What is Behind the Brazilian Stabilization?*

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This paper presents a two-country general equilibrium model to study the Brazilian stabilization, emphasizing role of the exchange rate policy before the stabilization. The model is able to reproduce, in qualitative terms, some of the facts that followed the Brazilian monetary reform.

Este artigo adota um modelo de equilíbrio geral com dois países para estudar a estabilização brasileira, enfatizando o papel da política cambial no período anterior ao Plano Real. O modelo reproduz, em termos qualitativos, alguns dos fatos observados após a reforma monetária.

1. Introduction

After decades of chronic high inflationary levels, the Brazilian economy experienced a successful and lasting stabilization. In July 1994, the inflation rate dropped from more than 40% per month to virtually zero. The program that led to the stabilization is known as *Real Plan.*[1]

According to the prevailing wisdom, the stabilization was due to clever heterodox policies plus orthodox fiscal policies and an exchange rate pegging that anchored the price of the tradable goods. However, this explanation is flawed. The notion that the heterodox policies mattered lacks scientific support. It is not easy to match the timing of the fiscal changes to the timing of the stabilization.

Calvo et alii (1995) showed that the Brazilian exchange rate policy closely followed a PPP rule in the period between 1990 and 1993. Indeed, that policy

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[1] Real *is also the name of the new currency introduced in July 1994.*

was in effect until the advent of the Real. After that the exchange rate was allowed to float, at least during the earlier stages of the stabilization.

Montiel and Ostry (1991) studied, at a theoretical level, the effects of a real exchange rate target over inflation and concluded that a real over-depreciation of the currency leads to higher inflation. The same question is studied by Calvo et alii (1995), who reached a similar conclusion. Empirically, those authors concluded that the tradeoff between inflation and a higher real exchange rate is likely to be steep.

This paper presents a dynamic two country general equilibrium model similar to the one by Lucas (1982) and studies the properties of the model under two distinct exchange rate regimes. In the first regime the exchange rate floats freely, while in the other regime the government pegs the real exchange rate above the level that would prevail in the free-floating regime. The model can reproduce the qualitative findings of Montiel and Ostry (1991) and Calvo et alii (1995).

The paper is organized as follows. Section 2 discusses the conventional wisdom on the Brazilian stabilization and the events that preceded the stabilization. Section 3 describes the model created to study the Brazilian stabilization; equilibrium properties are discussed and a policy exercise is performed in that section. Concluding remarks and suggestions for further research are found in section 4.

2. The Stabilization: Theories and Facts

The sound performance of the Real Plan is usually explained by the combination of:

a) heterodox policies;

b) fiscal policies;

c) a pegged nominal exchange rate that tied the price of tradable goods.

In this paper the relevance of parts (a) and (b) are found to be dubious.

Any study on the heterodox theory is built on old fashioned techniques. Optimization, the cornerstone of modern economics, is absent or, at best,

marginal. This does not imply that the heterodox components did not matter; but it does imply that there is no sound research to support this theory.

Azevedo et alii chronologically list the normative acts that constitute the Real Plan. During the second quarter of 1994 the only changes in the fiscal policy were new tax rates for few imported goods. Just two minor changes occurred in June. After the monetary reform, the first change in the fiscal policy took place on August 2nd, when tax rates on imported milk and its derivatives were raised.

It is well known that the major changes in the fiscal policy took place more than three months before the monetary reform. Moreover, the operational surplus was 3.7% of the GDP in the first quarter of 1994, 0.5% in the second, 1.4% in the third, and -1.8% in the final quarter.[2] As a result, it is difficult to conciliate the timing and magnitude of the observed fiscal adjustments with the observed inflation collapse.

We look at the events that took place on July 1st, when inflation collapsed, to identify factors that may explain the stabilization. Two events were highly recognized by Brazilian economists and the press: monetary reform and desindexation. Not as praised as the other two was the withdrawal of the BCB (Brazilian Central Bank) from the exchange market, allowing the exchange rate to float.

