Is There Any Rationale to the Brazilian Fiscal Policy?*

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Summary: 1. Introduction; 2. The model; 3. Test procedures; 4. The data; 5. Empirical results; 6. Concluding remarks.

Key words: tax smoothing; vector autoregression; Brazil.

JEL codes: H6; H62 and H63.

Barro's tax smoothing hypothesis (TSH) implies that, when faced by a temporary increase in expenditure, the government should issue debt in order to spread the increase in taxes over time and minimize the welfare costs of high tax rates. Changes in the tax rate should then be unpredictable. This paper performs a test of the TSH that goes beyond the random walk tests usually done in literature. It examines the implied restrictions of the TSH on a vector autoregression (VAR) model using Brazilian data for the period 1970-94. The tests reject the hypothesis for the full sample period. For the period 1987-92, however, disregarding the volatility of the actual budget surplus, the TSH seems to provide a better approximation to the historical movements of Brazilian fiscal data series.

A hipótese de suavizamento de impostos de Barro implica, que quando o Governo se defronta com um aumento temporário nos seus gastos, ele deve emitir dívida a fim de espalhar o aumento de impostos ao longo do tempo e minimizar os custos de bem estar associados com os impostos mais altos. Neste artigo, faz-se um teste da hipótese de suavizamento de impostos que vai além dos tradicionais testes de passeio aleatório feitos na literatura. Examina-se as restrições sobre os gastos, as receitas e os déficits brasileiros trazidas pela hipótese de suavizamento, usando-se um modelo de vetores autoregressivos para 1970-94. Os testes rejeitam a hipótese quando o período amostral completo é considerado. Contudo, para o período 1987-92, desconsiderando-se a volatilidade dos saldos orçamentários efetivos, a hipótese de suavizamento de impostos parece fornecer uma aproximação melhor para os movimentos históricos das séries fiscais brasileiras.

1. Introduction

It is hard to believe that any government, and in particular the Brazilian government, follows a debt management policy. Barro (1979) proposed a simple theory of how governments operate. Since the government has the objective of minimizing the present value of excess burden for a given present value of tax collections, it should spread the burden of raising taxes over time. Therefore, when faced with a temporary increase in expenditure it should finance the temporary difference between expenditures and revenues by issuing debt in order to reduce the welfare costs associated with high taxes.

The question of whether or not the time series properties of Brazilian fiscal data are consistent with the above observable implications has never been examined before. The purpose of this paper is to test the theory of optimal taxation over time using data for 1970-94 in order to verify if it gives a rationale to the pattern of budget deficits and taxes in Brazil over this period. The methodology used was initially developed by Campbell (1987) to analyze the optimal pattern of savings. The assumption of tax smoothing implies that the budget surplus should equal the expected present discounted value of the change in government expenditures. If the government expects a temporary increase in its expenditures, it should spread the tax burden over time in order to minimize the distortionary effects of taxes. In other terms the increase in expenditures should be financed by means of a budget deficit. If the government expects a permanent increase in its expenditures it should raise taxes immediately in order to minimize the tax rate when the higher expenditure is actually observed. Given the analogy between a consumer who tries to smooth consumption and a government who tries to smooth taxes, Ghosh (1995b) adapts Campbell's methodology to study the behavior of the optimal budget surplus.[1] The result that the budget surplus should equal the present discounted value of expected changes in government expenditures has two implications. First, it is possible to construct a time series for the optimal budget surplus and compare it to the actual surplus. Under the hypothesis that the government smoothes taxes these two series should differ only by sampling error. Second, as long as the government has more information about the path of future expenditure than is contained in past values of the expenditure series, the budget surplus will have this additional information and thus Granger-cause changes in government expenditures.

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[1] *Ghosh (1995a) applies the same idea to the behavior of current account surpluses.*

The paper is organized as follows. Section 2 presents Barro's model of optimal taxation over time. Section 3 discusses Ghosh's test. It tries to derive the implications of the smoothing hypothesis for the time series behavior of budget surpluses and government expenditures. Section 4 outlines the data to be used in the empirical analysis. Section 5 applies the test to the Brazilian fiscal data. Section 6 presents the conclusions and provides some suggestions for further research.

