Revista Brasileira de Economia. 2023, V. 77: e242023

An early warning test for the Brazilian inflation-targeting regime: An application to the COVID-19 Pandemic

Alain Hecq
Maastricht University, Netherlands

João Victor Issler
Brazilian School of Economics and Finance, FGV/EPGE, Brazil

Elisa Voisin
Maastricht University, Netherlands

We estimate in this paper a mixed causal noncausal model for Brazilian inflation year-over-year (YoY) and ask whether it could serve as an early-warning system for the Brazilian Central Bank during the COVID-19 pandemic era. We focus on forecasting inflation and forecasting the probability of staying within the bounds of the Inflation-Targeting Regime during the Covid-19 pandemic and its aftermath – namely, the sample from January 2020 to December 2022. We estimate a high probability that Brazilian inflation will leave the tolerance bounds of the Inflation-Targeting System in March 2021, using information up to February 2021. This is one month in advance compared to the *Consensus* of experts in the *Focus* database. For point forecasts we show that the mixed causal noncausal MAR(1,1) model has a significant improvement for 1 and 3-months ahead horizons compared to the forecast of these experts. This is an interesting finding, since our model only requires the estimation of a linear model with leads and lags under non-Gaussian disturbances. Although simple to estimate, it has the important feature of being a forward-looking model.

Keywords. Mixed causal non-causal (MAR) model, Inflation targeting regime, Brazilian inflation.

JEL classification. C22, E31, E42.

1. Introduction

The Brazilian economy has been plagued with hyperinflation since the 1970s all the way until the mid-1990s. So it is understandable that Brazilian society has developed

Alain Hecq : a.hecq@maastrichtuniversity.nl
João Victor Issler : joao.issler@fgv.br
Elisa Voisin : elisa.voisin@outlook.com
We thank CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico, FAPERJ, CAPES PrInt, and FGV for financial support on different grants. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. Hecq and Voisin thank the hospitality of FGV and Issler thanks the hospitality of Maastricht University, where parts of this paper were written using the CAPES PrInt grant.

 https://doi.org/10.5935/0034-7140.20230024

inflation-*phobia*. One of the tools to fight high inflation is inflation targeting, where central banks essentially write a contract with society to keep the level of inflation under control. The Brazilian Central Bank (BCB) has adopted an *Inflation-Targeting Regime* since 1999. The target variable is the Extended National Consumer Price Index (IPCA), relevant for families with household-income ranging from 1 to 40 minimum wages. In any given year, the BCB will fail its *mandate* with society if annual inflation, measured in December, falls outside a pre-specified range comprised of an upper and a lower bound, which includes the inflation target itself – call it the *tolerance bounds*. If it fails, the BCB has to write an open letter to society explaining why it did.

In a companion paper to this one (Hecq et al., 2022) we have asked whether it is sustainable for inflation to stay within the actual tolerance bounds set in advance by the Inflation-Targeting Regime of the BCB. We did this by computing the conditional probability that actual inflation stays within the bounds in the near future (1- to 6-months ahead) using a forward-looking approach where current inflation depends on future inflation as well on lagged inflation – a mixed causal-noncausal time-series MAR(r,s) model with a *lag* polynomial of order r but also with a *lead* polynomial of order s. This method was applied using monthly data, which serves the purpose of being an early-warning system for the BCB that could be applied in real time.[1]

We had focused on Brazil for two main reasons: (i) according to previous work, Brazilian inflation has been inside an outside the tolerance bounds of the Inflation-Targeting Regime on several occasions, which serves the purpose of testing our early-warning system; (ii) Brazil has a very good database of expectations – Focus database, which we could use as strictly-exogenous regressors in the model – See Hecq, Issler and Telg (2020). The MAR model allows to parsimoniously fit nonlinear features observed in inflation data and other financial time series. Moreover, the presence of a noncausal component is coherent with the existence of non-fundamental shocks when economic agents have more information about inflation than econometricians. This means that a MAR(r,s) model is a possible proxy for economic-based models that capture expectation beliefs.

In the current paper, we advance with respect to Hecq et al. (2022), since our focus is solely on the pandemic era behavior of inflation. Indeed, the sample analyzed by Hecq et al. (2022) ends in January 2020. The quick economic recovery observed in 2021 with the increase of demand in the aftermath of the lockdowns as well as the energy crisis inherited from the Ukraine/Russia conflict have induced an increase of inflation rate that is well above the BCB margins. The investigation of inflation is now of major importance for many applied econometricians around the globe. Inflation has rarely been a problem for developed countries since the last decades[2], but the pandemic era after the spread of COVID has been an exception on that regard, where some of them saw double-digit inflation levels.

Central banks of several countries were criticized for not *preempting* the rise of inflation. On a *CNBC* interview with Bernanke, on May, 16th, 2022, the headline says,

[1]There is a slight delay to observe IPCA data, which is usually available on the 10th of the subsequent reference month. See also Subsection 3.4.