According to the economic theory, replacing one type of paper with another cannot affect the prices. The available theory on desindexation lacks microfoundations. Thus, the focus will be placed in the exchange rate policy.

The exchange rate can be an effective stabilization tool. Most, if not all, of the stabilization programs that defeated major inflationary processes in this century adopted some type of exchange rate pegging. If the nominal exchange rate is fixed, the domestic prices of the tradables follow international prices. However, in the Brazilian case, the exchange rate policy was important not only for that reason, but also because the foreign assets of the BCB changed from US$9 billion at the beginning of 1992 to US$45 billion on July 1st, 1994. Real resources were needed to finance that asset expansion. To make this

[2] *These numbers can be obtained from the March 1995 issue of the* Boletim do Banco Central *in table III.19 on page 115. They were obtained by calculating the quarterly GDP and operational deficit from the yearly accumulated values of those variables.*

point clear, consider a closed economy. The government budget constraint is:

$$\frac{d_t}{p_t} + (i_t - \pi_t)\frac{B_{t-1}}{p_t} = \frac{M_t}{p_t} - \frac{M_{t-1}}{p_{t-1}} + \frac{B_t}{p_t} - \frac{B_{t-1}}{p_{t-1}} + \pi_t \frac{M_{t-1}}{p_t} \qquad (1)$$

where d_t is the primary deficit, M_t the money supply, B_t the public debt (both at the end of period t), i_t the nominal interest rate, p_t the price level, and π_t the inflation rate.

Suppose now that the government holds, at end of date t, an amount A_t of a foreign currency. Let E_t denote the price of the foreign currency at date t. The equivalent of equation (1) is:

$$\frac{d_t}{p_t} + (i_t - \pi_t)\frac{B_{t-1}}{p_t} + \frac{E_t}{p_t}(A_t - A_{t-1}) = \frac{M_t}{p_t} - \frac{M_{t-1}}{p_{t-1}} + \frac{B_t}{p_t} - \frac{B_{t-1}}{p_{t-1}} + \pi_t \frac{M_{t-1}}{p_t} \qquad (2)$$

In an open economy the inflation tax can finance the Treasury component of the operational deficit (the first two terms in the left-hand side) or the accumulation of international assets or a reduction in the real values of money and/or public debt holdings.

The data in the appendix show that if the BCB had not accumulated foreign assets in the first semester of 1994 the inflationary revenue could have been dispensed by the public sector without any fiscal changes. The same is true for the last quarter of 1993. So, it seems that at least in the final stages of the inflationary process, the Treasury budget was already consistent with lower inflationary levels.

3. The Model Economy

There exist two countries, 1 (US) and 2 (Brazil), each populated by a single infinitely lived household and an agent called government. A household is composed of one shopper and one worker, who is endowed with one unit of time. Nation i produces a non-storable country-specific good c_i, which can be domestically consumed (c_{ii}) or exported (c_{ij}). Labor does not migrate across countries.

The production of c_i is carried out by a single competitive firm. Let n_i denote the amount of time worked by household i. The technology available

in country i is described by the production function $f_i(n_i) = b_i n_i$, where b_i is a positive constant. Therefore, feasibility requires:

$$c_{11t} + c_{12t} = b_1 n_{1t} \qquad \text{and} \qquad c_{21t} + c_{22t} = b_2 n_{2t} \tag{3}$$

At each date t government 1 transfers a lump-sum amount T_{1t} of currency 1 to household 1. Government 1 can also purchase and sell currency 2. Its date t budget constraint is:

$$E_t(M_{11t}+M_{12t}+A_{12t}-M_{11t-1}-M_{12t-1}-A_{12t-1}) = E_t T_{1t}+A_{21t}-A_{21t-1} \tag{4}$$

where M_{11t-1} and M_{12t-1} are the balances of currency 1 held, respectively, by household 1, household 2 and government 2 at the end of period $t-1$. E_t is the date t nominal exchange rate (price of currency 1 in terms of currency 2) and A_{ijt-1} is the amount of currency i held by government j at the end of period $t-1$. Government 2 is in a symmetric situation. Its budget constraint is:

$$M_{21t}+M_{22t}+A_{21t}-M_{21t-1}-M_{22t-1}-A_{21t-1} = T_{2t}+E_t(A_{12t}-A_{12t-1}) \tag{5}$$