2. The Model

Barro's tax smoothing model establishes that if a government must rely on distortionary taxes, the tax rate should be set on the basis of permanent (noninterest) government expenditures with transitory expenditure fluctuations financed by issuing debt. The purpose of this section is to derive the optimal path of taxes associated with the financing of a given path for government expenditure.

The problem facing the government is:

$$V = \text{Max}(-1/2) \sum_{i=0}^{\infty} \beta^i E\{\tau_{t+1}^2 | \Omega_t\} \qquad 0 < \beta < 1 \qquad (1)$$

where the distortionary costs are assumed proportional to the square of the tax rate and where β is the government's subjective discount rate. $E\{\cdot | \Omega_t\}$ is the expectation conditional on the government's information set at time t, Ω_t.

The government maximizes (1) subject to its budget constraint:

$$B_{t+1} = (1 + r)B_t + G_t - \tau_t Y_t$$

where B is the stock of government debt, G is government expenditure, r is a real rate of interest (which is assumed, for the sake of simplicity, to be constant over time), τ is the tax rate, and Y is output.

Or:

$$(1 + n)b_{t+1} = (1 + r)b_t + g_t - \tau_t \qquad (2)$$

where lowercase letters denote corresponding variables expressed as a fraction of the GDP, and where n is the rate of growth of output (which is also assumed to be constant for the sake of analytic convenience).

The government's intertemporal budget constraint states that the present value of government expenditures must equal the present value of tax revenues, net of initial indebtedness:

$$\sum_{i=0}^{\infty} G_{t+i}/(1+r)^i = \sum_{i=0}^{\infty} \tau_{t+i} Y_{t+i}/(1+r)^i - (1+r)B_t$$

or

$$\sum_{i=0}^{\infty} g_{t+i}\left[(1+n)/(1+r)\right]^i = \sum_{i=0}^{\infty} \tau_{t+i}\left[(1+n)/(1+r)\right]^i - (1+r)b_t \qquad (3)$$

Solving the optimization problem (1), subject to (2) and (3), yields the optimal tax rate in each period:

$$\tau_t = \gamma\{1 - R\} \sum_{i=0}^{\infty} R^i E\{g_{t+i}|\Omega_t\} + (1+r)b_t\gamma\{1 - R\}$$

$$= \gamma\left\{(1 - R) \sum_{i=0}^{\infty} R^i E\{g_{t+i}|\Omega_t\} + (r - n)b_t\right\} \qquad (4)$$

where $R = (1+n)/(1+r)$, and where $\gamma = [(1 - (R/\beta)R/(1 - R)]$.

Therefore, taxes are proportional to the present discounted value of government expenditure plus the effective real interest cost of servicing the initial debt. Assuming the government's subjective discount rate, β, is equal to its effective interest rate, R (so that γ is equal to unity), the optimality condition (4) establishes that taxes should equal the expected present value of government expenditure and not just government expenditure in the current period.

One implication of (4) is that optimally set tax rates should follow a random walk:

$$\tau_t - \tau_{t-1} = (1 - R) \sum_{i=0}^{\infty} R^i E\{g_{t+i}|\Omega_t\} + (r - n)b_t$$

$$- (1 - R) \sum_{i=0}^{\infty} R^i E\{g_{t+i-1}|\Omega_{t-1}\} + (r - n)b_{t-1}$$

$$= (1 - R) \sum_{i=0}^{\infty} R^i E\{g_{t+i}|\Omega_t\} + (r - n)b_t$$

$$- \left\{ (1-R) \sum_{i=1}^{\infty} R^{i-1} E\{g_{t+i-1}|\Omega_{t-1}\} + (r-n)b_t \right\}$$

$$= (1-R) \sum_{i=0}^{\infty} R^i \left[E\{g_{t+i}|\Omega_t\} - E\{g_{t+i}|\Omega_{t-1}\} \right] \qquad (5)$$

Since the right-hand side of (5) is an expectational error, it should be unpredictable based on information available in period $t-1$ or earlier. Thus, (5) states that under tax smoothing, tax rates should follow a random walk.