[2]The high inflation of the 1970s and early 1980s in the U.S. and in many European countries is an example of exceptional times.

"Bernanke says the Fed's slow response to inflation 'was a mistake.'" Also, Luis de Guindos, the Vice-President of the European Central Bank (ECB), was interviewed on September, 11th, 2022, where he was asked the following question: "Some critics say the ECB was slow to react to rising inflation. What is your answer to those critics, given that, as you said, inflation is well above the target of 2%?" Of course, this type of critique suggests that the warning system of the *Fed* or of the *ECB* were not working properly. Our method allows answering whether or not the BCB was slow or not in preempting Brazilian inflation, using the short-term probability of staying within the tolerance bounds as the major criterion.

The rest of the paper is as follows. Section 2 provides a summary of the model and estimation methods used here, including the methods that have been developed to forecast with MAR models. Section 3 presents the estimated probabilities for the inflation to stay within announced bounds as well as point forecasts at various horizons. Section 4 concludes.

2. Model

Due to the current controversy about inflation, a key motivation for this paper is to investigate the *early-warning* performance of the mixed causal-noncausal time-series MAR(r,s) model used in Hecq et al. (2022) during the pandemic era in Brazil. Since early-warning systems have apparently failed in several countries, this is an important test on whether or not it serves its purpose of being an early-warning system that could enable the BCB to preempt the rise of Brazilian inflation.

2.1 *Notation*

A MAR model with additional exogenous variables has been studied by Hecq et al. (2020). This is the so-called MARX model. The MARX(r,s,q) for a stationary time series π_t (here the 12-months YoY IPCA inflation rate) reads as follows:

$$\phi(L)\varphi(L^{-1})\pi_t - \sum_{j=0}^{q} \beta_j' X_{t-j} = \varepsilon_t, \tag{1}$$

where $\phi(L)$ and $\varphi(L^{-1})$ are, respectively, the lag and the lead polynomials of order r and s with $r+s=p$, and q is the number of lags for the strictly exogenous variables collected in a column vector X_t. When we exclude the presence of the strictly-exogenous variables, the process reduces to a MAR(r,s) model:

$$\phi(L)\varphi(L^{-1})\pi_t = \varepsilon_t. \tag{2}$$

Note that when $\varphi(L^{-1}) = (1-\varphi_1 L^{-1} - \cdots - \varphi_s L^{-s}) = 1$, namely when $\varphi_1 = \cdots = \varphi_s = 0$, the process π_t is a purely causal autoregressive process, denoted $AR(r,0)$ or simply $AR(r)$ model $\phi(L)\pi_t = \varepsilon_t$. The purely noncausal model $AR(0,s)$ is $\varphi(L^{-1})\pi_t = \varepsilon_t$, when $\phi_1 = \cdots = \phi_r = 0$ in $\phi(L) = (1-\phi_1 L - \cdots - \phi_r L^r)$. The roots of both the causal and noncausal polynomials are assumed to lie outside the unit circle, that is $\phi(z) = 0$ and $\varphi(z) = 0$ for

$|z| > 1$ respectively. These conditions imply that the series π_t admits a two-sided moving average (MA) representation $\pi_t = \sum_{j=-\infty}^{\infty} \psi_j \varepsilon_{t-j}$, such that $\psi_j = 0$ for all $j < 0$ implies a purely causal process π_t (with respect to ε_t) and a purely noncausal model when $\psi_j = 0$ for all $j > 0$ (Lanne and Saikkonen, 2011). Error terms ε_t are assumed iid and with a non-Gaussian distribution to ensure the identifiability of the causal and the noncausal part (Breidt et al., 1991). Note that this is a stronger condition than the usual white noise assumption in standard time-series models of the ARMA class.

We will use the MAR(r,s) instead of the MARX(r,s,q) specification in this paper for several reasons. First, although the covariates detected in Hecq et al. (2022) (Brazilian industrial production index, dollar exchange rate and the Selic interest rate) are significantly different from zero, they do not provide any improvements for forecasting inflation compared to the MAR model. In the MARX class, one has indeed to replace the regressors by their forecasts (e.g. using ARMA models) as the forecast horizon moves forward. Hecq, Issler and Voisin (2022) use the Focus database on expert forecasts for the regressors instead of statistical models. This approach reaches some limits when forecasters do not have a clear idea about the future of macroeconomic variables for a medium run horizon. Moreover, some of these series were discontinued in the $Focus$ database. So, we use a *parsimony* criterion to employ a simpler and more accurate forecasting model.