Transactions take place in this world in a particular way. At the first stage of date t, a spot market for goods and labor services opens and closes in each country. Goods and labor services have to be paid with the domestic currency. At the second stage, a centralized currency market operates. At the first stage, shoppers buy both goods, and workers sell labor services for the domestic firms. At the second stage, people and governments convene in the financial market. Transfers happen in the second stage.

The function $u^1(c_{11}, c_{21}, n_1) = [c_{11t}^{\alpha_{11}} c_{21t}^{\alpha_{21}}(1-n_{1t})^{\gamma_1}]^{1-\sigma_{1t}}/(1-\sigma_1)$ is the period utility function of household 1. Parameters satisfy $\alpha_{11}, \alpha_{21}, \gamma_1, \sigma_1 > 0$ and $\alpha_{11} + \alpha_{21} + \gamma_1 = 1$. Let p_{it} and w_{it} denote the date t price of good i and wage rate in country i, both in terms of currency i. Household 1 chooses a sequence $\{c_{11t}, c_{21t}, n_{1t}, M_{11t}, M_{21t}\}_{t=1}^{\infty}$ to maximize

$$\sum_{t=1}^{\infty} \beta_1^{t-1} u^1(c_{11t}, c_{21t}, n_{1t}) \tag{6}$$

subject to

$$M_{11t}+M_{21t}/E_t \leq (M_{11t-1}-p_{1t}c_{11t})+(M_{21t-1}-p_{2t}c_{21t})/E_t+w_{1t}n_{1t}+T_{1t} \quad (7)$$

$$p_{1t}c_{11t} \leq M_{11t-1} \qquad \text{and} \qquad p_{2t}c_{21t} \leq M_{21t-1} \quad (8)$$

$$c_{11t}, c_{21t}, n_{1t}, M_{11t}, M_{21t} \geq 0; \;\; n_{1t} \leq 1; \;\; M_{11,0} \quad \text{and} \quad M_{21,0} \text{ given} \quad (9)$$

Similarly, household 2 chooses $\{c_{12t}, c_{21t}, n_{1t}, M_{12t}, M_{22t}\}_{t=1}^{\infty}$ to maximize

$$\sum_{t=1}^{\infty} \beta_2^{t-1} u^2(c_{12t}, c_{22t}, n_{2t}) \quad (10)$$

subject to

$$p_{1t}c_{12t} \leq M_{12t-1} \qquad \text{and} \qquad p_{2t}c_{22t} \leq M_{22t-1} \quad (11)$$

$$M_{22t}+E_tM_{12t} \leq (M_{22t-1}-p_{2t}c_{22t})+E_t(M_{12t-1}-p_{1t}c_{12t})+w_{2t}n_{2t}+T_{2t} \quad (12)$$

$$c_{22t}, c_{12t}, n_{2t}, M_{22t}, M_{12t} \geq 0; \;\; n_{2t} \leq 1; \;\; M_{22,0} \quad \text{and} \quad M_{12,0} \text{ given} \quad (13)$$

At each date t, firm i hires a non-negative amount of labor n_{it} to maximize $p_{it}b_in_{it} - w_{it}n_{it}$.

A *competitive equilibrium* (CE) is a sequence of prices $\{p_{1t}, p_{2t}, w_{1t}, w_{1t}, w_{2t}, E_t\}_{t=1}^{\infty}$ plus sequences of allocations and cash holdings $\{c_{11t}, c_{21t}, n_{1t}, M_{11t}, M_{21t}, c_{12t}, c_{22t}, n_{2t}, M_{12t}, M_{22t}\}_{t=1}^{\infty}$ plus a sequence of foreign assets and transfers $\{A_{12t}, A_{21t}, T_{1t}, T_{2t}\}_{t=1}^{\infty}$, such that:

a) $p_{it}(c_{iit} + c_{ijt}) = w_{it}n_{it}$;

b) given prices and transfers, $\{c_{iit}, c_{jit}, n_{it}, M_{iit}, M_{jit}\}_{t=1}^{\infty}$ solves the problem of household 1;

c) (3), (4) and (5) hold.