Barro (1981) and Sahasakul (1986) used this random walk property to test whether governments have tried to smooth taxes. Barro found that US public debt issue responds positively to temporary increases in government expenditures but responds negatively to temporary increases in aggregate output. Sahasakul made a joint test of the hypotheses of uniform taxation over time and the reliability of a specific forecasting equation for the permanent government spending variable. The joint hypotheses imply that only permanent government spending and initial public debt determine the marginal tax rates. Since there are other variables in the current information set that help to determine the marginal tax rate (temporary defense purchases, the general price level, and a time trend), he rejects the tax smoothing hypothesis. Although Barro and Sahasakul came to different conclusions, their econometric analyses require that government spending and aggregate output be empirically decomposed into permanent and temporary components. So, whether or not their empirical finding is valid depends upon the specification of variables and equations in their construction of the temporary or permanent components of the series.

Trehan and Walsh (1988) argued that if expenditures are nonstationary, taxes will also be nonstationary even in the absence of smoothing. Therefore, in order to find support to the smoothing hypothesis one should check if the difference between tax receipts and net of interest expenditures is nonstationary. The authors' empirical tests, however, showed that this difference is stationary for the United States for 1890-1928, implying rejection of the tax smoothing hypothesis.

Huang and Lin (1993) focused on the budget surplus instead of the tax rate. They derived and tested the restrictions imposed by the tax smoothing hypothesis on the process of the budget deficit (surplus), government expenditure, and aggregate output using a vector autoregression (VAR). They

concluded that the tax smoothing hypothesis provides a good approximation for US fiscal data from 1929 to 1988.

3. Test Procedures

This section presents Ghosh's approach (1995b) to test the theory of optimal taxation over time. It will be used to analyze Brazilian fiscal policy in the next section. There are several reasons that justify this choice.

First, the test goes beyond the random walk tests of tax smoothing models. Since tax smoothing is only one of many possible explanations for tax rates following a random walk, it is important to try to test all of the time series implications of the optimal tax path.

Second, the tax smoothing hypothesis is tested directly without any empirical pre-decomposition of data into permanent and temporary components. Therefore, it avoids any specification error derived from inappropriate data decomposition.

Third, when the government runs a budget deficit it is (potentially) motivated by two considerations: tax tilting and tax smoothing. Tax smoothing is the intertemporal substitution of taxes in order to give them a relatively flat time profile. Tax tilting occurs when the burden of taxation is shifted either toward or away from the present, depending on the government's subjective discount rate, but in a manner which is consistent with intertemporal solvency. If the ratio between government expenditure and GDP is constant over time, there is no tax smoothing component of budget deficits. However, a tax tilting component may exist. When $\beta < R$, the government has a high discount rate relative to the market interest rate. It runs large deficits early on and increases taxes over time to serve its accumulating debt. In this case, γ is less than unity. When $\beta > R$, the government has a low discount rate relative to the market interest rate. It chooses relatively high rates early on in order to build up a stock of assets, and plans to lower taxes over time. If the ratio between government expenditure and GDP follows a perfect cycle, and if $\beta = R$, there is no tax tilting motive for budget deficits but there would be a strong tax smoothing motive. Ghosh's method distinguishes between these two motivations.[2] The primary focus of the analysis is on the

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[2] *Huang and Lin's (1993) approach is very similar to Ghosh's but uses a log-linear version of the model and does not allow for tax tilting.*

tax smoothing component of the budget deficits. Therefore, the long-term trend in the public saving does not matter, but only the short run dynamics of the budget deficits around their trend. Intertemporal solvency constraint is, then, imposed. Therefore, it is necessary initially to remove the tax tilting component. After that it is possible to identify correctly the consumption smoothing component of the tax rate with which the model is concerned.

The procedure is as follows.

