree of inequality aversion ε				
	0.1	0.5	1.0	2.0	5.0
Food	0.0	0.0	0.0	0.0	0.0
Clothing	13.0	5.9	0.4	1.8	2.6
Housing	14.6	11.3	5.3	-1.0	-1.3
Durables	17.5	29.2	43.9	49.4	47.4
Personal care	13.7	9.5	7.1	11.2	12.9
Transport	16.3	24.2	34.0	41.5	41.9
Recreation	16.2	22.9	30.7	37.0	37.5
Beverages & tobacco	7.9	-19.7	-44.6	-38.6	-28.6
Miscellaneous	15.5	20.5	24.7	26.1	25.4

5.4 Case 4: housing can be taxed differently across rural-urban locations

In case 4 the arbitrage restriction is relaxed to make allowance for the fact that certain goods such as electricity need not be subject to that restriction. Here housing (which includes electricity) is taken to represent such goods, and

its price is allowed to vary across rural and urban areas. The results yielded by this case are displayed in table 4.

A comparison of this table with table 3 shows that, as one would expect from the results in case 1, allowing dual pricing for housing produces a substantial increase at all levels of ε in the taxation of urban housing, which is used mainly to finance a subsidy for rural housing. This is due not only to the relative poverty of rural residents, but also to the fact that in the rural sector housing tends to be more important in the budget of poor than rich households, whereas the reverse is true in the urban sector. This means that the ability to set a different tax rate for housing in each sector is likely to increase significantly the distributional capabilities of the tax system in relation to case 3.

Table 4

Optimal tax rates with varying degrees
of inequality aversion: case 4 (%)

Commodity group	Degree of inequality aversion ε				
	0.1	0.5	1.0	2.0	5.0
Food	0.0	0.0	0.0	0.0	0.0
Clothing	12.7	4.5	-2.3	-1.5	2.9
Urban housing	16.9	22.7	28.1	29.8	24.5
Rural housing	-1.3	-63.1	-135.5	-199.9	-190.5
Durables	17.3	27.8	40.7	50.4	52.0
Personal care	13.4	8.3	4.3	5.6	8.5
Transport	16.2	23.7	32.4	39.8	40.9
Recreation	16.1	22.1	28.1	34.4	36.5
Beverages & tobacco	7.1	-23.9	-57.5	-74.7	-64.9
Miscellaneous	15.4	19.9	23.6	25.4	25.7

As the degree of inequality aversion increases over the range 0.1 to 2.0, the tax on urban housing, as well as the subsidy on rural housing, increases, but these increases are reversed at higher levels of ε. This is because, though more important in the budget of the urban rich, housing also figures heavily in the budget of the urban poor. The rates on all the other goods behave as in case 3 with respect to changes in ε.

5.5 Case 5: the government can pay a uniform subsidy to all households

The results for the case where the government can make uniform lump-sum transfers to households are presented in table 5.

Table 5

Optimal tax rates and poll subsidy with varying degrees of case 5 (tax rates in % and inequality aversion: poll subsidy in thousands of 1974 cruzeiros per year)

Commodity group	Degree of inequality aversion ε		
	0.1	1.0	2.0
Food	10.4	-8.3	-14.7
Clothing	29.7	45.1	48.5
Housing	30.6	46.0	48.3
Durables	35.0	65.2	70.9
Personal care	30.6	47.8	51.4
Transport	33.1	55.2	59.5
Recreation	32.8	53.4	57.2
Beverages & tobacco	26.8	36.9	40.0
Miscellaneous	31.3	48.3	50.8
Poll subsidy	3.956	6.821	7.143

Note: It may be convenient to compare the value of the optimal poll subsidy with the minimum wage in fore in August 1974: Cr$376,80, which corresponds to approximately Cr$4,500.00 yearly.

Comparison of this table with table 2 shows that, as expected, the payment of an optimal subsidy to all households requires a substantial increase in all the tax rates at the same time that it drastically reduces the difference among them. This is because lump-sum subsidies are more efficient instruments to achieve redistribution (they are actually non-distortionary, by definition) than are differential commodity taxes, and hence it is desirable to rise commodity taxation (and reduce rate differentiation) in order to finance lump-sum subsidies to households. Actually, Atkinson (1977) shows that the presence of a uniform payment to all households, where households have identical preferences represented by the linear expenditure system, implies that indirect

taxes should be uniform.[17] We are not, however, assuming identical prefer-
ences,[18] so that there remains some room for redistribution through indirect
taxes are differentiation. Accordingly, table 5 suggests that at higher levels of
inequality aversion, food should still be subsidised and durables taxed more
heavily.

5.6 The effect of using Cobb-Douglas utility functions

The optimal taxes for the Cobb-Douglas specification of the households'
preferences are reported in table 6. This table is comparable to table 2.

Overall, the results for the Cobb-Douglas case do not appear to be very
different from those yielded by the linear expenditure system. There is, how-
ever, a closer agreement between the estimates for the two preference specifi-
cations at low levels of inequality aversion than at higher levels. This should
be expected since, as observed before, the assumption of fixed incomes implies
that if redistribution is not a major concern, the optimal tax structure will
resemble unformity regardless of the choice of the demand system.

Nevertheless, it is noticeable that at $\varepsilon = 0.1$ the tax structure is closer to
uniformity under the Cobb-Douglas case than under the linear expenditure
system. The reason for this is that the Cobb-Douglas formulation does not in-
volve committed quantities, which has two implications. First, the differences
in utility levels among households is less accentuated unde the Cobb-Douglas
specification than under the linear expenditure system, for in the former util-
ity is a function of total expenditure while in the latter it is a function of
supernumerary expenditure. Second, the absence of committed quantities
means that consumers have more flexibility in allocating consumption among
goods, and consequently the distortionary cost of rate differentiation is higher
under the Cobb-Douglas formulation.