A *stationary competitive equilibrium* (SCE) is a CE in which the real balances M_{11t-1}/p_{1t}, M_{12t-1}/p_{1t}, M_{21t-1}/p_{2t} and M_{22t-1}/p_{2t}, and real foreign assets A_{12t-1}/p_{1t} and A_{21t-1}/p_{2t} are constant over time.

Adding government 2's budget constraint to the budget constraint of household 2 taken as equality, and combining the resulting equation to firm 2 zero profit condition, one gets the balance-of-payments:

$$p_{2t}c_{21t} - E_t p_{1t} c_{12t} + M_{21t} + A_{21t} - M_{21t-1} - A_{21t-1} - \\ - E(M_{12t} + A_{12t} - M_{12t-1} - A_{12t-1}) = 0 \tag{14}$$

We now focus on SCE in which the supply of each currency grows at a constant rate. Then we will study a SCE in which government 2 pegs the real exchange rate. After we will compare the equilibria.

To simplify the notation, denote the real exchange $E_t p_{1t}/p_{2t}$ rate by e_t, the real foreign assets A_{ijt-1}/p_{it} by a_{ijt}, the growth rate of the supply of currency i by μ_i, and the growth rate of p_{it} by π_{it}. Each μ_i is constrained to be positive. Assume that each government holds a constant nominal amount of foreign assets. It can be shown that in a SCE $\pi_{it} = \mu_i$ and all cash-in-advance constraints bind (otherwise, some transversality condition would not hold). So, the c's and the n's are constant. In a SCE with a floating exchange rate, both a_{ijt} and A_{ijt} are constant. Since p_{it} is growing, it follows that $A_{ijt} = a_{ijt} = 0$.

Proposition 1: *If both governments follow a μ% rule and the exchange rate floats, then the economy has a unique SCE, which is fully characterized by:*

$$c_{11}^* = \frac{b_1\beta_1\alpha_{11}}{\gamma_1(1+\mu_1) + \beta_1(\alpha_{11}+\alpha_{21})}, \quad c_{12}^* = \frac{b_1\beta_1\alpha_{21}}{\gamma_1(1+\mu_1) + \beta_1(\alpha_{11}+\alpha_{21})}$$

$$c_{21}^* = \frac{b_2\beta_2\alpha_{12}}{\gamma_2(1+\mu_2) + \beta_2(\alpha_{12}+\alpha_{22})}, \quad c_{22}^* = \frac{b_2\beta_2\alpha_{22}}{\gamma_2(1+\mu_2) + \beta_2(\alpha_{12}+\alpha_{22})}$$

$$n_1^* = \frac{\beta_1(\alpha_{11}+\alpha_{21})}{\gamma_1(1+\mu_1) + \beta_1(\alpha_{11}+\alpha_{21})}, \quad n_2^* = \frac{\beta_2(\alpha_{12}+\alpha_{22})}{\gamma_2(1+\mu_2) + \beta_2(\alpha_{12}+\alpha_{22})}$$

$$e^* = \frac{b_2\beta_2\alpha_{12}(1+\mu_2)[\gamma_1(1+\mu_1) + \beta_1(\alpha_{11}+\alpha_{21})]}{b_1\beta_1\alpha_{21}(1+\mu_1)[\gamma_2(1+\mu_2) + \beta_2(\alpha_{12}+\alpha_{22})]} \tag{15}$$

Proof: *See the appendix.*

Consider now a situation in which the supply of currency 1 grows at a constant rate μ_1. As before, government 1 will use its seignorage revenue to finance lump-sum transfers to household 1. Government 2 will make a lump-sum transfer in the amount of $\mu_2\%$ of the previous supply of currency 2. At the beginning of each date t, government 2 announces that it will buy and sell any amount of currency 1 at some price. This price is chosen to lead to a real exchange rate $\overline{e}$ that does not vary over time. Since we are concerned with the case in which currency 2 is over depreciated, we assume that $\overline{e} \geq e^*$.