First, the budget surplus is defined as $SUR_t = (1 + n)(b_t - b_{t+1})$.

Using (2) and (4) we have:

$$
\begin{aligned}
SUR_t &= \tau_t - g_t - (r - n)b_t \\
&= (1 - R) \sum_{i=0}^{\infty} R^i E\{g_{t+i}|\Omega_t\} - g_t \\
&= (1 - R) \sum_{i=0}^{\infty} R^i \left\{ \sum_{j=1}^{i} E\{\Delta g_{t+j}|\Omega_t\} \right\} \\
&= (1 - R) \sum_{j=1}^{\infty} \left\{ \sum_{i=j}^{\infty} R^i \right\} E\{\Delta g_{t+j}|\Omega_t\} \\
&= (1 - R) \sum_{j=1}^{\infty} \frac{R^j}{1 - R} E\{\Delta g_{t+j}|\Omega_t\} \\
&= \sum_{j=1}^{\infty} R^j E\{\Delta g_{t+j}|\Omega_t\}
\end{aligned}
\tag{6}
$$

where $\Delta g_{t+j} = g_{t+j} - g_{t+j-1}$ is the backward difference.

Expression (6) embodies all the implications of optimal intertemporal tax smoothing. The budget surplus is equal to the expected present discounted value of changes in government expenditure. Assuming that the government expects a future increase in its expenditures, waiting to raise taxes would result in relatively large tax rates and distortionary cost. The optimal strategy requires that the government raise taxes immediately in order to minimize the distortion in any period. Because taxes increase prior to the rise in expenditures, the government runs a larger budget surplus (smaller deficit). Expression (6) gives the optimal budget surplus, and it is going to be written hereafter as SUR_t^*.

The tax smoothing hypothesis is then tested by comparing the optimal budget surplus to the actual budget surplus. In order to do so, it is first necessary to create the right hand side of (6a). The difficulty is that the path of the government expenditure may depend on institutional and political factors that are not completely known by the econometrician. However, if the government has more information about the path of future expenditure than is contained in lagged values of Δg_t, the budget surplus, which is equal to the expected future changes in g, will contain this additional information. Thus, the first implication of the tax smoothing hypothesis is that the budget surplus should Granger-cause subsequent changes in government expenditure.[3]

The estimation proceeds as follows. To create the optimal budget surplus it is necessary to calculate the expected present discounted value of changes in government expenditure, where the expectation is conditional on information available at time t. According to Campbell (1987), the following unrestricted vector autoregression (VAR) model in Δg_t and SUR_t is estimated, where SUR_t is the actual tax smoothing component of the budget surplus defined as $SUR_t = (1/\gamma)\tau_t - [g_t + (r-n)b_t]$:

$$\begin{bmatrix} \Delta g_t \\ SUR_t \end{bmatrix} = \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_3 & \psi_4 \end{bmatrix} \begin{bmatrix} \Delta g_{t-1} \\ SUR_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{7}$$

or $Z_t = \Psi Z_{t-1} + U_t$, where $Z = [\Delta g_t, SUR_t]'$, Ψ is the transition matrix of the VAR, and U_t is a white noise errors vector. A first order VAR is used because, as will be seen soon, one lag suits better the Brazilian data. However, it is possible to generalize this expression for higher order VARs just stacking the lagged variables. The optimal forecast of Z_t k periods ahead, given $\{Z_t, Z_{t-1}, \cdots\}$, should satisfy $E_t Z_{t+k} = \Psi^k Z_t$ for $k \geq 1$. Using this formula, (6a) can be translated into the following expression:

$$[01]Z_t = \sum_{i=1}^{\infty} R^i \Psi^i [10] Z_t \tag{8}$$

Since Δg_t and SUR_t are both stationary, the infinite sum in (8) converges to:

$$SUR_t^* = [10] R\Psi [I - R\Psi]^{-1} Z_t = \Lambda Z_t \tag{9}$$

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[3] *As shown by Campbell (1987), under the null hypothesis SUR_t Granger-causes Δg_t unless SUR_t is an exact function of current and lagged Δg_t.*

Assuming that g_t is $I(1)$ its first difference will be stationary, and since the budget surplus is a discounted sum of Δg_t (by expression (6)), it will also be stationary.