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[17] *This result was generalised for a broader class of preferences by Deaton (1979).*

[18] *The parameters of the linear expenditure system were estimated separately for each income
class in our model. A remark should be made, however, that in this case differences in con-
sumption patterns cannot be explained by differences in income, but must be accounted for by
differences in correlated household characteristics (see Coulter et alii, 1995).*

While at higher levels of inequality aversion there are more significant differences between the rates yielded by the two demand systems, the basic lessons that emerge from the results in each case are fairly similar. For instance, as in table 2, the results in table 6 indicate that for moderate and high levels of inequality aversion, the optimal tax system involves a subsidy for food, a low tax on alcoholic beverages and tobacco, and high taxes on all other commodities, particularly on durables, transport and recreation.

Table 6

Optimal tax rates with varying degrees of inequality aversion: case 2 with Cobb-Douglas utility functions (%)

Commodity group	Degree of inequality aversion ε		
	0.1	1.0	2.0
Food	5.8	-27.0	-49.3
Clothing	10.3	15.8	24.7
Housing	11.6	22.7	28.3
Durables	14.3	44.4	62.3
Personal care	10.8	19.5	29.7
Transport	14.0	45.7	66.8
Recreation	14.0	43.6	63.0
Beverages & tobacco	8.0	-0.6	0.7
Miscellaneous	14.0	42.7	60.6

5.7 The effect of increasing revenue requirement

Table 7 shows the optimal tax structure for two different levels of the government revenue requirement, corresponding to 15% and 20% of the households' total expenditure. The results in this table and those in table 2 (where an amount of revenue equal to 10% of total household expenditure is raised) are qualitatively similar for all values of the inequality aversion parameter, though, as one would expect, larger revenue requirements bring about a reduction in the subsidy for food and increase the rate of tax for the other goods.

Table 7

Optimal tax rates with varying degrees of inequality aversion
and different revenue requirements: case 2 (%)

| Commodity group | Degree of inequality aversion ε | | | | | |
| | 0.1 | | 1.0 | | 2.0 | |
	a	b	a	b	a	b
Food	4.6	9.6	-48.8	-40.9	-64.0	-53.2
Clothing	18.4	23.7	31.6	36.8	38.8	43.0
Housing	19.9	25.1	33.7	37.9	35.0	39.0
Durables	22.9	28.2	61.9	67.0	68.2	70.9
Personal care	19.0	24.3	34.6	39.5	42.1	46.2
Transport	21.4	26.5	46.8	50.1	53.3	55.3
Recreation	21.3	26.3	44.5	47.8	50.6	52.8
Beverages & tobacco	13.7	19.5	5.5	17.2	18.9	33.5
Miscellaneous	20.6	25.7	38.6	42.1	41.4	44.1

Note: a refers to 15% of expenditure, while b refers to 20% of expenditure.

6. Conclusions

This paper attempted to characterise the likely structure of optimal indirect taxes for Brazil under alternative assumptions about taxation possibilities and the government's attitudes towards inequality. It also examined the robustness of the results with respect to the specification of household preferences and the required level of revenue. The following broad conclusions emerge from this experiment.

The optimal redistributive indirect tax system involves a subsidy for food, a low tax on alcoholic beverages and tobacco,[19] and high taxes on all other commodities, particularly on durables, transport and recreation. If food cannot be subsidised, then the optimal tax estimates suggest a subsidy for beverages and tobacco and exemptions for clothing and housing. These patterns are accentuated as the concern with inequality becomes stronger. In addition, the results indicate that even if the government could pay an optimal uniform subsidy to all households, there would still be room for further redistribution

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[19] *It may be appropriate to observe that the result that alcoholic beverages and tobacco should be lightly taxed is based solely on equity considerations, ignoring the negative effects associated with the consumption of these goods (on which grounds the heavy taxes commonly levied on them are justified).*

through food subsidies and higher rates on luxuries (notably, durables). Further, the optimal tax results point to a substantially lower tax burden on rural households than on urban residents.

These conclusions have been shown to be robust with respect to the specification of household preferences and the level of government revenue. It should be noted, however, that choices between work and leisure have not been allowed for, and hence effects of the estimated tax structures on work effort have been ignored.[20]

Two final remarks should be made about the present study. First, the data used refer to 1975, and although in some respects they still may be seen as a reasonable representation of the current state of affairs (concerning the disparities in the distribution and patterns of expenditure among households, for example), the proportion of the population living in urban areas has increased significantly, weakening the conclusions about policy discrimination between rural and urban locations. Second, full implementation of the optimal tax system may be undesirable or infeasible, since it represents a large departure from the current tax structure and may thus involve high administrative and political costs. However, the optimal tax estimates can still be useful as a guide to the formulation of politically and administratively acceptable tax reforms. A comparison of the distributional effects of such reforms with the effect of imposing a flat rate of tax on all commodities can provide an idea of potential welfare improvements from indirect taxation.[21]

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[20] *Empirical studies suggest that the assumption of fixed labour supply is not too far from the truth for adult male workers, since this group generally has a very low elasticity of labour supply. Married women, however, show greater responsiveness to changes in wages. See, for example, Blundell (1996).*

[21] *An investigation of the welfare effects of alternative partial reform packages elaborated in the light of the results presented here is provided in Siqueira (1997).*

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