Again, in a SCE all cash-in-advance constraints bind. The US inflation will be equal to μ_1. Inflation in country 2 will be constant, but might differ from μ_2.

Proposition 2: *If both governments follow a $\mu\%$ transfer rule and government 2 pegs the real exchange rate at some fixed level $\overline{e} \geq e^*$, then the economy has a unique SCE, which is fully characterized by:*

$$n_2' = \frac{\beta_2(\alpha_{12}+\alpha_{22})}{\gamma_2(1+\mu_2)+\beta_2(\alpha_{12}+\alpha_{22})}, c_{12}' = \frac{b_2\beta_2\alpha_{12}(1+\mu_2)}{(1+\mu_1)\overline{e}[\gamma_2(1+\mu_2)+\beta_2(\alpha_{12}+\alpha_{22})]}$$

$$n_1' = \frac{\beta_1\alpha_{11}}{\beta_1\alpha_{11}+\gamma_1(1+\mu_1)} + \frac{\gamma_1 b_2\beta_2\alpha_{12}(1+\mu_2)}{\overline{e}b_1[\beta_1\alpha_{11}+\gamma_1(1+\mu_1)][\gamma_2(1+\mu_2)+\beta_2(\alpha_{12}+\alpha_{22})]}$$

$$c_{11}' = \frac{\beta_1\alpha_{11}(b_1 - c_{12}')}{\beta_1\alpha_{11}+\gamma_1(1+\mu_1)}, a_{12}' = \frac{1+\mu_1}{\alpha_{11}\mu_1}(\alpha_{21}c_{11}' - \alpha_{11}c_{12}'), \pi_2' = \mu_2 + \frac{\overline{e}\mu_1}{b_2 n_2'}\, a_{12}'$$

$$c_{22}' = \frac{b_2\beta_2\alpha_{22}(1+\mu_2)}{(1+\pi_2')[\gamma_2(1+\mu_2)+\beta_2(\alpha_{12}+\alpha_{22})]}, c_{21}' = \frac{b_2\beta_2(\alpha_{12}+\alpha_{22})}{[\gamma_2(1+\mu_2)+\beta_2(\alpha_{12}+\alpha_{22})]} - c_{22}' \tag{16}$$

Moreover, $c_{11}', c_{21}', a_{21}'$, and π_2' are increasing and n_1', c_{12}', and c_{22}' are decreasing functions of $\overline{e}$; if $\overline{e} = e^*$, then $(a_{12}', c_{11}', c_{12}', c_{21}', c_{22}', n_1', n_2', \pi_2') = (0, c_{11}^*, c_{12}^*, c_{21}^*, c_{22}^*, n_1^*, n_2^*, \mu_2)$; if $\overline{e} > e^*$, then $a_{12}' > 0$.

Proof: *See the appendix.*

Intuition for the above results is very simple. Substitution effects explain the behavior of the c's with respect to the real exchange rate. If government 2 pegs the real exchange rate at the level e^*, the stationary effects of such a policy are null. If it pegs e above e^*, the consequence will be the existence of real transfers of resources, due to a trade balance surplus, from Brazil to the US. This transfer of resources is mirrored by the inflationary tax $\mu_1 a_{12}$ paid by country 2 to country 1. This additional expenditure by government 2 generates the extra factor that shows up in the expression for π_2'.

Calvo et alii (1995) argue that a government can successfully over depreciate its currency only temporarily. Had we assumed that $\mu_1 = 0$ we would have obtained the same conclusion. Inflationary transfers across governments is essential for the existence part of proposition 2.

Assume that the Brazilian economy was in a stationary competitive equilibrium with a pegged real exchange rate before the monetary reform and afterwards started to converge smoothly to a stationary equilibrium with $\mu\%$ transfers. The following fact should be observed: higher imports, lower exports, and lower inflation. It is well known that the Brazilian economy displayed these types of movements after the stabilization. So, at least at a qualitative level, the model accounts for some of the facts that followed the monetary reform.