Once the optimal budget surplus series SUR_t^* has been calculated, a number of tests may be performed. First, as observed previously, the model predicts that the budget surplus should Granger-cause changes in government expenditure. This provides a simple test of the model.

Second, a more formal test of the model can be done rewriting equation (9) as:

$$SUR_t^* = \Lambda Z_t = [\lambda_1 \ \lambda_2] \begin{bmatrix} \Delta g_t \\ SUR_t \end{bmatrix} \qquad (10)$$

If the null hypothesis is correct, the weight λ_1 on Δg_t should be zero and the weight λ_2 on SUR_t should be unity. If the VAR has an order higher than 1, the coefficient on the contemporaneous budget surplus (SUR_t) should be unity and the coefficients on all other variables should be zero.[4]

Third, the above VAR model can also be used for informally evaluating the performance of the tax smoothing hypothesis. SUR_t^* is the "theoretical" budget surplus, that is, the optimal VAR forecast of the present value of future growth rates of government expenditures. Since SUR_t is included in the current information set, according to (6), under the tax smoothing hypothesis SUR_t^* should differ from SUR_t only by sampling error. Therefore, the plausibility of the tax smoothing hypothesis can be informally evaluated by comparing the predicted surplus, SUR_t^*, to the actual surplus, SUR_t.

Fourth, the variances and the sample correlation between SUR_t^* and SUR_t should be compared. If the variance of the optimal budget surplus is larger than the variance of the actual (tax smoothing) budget surplus, the actual surplus has not changed enough to smooth taxes in face of shocks to expenditures.

Finally, it is necessary to explain how the tax tilting parameter is calculated. So far it has been assumed that the government does no tax tilting, that is, $\gamma = 1$. However, now it is necessary to estimate γ in order to eliminate

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[4]*It is important to call attention to the fact that (10) is not a regression equation. The model determines λ_1 and λ_2 as nonlinear functions of the VAR parameters and implies the especified restrictions.*

the tax tilting component of the actual data on the budget surplus. When γ is different from unity the tax smoothing component of the budget surplus can be defined as:

$$SUR_t = (1/\gamma)\tau_t - [g_t + (r-n)b_t] \qquad (11)$$

As argued previously, the optimal budget surplus series will be an $I(0)$ process. Under the null hypothesis that the actual tax smoothing component of the surplus is equal to the optimal budget surplus, the actual budget surplus is also $I(0)$. This means that the left hand side of equation (11) is $I(0)$. If τ_t and $[g_t + (r-n)b_t]$ are $I(1)$, (11) defines a cointegrating regression and $(1/\gamma)$ can be obtained by regressing $[g_t + (r-n)b_t]$ and τ_t. $(1/\gamma)$ can also be obtained as the cointegrating vector of the cointegration regression between $[g_t + (r-n)b_t]$ and τ_t.

4. The Data

The data on expenditures and on revenues are from *Anuário Estatístico do Brasil*, several issues. Since the tax smoothing hypothesis is tested using expenditures other than interest payments, interest payments (also from *Anuário Estatístico do Brasil*) are subtracted from expenditures when the VAR is estimated. The tax tilting parameter test, however, uses the government expenditure including real interest rate payments. Given that the data on expenditures includes payments of nominal interest on public debt, the actual series differs from the theoretical one. As seigniorage is an important source of revenue to the Brazilian government, it is added to tax revenues . It is calculated as ΔB_t, where B_t is the monetary base (end-of-the-year values) extracted from *Boletim do Banco Central*, several issues. All data are annual series for the period 1970-94. All variables are divided by GDP.[5]

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[5] *Some could argue that there are too few observations to make the regressions meaningful. The tax smoothing test requires a series of expenditures excluding interest payments and the tax tilting estimate requires a series of expenditures including interest payments. Since it was possible to obtain the two different series of expenditures only for the period after 1970, the span of the data used is really short. The estimations, however, were also performed using data from 1947-94, where the series from 1947-92 was kindly provided by João Victor Issler. As the main conclusions do not change, only the results for the period 1970-94 are presented since the variables used match the theoretical model.*

5. Empirical Results

Table 1 reports augmented Dickey-Fuller statistics on g_t (expenditures excluding interest payments), g_t^* (expenditures including interest payments), τ_t (tax revenues plus seigniorage), and SUR_t (budget surplus).