2.2 *Forecasting with a MAR model*

There are important features of the approach in Hecq et al. (2022) that are worth emphasizing. First, it employs a forward-looking model, since in the MAR(r,s) model current inflation depends on future inflation. Second, only data on inflation is used to compute the conditional short-term probability that inflation is within the bounds that the BCB must obey. Third, the whole analysis does not involve subjective decisions. Indeed, it requires computing the conditional distribution of inflation. So, a positive side effect is that several other relevant statistics for decision-making can also be calculated, e.g., value-at-risk measures. This is an important character of the approach, since the central-bank-credibility literature is plagued by ad-hoc statistics that do not have a theoretical strong footing; see the critique in Issler and Soares (2023).

We must stress that, when using the MAR model for forecasting, we do not to use future information to forecast the present as the lead polynomial seems to imply, but, we derive and compute the non-linear conditional expectation of the series given its past by using several times Bayes' Theorem. Here, to save space, we do not describe the methods used in details in this analysis, but, the interested readers are referred to Hecq and Voisin (2021; 2023) and Hecq et al. (2022). In the absence of closed-form expressions for the predictive density, two approximation methods have been developed in the literature. The first one is based on simulations and was proposed by Lanne et al. (2012). The second one employs past realized values instead of simulations and was proposed by Gouriéroux and Jasiak (2016). However, as the latter becomes too computationally demanding when the forecast horizon increases, Gouriéroux and Jasiak (2016) proposed a Sampling Importance Resampling algorithm (SIR) facilitating longer horizon forecasts with their method. Both approaches use the decomposition of the mixed process into a causal and a noncausal component as such

$$
\begin{aligned}
\phi(L)\pi_t &= u_t \\
\varphi(L^{-1})\pi_t &= v_t,
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
\varphi(L^{-1})u_t &= \varepsilon_t, \\
\phi(L)v_t &= \varepsilon_t.
\end{aligned}
$$

The process u_t is the purely noncausal component of the errors, on which we will focus. In this analysis, since the inflation series is an MAR$(1,1)$ process, $u_t = \psi u_{t+1} + \varepsilon_t$.

In the Lanne, Luoto and Saikonen approach (LLS hereafter) the purely noncausal component of the errors, u_t, can be expressed as an infinite sum of future error terms. Lanne et al. (2012) base their methodology on the fact that there exists an integer M large enough so that any future point of the noncausal component can be approximated by the following finite sum,

$$
u_{T+h} \approx \sum_{i=0}^{M-h} \psi^i \varepsilon_{T+h+i}, \tag{4}
$$

for any forecast horizon $h \geq 1$.

Let $\varepsilon_+^{(j)} = \left(\varepsilon_{T+1}^{(j)}, \ldots, \varepsilon_{T+M}^{(j)}\right)$, with $1 \leq j \leq N$, be the j-th simulated series of M independent errors, randomly drawn from the errors distribution, here a Student $t(3.30)$ distribution[3], which *pdf* is denoted by g. Let lb_t and ub_t be the lower and upper bound for inflation in Brazil at time t. We are interested in the conditional probabilities that inflation will be within the bounds at a given horizon h,

$$
\begin{aligned}
\mathbb{P}\left(lb_{T+h} \leq \pi_{T+h} \leq ub_{T+h} \middle| \mathcal{F}_T\right) &= \mathbb{P}\Big(\pi_{T+h} \leq ub_{T+h} \Big| \mathcal{F}_T\Big) - \mathbb{P}\Big(\pi_{T+h} \leq lb_{T+h} \Big| \mathcal{F}_T\Big) \\
&= \mathbb{E}_T\left[\mathbf{1}\big(\pi_{T+h} \leq ub_{T+h}\big) - \mathbf{1}\big(\pi_{T+h} \leq lb_{T+h}\big)\right],
\end{aligned}
\tag{5}
$$

where $\mathbb{E}_T[\cdot]$ is the conditional expectation given information up to time T.

Since $\pi_t = \phi\pi_{t-1} + u_t$, by recursive substitution and using the approximation equation (4), we obtain,

$$
\begin{aligned}
\pi_{T+h} &= \phi^h \pi_T + \sum_{i=0}^{h} \phi^i u_{T+h-i} \\
&\approx \phi^h \pi_T + \sum_{i=0}^{h} \sum_{j=0}^{M-h-i} \phi^i \psi^j \varepsilon_{T+h-i+j},
\end{aligned}
\tag{6}
$$

[3] These are the degrees of freedom estimated using our data.

where M is the truncation parameter introduced in equation (4). Substituting this approximation in (5), an approximation of the conditional probabilities is the following,

$$\begin{aligned}\mathbb{P}\Big(lb_{T+h} \leq \pi_{T+h} \leq ub_{T+h} \big| \mathcal{F}_T\Big) \approx \mathbb{E}_T\Big[&\mathbf{1}\big(\pi_T + \sum_{i=0}^{h}\sum_{j=0}^{M-h-i}\phi^i\psi^j\varepsilon_{T+h-i+j} \leq ub_{T+h}\big)\\ &-\mathbf{1}\big(\pi_T + \sum_{i=0}^{h}\sum_{j=0}^{M-h-i}\phi^i\psi^j\varepsilon_{T+h-i+j} \leq lb_{T+h}\big)\Big]\end{aligned} \tag{7}$$