4. Concluding Remarks

The Real Plan led to a spectacular reduction of the inflation rates in Brazil. A puzzling fact about this phenomenon is that it is not linked in an obvious way to any change in the Treasury deficit. A possible answer for this puzzle is the accumulation of foreign assets that was happening before the stabilization, which had to be financed with real resources.

In this paper we studied a simple two-country monetary model that accounted for the qualitative changes experienced by the Brazilian economy. We showed, as previously done by Montiel and Ostry (1991) and Calvo et alii (1995), that a policy that pursues a real over depreciation of the domestic currency will lead to higher inflation.

An obvious shortcoming of the paper is that it focuses only on stationary equilibria. A fruitful extension of this line of research is to study the transition dynamics. In addition, the public debt is a feature that deserves to be incorporated into the model.

A not so obvious shortcoming of the paper is the way the government behavior was modeled. Focusing on an exogenous change of policies prevents people's expectations to play a major role. The observed inflation rates are outcomes of a game of policy selection in which the largest player is the government. Chari and Kehoe (1993) provide a good example of such a game. In an environment where government behaves in an optimal fashion, people's reaction will change according to their beliefs of future government actions. The feedback between government actions and the expectations of private agents allows for several interesting events, as self fulfilling inflationary expectations. We believe that the most promising avenue for future studies is to endogenize the government policies.

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Appendix

A.1 Households' first order conditions

The first order necessary and sufficient conditions for household 1 maximization problem are:

$$\beta_1^{t-1}\alpha_{11}c_{11t}^{\alpha_{11}(1-\sigma_1)-1}c_{21t}^{\alpha_{21}(1-\sigma_1)}(1-n_{1t})^{\gamma_1(1-\sigma_1)} = \theta_{1t}p_{1t} + \lambda_{1t}p_{1t} \tag{17}$$

$$\beta_1^{t-1}\alpha_{21}c_{11t}^{\alpha_{11}(1-\sigma_1)}c_{21t}^{\alpha_{21}(1-\sigma_1)-1}(1-n_{1t})^{\gamma_1(1-\sigma_1)} = \psi_{1t}p_{2t} + \lambda_{1t}p_{2t}/E_t \tag{18}$$

$$\beta_1^{t-1}\gamma_1 c_{11t}^{\alpha_{11}(1-\sigma_1)}c_{21t}^{\alpha_{21}(1-\sigma_1)}(1-n_{1t})^{\gamma_1(1-\sigma_1)-1} = \lambda_{1t}w_{1t} \tag{19}$$

$$M_{11t-1} \geq p_{1t}c_{11t} \qquad \text{and} \qquad M_{21t-1} \geq p_{2t}c_{21t} \tag{20}$$

$$[M_{11t-1} > p_{1t}c_{11t} \Rightarrow \theta_{1t} = 0] \quad \text{and} \quad [M_{21t-1} > p_{2t}c_{21t} \Rightarrow \psi_{1t} = 0] \tag{21}$$

$$\theta_{1t+1} - \lambda_{1t} + \lambda_{1t+1} = \psi_{1t+1} - \lambda_{1t}/E_t + \lambda_{1t+1}/E_{t+1} = 0 \tag{22}$$

$$M_{11t} + M_{21t}/E_t = (M_{11t-1} - p_{1t}c_{11t}) + (M_{21t-1} - p_{2t}c_{21t})/E_t + w_{1t}n_{1t} + T_{1t} \tag{23}$$

$$\lim_{t\to\infty} \lambda_{1t}M_{11t} = \lim_{t\to\infty} \lambda_{1t}M_{21t}/E_t = 0 \tag{24}$$

where θ_{1t} and ψ_{1t} are Lagrange multipliers for, respectively, the first and the second cash-in-advance constraints and λ_{1t} for the budget constraint.