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Table 1

Unit root tests

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Series	Lags	ADF Test
g_t	2	-1.47
g_t^*	2	-2.04
τ_t	2	-0.33
SUR_t	1	-2.45*

Notes: The number of lagged terms was chosen to ensure that the errors are uncorrelated. For g, g^*, and τ, a constant and a time trend were included. Critical values are -3.53 and -4.20 for the 5% and 1% significance levels, respectively. Tests for two unit roots, not shown here, reject the null hypothesis for all three series.
For SUR, the nonconstant specification was used. Critical values are -1.95 and -2.62 for the 5% and 1% significance levels, respectively.
*represents rejection of the null of a unit root at the 5% significance level.

As previously observed, as long as g_t follows an $I(1)$ process, its first difference will be stationary, and from (6), the budget surplus, which is the discounted value of a stationary series, will also be stationary. Therefore, one implication of the model is that SUR_t should be stationary, even if g_t is not. The unit root test on g_t cannot reject the null of nonstationarity. The null of a unit root in SUR_t, on the other hand, can be rejected at the 5% level.

The estimated tax tilting parameter $(1/\gamma)$ is 0.944 (s.e. = 0.039).

The regression of $[g_t + (r - n)b_t]$ on τ_t, therefore, results in a coefficient not significantly different from unity. A value of $(1/\gamma)$ equal to 1 is consistent with the theoretical model, that is, there is no tax tilting component to the budget surplus. The budget surplus movements associated with the tax tilting motive are not included in the analysis.[6]

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[6] As observed, the tax tilting parameter can also be estimated as the cointegrating vector between expenditures including interest payments and revenues including seigniorage. Pastore (1995), Rocha (1997), and Issler and Lima (2000) test for cointegration between g^* and τ in order to verify if the debt is sustainable. They find that in fact the two series cointegrate and, therefore, the Brazilian government is solvent. The same result is not obtained when seigniorage is excluded.

In order to choose the lag length for the VAR, the Schwarz criteria is evaluated (table 2).[7]

Table 2

VAR truncation

VAR order		Constant	Schwarz criteria
a)			
	1	Unrestricted	-7.267
	2	Unrestricted	-7.133
	3	Unrestricted	-6.613
b)			
	1	No constant	-7.492
	2	No constant	-7.269
	3	No constant	-6.723

Note: The VARs were estimated without a linear trend.

The optimal lag length is 1. Lagrange multiplier tests for serial correlation of second order indicate that one lag is sufficient to guarantee that the residuals are white noise. For the first equation (Δg_t) the test statistic is 2.55 with a p-value equal to 0.11. For the second equation (SUR_t) the test statistic is 1.96 with a p-value equal to 0.17.

Table 3 reports the VAR coefficients. It is possible to verify if the budget surplus Granger-cause changes in the government expenditures by evaluating the coefficient of SUR_{t-1} in the equation for Δg_t. This coefficient is not statistically significant.

Table 3

VAR coefficients

	Δg_t	SUR_t
Δg_{t-1}	-0.281	0.875
	(0.506)	(0.496)
SUR_{t-1}	-0.118	0.914
	(0.224)	(0.233)

Note: White heteroskedasticity-consistent standard errors in parentheses. SUR_t does not Granger-cause Δg_t.

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[7]It is important to remember that in the VAR, $SUR_t = (1/\gamma)\tau_t - [g_t + (r-n)b_t]$ therefore, it is still necessary to check if the actual (detrended) surplus is stationary. The Dickey-Fuller statistic -2.37 is for a regression including only the constant and one lag. Since the 5% critical value is -1.95 it is possible to reject the null, and in fact SUR_t is $I(0)$.

Table 4 presents formal Granger causality tests (which are just F-tests that all coefficients on lagged budget surplus in the government expenditure are jointly zero). Lags from 2 to 4 are used in order to check the robustness of the results. For none of the lags it is possible to reject the null of no Granger causality, so the data is inconsistent with the most basic implication of the tax smoothing behavior.

Table 4

Granger causality tests

Lags	F statistic	p-value
2	0.785	0.471
3	1.222	0.336
4	0.851	0.520

Note: Tests that SUR_t Granger-causes Δg_t.

As discussed previously, if the tax smoothing hypothesis is correct, λ_1 should equal zero, and λ_2 should equal 1. This is equivalent to a simple condition on the VAR transition matrix Ψ. Given that $SUR_t^* = [10]R\Psi$ $[I - R\Psi]^{-1} Z_t$, and $SUR_t = [01]Z_t$, $SUR_t^* = SUR_t$ if $[10]R\Psi[I - R\Psi]^{-1} = [01]$.

Post-multiplying by $[I - R\Psi]$ and adding $[01]R\Psi$ yields:

$$[10]R\Psi + [01]R\Psi = [01]$$

or

$$[11]R\Psi = [01]$$

Thus, the sum of the elements of the first column of $R\Psi$ should be zero and the sum of the elements of the second column of $R\Psi$ should be 1.

Since R is approximately equal to 1, the sum of the elements of the first column of Ψ should be approximately equal to zero, or $\psi_1 + \psi_3 \cong 0$, and the sum of the elements of the second column of Ψ should be approximately close to 1, or $\psi_2 + \psi_4 \cong 1.$[8] According to table 3, $\psi_1 + \psi_3 = 0.574$, which is significantly different from zero [$\chi^2_{(1)} = 11.36$], and $\psi_2 + \psi_4 = 0.796$ which is significantly different from one [$\chi^2_{(1)} = 38.71$].

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[8] *In fact, using actual Brazilian data for the period 1970-94, R=(1+n)/(1+r) is very close to 1.*

The performance of the model is next evaluated by comparing the theoretical surplus SUR_t^* with the actual (detrended) budget surplus, $SUR_t = (1/\gamma)\tau_t - [g_t + (r-n)b_t]$. The relation between SUR_t^* and SUR_t is summarized by the correlation coefficients and the standard deviation ratios of the two series, which are reported in table 5. The two series have a negative correlation coefficient of -0.684 for the period 1970-94. Besides, actual budget movements have been two times as large as would have been necessary for tax smoothing. If, however, attention is focused on the period 1987-92, although the standard deviation ratio increases, the tax-smoothing model tracks much better budget surplus movements.

Although the actual budget has been almost six times as large as would have been necessary for tax smoothing, the two series now become positively correlated. In other terms, for this subperiod the model explains better the budget surplus movements though not necessarily the magnitudes of those movements.

Table 5

Summary statistics of theoretical and actual (detrended) surplus

Period	$\sigma(SUR_t)$	$\sigma(SUR_T^*)$	$\sigma(SUR_t)/\sigma(SUR_T^*)$	Corr
1970-94	0.057	0.026	2.16	-0.684
1970-86	0.040	0.023	1.74	-0.926
1987-92	0.078	0.013	6.00	0.121

A visual impression of these results is given in figure 1.

The theoretical budget surplus generally does not match the tax smoothing component of the budget surplus , indicating absence of evidence supporting the tax smoothing hypothesis. Till 1983 the actual surpluses are positive, indicating taxes greater than expenditures, while the optimal surpluses are negative. In other terms, the government was running a surplus when the optimal, in order to smooth taxes, would be to run a deficit. After 1983, the reverse happens. The optimal surpluses are mostly positive and the actual surpluses are mostly negative. If, however, attention is focused on the period 1987-92, disregarding the magnitude of actual flows, i.e., the volatility of the

actual budget surpluses, the two series follow each other movements.[9] It is important to observe that the results for the 1987-92 period are based only on the visual impression conveyed by the picture, and confirmed by the standard deviation ratios and the correlation coefficients. There are no sufficient observations to perform a formal test. Once again, during this period actual and predicted budget surpluses still do not move closely together but at least they follow partly the same movements.