Similarly, the necessary and sufficient conditions of the maximization problem of household 2 are:

$$\beta_2^{t-1}\alpha_{22}c_{22t}^{\alpha_{22}(1-\sigma_2)-1}c_{12t}^{\alpha_{12}(1-\sigma_2)}(1-n_{2t})^{\gamma_2(1-\sigma_2)} = \theta_{2t}p_{2t} + \lambda_{2t}p_{2t} \tag{25}$$

$$\beta_2^{t-1}\alpha_{12}c_{22t}^{\alpha_{22}(1-\sigma_2)}c_{12t}^{\alpha_{12}(1-\sigma_2)-1}(1-n_{2t})^{\gamma_2(1-\sigma_2)} = \psi_{2t}p_{1t} + \lambda_{2t}E_t p_{1t} \tag{26}$$

$$\beta_2^{t-1}\gamma_2 c_{22t}^{\alpha_{12}(1-\sigma_2)}c_{12t}^{\alpha_{12}(1-\sigma_2)}(1-n_{2t})^{\gamma_2(1-\sigma_2)-1} = \lambda_{2t}w_{2t} \tag{27}$$

$$M_{22t-1} \geq p_{2t}c_{22t} \qquad \text{and} \qquad M_{12t-1} \geq p_{1t}c_{12t} \tag{28}$$

$$[M_{22t-1} > p_{2t}c_{22t} \Rightarrow \theta_{2t} = 0] \quad \text{and} \quad [M_{12t-1} > p_{1t}c_{12t} \Rightarrow \psi_{2t} = 0] \tag{29}$$

$$\theta_{2t+1} - \lambda_{2t} + \lambda_{2t+1} = \psi_{2t+1} - \lambda_{2t}E_t + \lambda_{2t+1}E_{t+1} = 0 \tag{30}$$

$$M_{22t} + E_t M_{12t} = (M_{22t-1} - p_{2t}c_{22t}) + E_t(M_{12t-1} - p_{1t}c_{12t}) + w_{2t}n_{2t} + T_{2t} \tag{31}$$

$$\lim_{t\to\infty} \lambda_{2t}M_{22t} = \lim_{t\to\infty} \lambda_{2t}E_t M_{12t} = 0 \tag{32}$$

A.2 Proof of Proposition 1

Consider the equations $e(1+\mu_1)c_{12} = (1+\mu_2)c_{21}$, $b_1\beta_1\alpha_{11}(1-n_1) = \gamma_1(1+\mu_1)c_{11}$, $\alpha_{11}(1+\mu_2)c_{21} = \alpha_{21}(1+\mu_1)ec_{11}$, $b_2\beta_2\alpha_{22}(1-n_2) = \gamma_2(1+\mu_2)c_{22}$, and $\alpha_{12}(1+\mu_2)c_{22} = \alpha_{22}(1+\mu_1)ec_{12}$, plus the two equations in (3) without the subscript t. The first of those equations is the balance-of-payments in a SCE. The second and the third are obtained from a combination of (17), (18), (19), and (22). The fourth and fifth are derived from (25), (26), (27), and (30). The unique solution for that system of equations is given by (15). This establishes the uniqueness part. We finish the proof showing that from (16) we can construct the sequences mentioned in the definition of a CE. Set the sequences of consumption and labor as given in (15). Set $M^*_{ij0} = c^*_{ij}$ and $p^*_{i1} = 1$ and let those variables grow at the rate μ_i. Choose wages by letting the real wage in country i be equal to b_i. Foreign assets must be set at zero. Set the transfers by $T^*_{it} = \mu_i(M^*_{iit-1} + M^*_{ijt-1})$. Finally, $E^*_t = M^*_{21t}/M^*_{12t}$. Now, choose the Lagrange multipliers as given in equations (17), (18), (19), (25), (26), and (27). It is a long, but straightforward, exercise to show that all conditions for a CE are satisfied.