Figure 1
Actual (detrended) budget surplus and optimal budget surplus
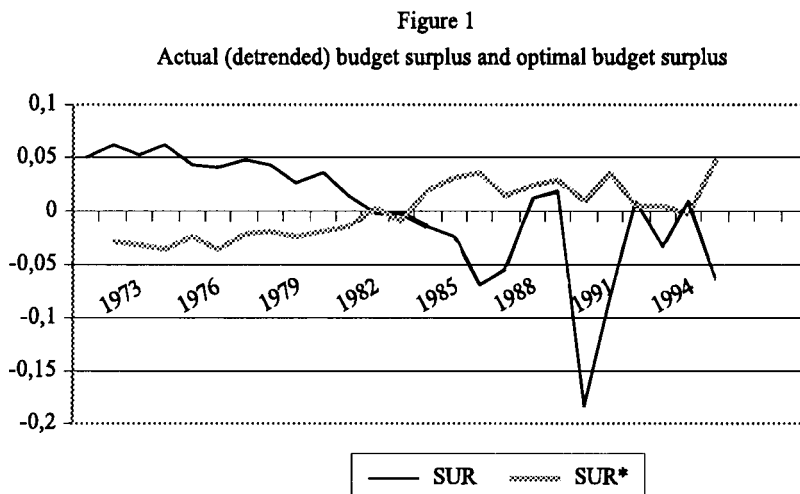
6. Concluding Remarks

The purpose of the paper was to test Barro's theory of optimal taxation over time, in order to verify if it provides a rationale to the pattern of budget deficits and taxes in Brazil over the period 1970-94. The assumption of tax smoothing implies that the budget surplus should equal the expected present discounted value of the change in government expenditures. Therefore, it is possible to construct a time series for the optimal budget surplus.

Under the hypothesis that the government smoothes taxes, the optimal budget surplus and the actual surplus should differ only by sampling error. Summarizing, and following Otto (1992), there are four implications of the model:

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[9]If, instead, the raw (undetrended) actual budget surplus $[SUR_t = \tau_t - g_t - (r-n)b_t]$ is compared to the optimal budget surplus the results are exactly the same, and the movements during 1987-92 are most clearly observed. The correlation coefficient between the actual and optimal surplus increases to 0,5.

a) the government surplus is stationary;

b) budget surplus Granger-causes changes in government expenditures;

c) the restrictions on the coefficients of Δg_t and SUR_t are observed;

d) the theoretical budget surpluses match the actual ones.

Since only (a) is confirmed, it is possible to conclude that the Brazilian government was not able to smooth taxes over the period 1970-94. However, over the period 1987-92, disregarding the volatility of the actual budget surplus, the model tracks budget surplus movements better. Therefore, maybe the tax smoothing approach would offer a reasonable explanation of how the government operated after 1987, although it seems a poor explanation when the whole period is considered. It is worth it to observe that this conclusion is based only in the visual inspection of figure 1. There are not enough observations to formally test the model.

Another possibility is that the tax smoothing approach does not explain why the Brazilian economy has been experiencing large budget deficits for several years or, in other terms, it does not provide a rationale to the Brazilian fiscal policy at all. The tax smoothing could be taken as a normative benchmark and it is still necessary to find an explanation of observed deviations from tax smoothing. It provides a theory of public debt and budget deficits which views the government as a "benevolent social planner" who maximizes the utility of the representative agent when in fact debt issue would be better explained as the outcome of a conflict between groups with different interests (see, for example, Cukierman & Meltzer, 1989, Persson & Svensson, 1989 and Alesina & Tabellini, 1990).

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