A.3 Proof of Proposition 2

Consider the following equations:

$$b_2\beta_2\alpha_{22}(1-n_2) = \gamma_2(1+\pi_2)c_{22} \tag{33}$$

$$\alpha_{12}(1+\pi_2)c_{22} = \alpha_{22}(1+\mu_1)\overline{e}c_{12} \tag{34}$$

$$b_1\beta_1\alpha_{11}(1-n_1) = \gamma_1(1+\mu_1)c_{11} \tag{35}$$

$$\alpha_{11}(1+\pi_2)c_{21} = \alpha_{21}(1+\mu_1)\overline{e}c_{11} \tag{36}$$

$$(1+\pi_2)c_{22} + (1+\mu_1)\overline{e}c_{12} = b_2n_2 + \mu_2(c_{21}+c_{22}) \tag{37}$$

$$(1+\pi_2)c_{21} = \overline{e}[(1+\mu_1)c_{12} + \mu_1a_{12}] \tag{38}$$

plus the two resource constraints in (3). Equations (33) and (34) are first order conditions of household 1; (35) and (36) are first order conditions of household 2; (36) consolidates the stationary budget constraints of household 2 and government 2; (38) is the stationary balance-of-payments. From (33) and (34) we get $b_2\beta_2\alpha_{12}(1-n_2) = \gamma_2(1+\mu_1)\overline{e}c_{12}$; (34), (37) plus the resource

constraint for c_2 yield $\alpha_{12}(1+\mu_2)b_2n_2 = (\alpha_{12}+\alpha_{22})(1+\mu_1)\overline{e}c_{12}$. Solving those two equations simultaneously, we get n_2' and c_2'.

Multiplying the resource constraint for c_1 by $\gamma_1(1+\mu_1)$ and using (35), we get $b_1\beta_1\alpha_{11}(1-n_1)+\gamma_1(1+\mu_1)c_{12} = \gamma_1(1+\mu_1)b_1n_1$. Solving for n_1 and using the expression just obtained for c_{12}', we get n_1'. Using the resource constraint for c_1, we obtain c_{11}'. Multiplying both sides of (38) by α_{11} and using (36), we get a_{12}'. From (37) and (38) we obtain $(\pi_2-\mu_2)(c_{21}+c_{22}) = \overline{e}\mu_1 a_{12}$. This equation combined with the resource constraint for c_2 yields π_2'. We obtain c_{22}' by plugging the expression for c_{12}' into (34). Finally, c_{21}' can be obtained combining the resource constraint for c_2 and the expressions for n_2' and c_{22}'. The reasoning used in the previous proof shows that (16) generates a SCE. The increasing and decreasing part is trivial. Setting $\overline{e} = e^*$ in (16) and carrying out some algebra we conclude that $(a_{12}', c_{11}', c_{12}', c_{21}', c_{22}', n_1', n_1', \pi_2') = (0, c_{11}^*, c_{12}^*, c_{21}^*, c_{22}^*, n_1^*, n_2^*, \mu_2)$. The last part follows from the fact that a_{12}' is increasing on $\overline{e}$.

A.4 Inflation Tax and International Assets

Year/ Month	Inflation tax (US$ million) (1)	Variation of international assets (US$ million)			Ratios		
		cash (2)	liquidity (3)	BOP (4)	(2)/(1) (5)	(3)/(1) (6)	(4)/(1) (7)
1993							
October	1236	2113	2071	2069	1.71	1.68	1.67
November	1304	2061	1992	2004	1.58	1.53	1.54
December	1736	1588	1200	1210	0.91	0.69	0.70
Total	4276	5762	5263	5283	1.35	1.23	1.24
1994							
January	1626	3260	3179	3190	2.00	1.96	1.96
February	1653	1387	1152	1311	0.84	0.70	0.79
March	1533	1770	1740	1751	1.15	1.14	1.14
April	1403	2787	7	10	1.99	0.01	0.01
May	1364	3188	3119	3120	2.34	2.29	2.29
June	1631	1861	1473	1480	1.14	0.90	0.91
Total	9213	14253	10670	10862	1.55	1.16	1.18

Sources: (1) author's computation; (2), (3), and (4) Brazilian Central Bank.