Given the information set known at time T, the indicator functions in (7) are only function of the M future errors, ε_+. Let $q(\varepsilon_+)$ be the function providing the difference between the value of the upper bound indicator function and the value of the lower bound indicator function. The probability that the inflation rate remains within the bounds in h months can be approximated as (Lanne et al., 2012),

$$\begin{aligned}\mathbb{P}\Big(lb_{T+h} \leq \pi_{T+h} \leq ub_{T+h} \big| \mathcal{F}_T\Big) &\approx \mathbb{E}_T\Big[q\big(\varepsilon_+\big)\Big]\\ &\approx \frac{N^{-1}\sum_{j=1}^{N} q\Big(\varepsilon_+^{(j)}\Big) g\Big(u_T - \sum_{i=1}^{M}\psi^i\varepsilon_{T+i}^{(j)}\Big)}{N^{-1}\sum_{j=1}^{N} g\Big(u_T - \sum_{i=1}^{M}\psi^i\varepsilon_{T+i}^{(j)}\Big)},\end{aligned} \tag{8}$$

where g is the *pdf* of the Student $t(3.30)$ distribution and N is the total number of simulations.

In the approach of Gouriéroux and Jasiak (GJ hereafter), the h-step ahead predictive density of the purely noncausal MAR(0,1) process u_t is given by

$$l(u_{T+1},\ldots,u_{T+h}|u_T) = g(u_T - \psi u_{T+1})\ldots g(u_{T+h-1} - \psi u_{T+h}) \times \frac{l(u_{T+h})}{l(u_T)},$$

where in this analysis g is the *pdf* of the Student-*t*(3.30)-distributed errors and l denotes the unknown densities related to the process u_t. When the errors follow a Student $t-$ distribution, we employ all past observed values of the process to approximate the marginal distributions of the process u_t. The sample-based approximation of the predictive density is:

$$\begin{aligned}&l(u_{T+1},\ldots,u_{T+h}|\mathcal{F}_T)\\ &\approx g(u_T - \psi u_{T+1})\ldots g(u_{T+h-1} - \psi u_{T+h})\frac{\sum_{i=2}^{T} g(u_{T+h} - \psi u_i)}{\sum_{i=2}^{T} g(u_T - \psi u_i)}.\end{aligned} \tag{9}$$

Since evaluating the density over all possible outcome becomes considerably computationally demanding as the forecast horizon increases Gouriéroux and Jasiak (2016) developed a Sampling Importance Resampling (SIR) algorithm. Based on the equivalence of information sets $(\pi_1,\ldots,\pi_T,\pi_{T+1},\ldots,\pi_{T+h})$ and $(v_1,\varepsilon_2,\ldots,\varepsilon_{T-1},u_T,u_{T+1},\ldots,u_{T+h})$, where $v_t = \phi v_{t-1} + \varepsilon_t$, the estimator for inflation directly is,

$$l\big(\pi_{T+1}|\mathcal{F}_T\big) \approx g\big((\pi_T - \phi\pi_{T-1}) - \psi(\pi_{T+1} - \phi\pi_T)\big)\frac{\sum_{i=2}^{T} g\big(\pi_{T+1} - \phi\pi_T - \psi(\pi_i - \phi\pi_{i-1})\big)}{\sum_{i=2}^{T} g\big(\pi_T - \phi\pi_{T-1} - \psi(\pi_i - \phi\pi_{i-1})\big)}. \quad (10)$$

The SIR consists in simulating potential paths of future noncausal components u_t's with an instrumental misspecified model from which it is easier to simulate than the distribution it intends to recover. The distribution of interest is then recovered using a weighted resampling of the simulations. Since in this analysis u_t is a non-causal MAR$(0,1)$ process, it can be expressed as a causal $AR(1)$ process, with non-linear dynamics. We employ a Gaussian $AR(1)$ model as the instrumental model for the algorithm,

$$u_t = \tilde{\rho} u_{t-1} + \tilde{\varepsilon}_t. \quad (11)$$

The parameter $\tilde{\rho}$ is consistently estimated using standard OLS on the observed values u_t filtered from the initial MAR$(1,1)$ process π_t. The errors $\tilde{\varepsilon}_t \sim IIN(0,\hat{\sigma}^2)$, where $\hat{\sigma}^2$ is the MAR residuals variance, $\tilde{}$ indicates estimation from the instrumental model, and $\hat{}$ from the initial MAR$(1,1)$ model. The conditional predictive density for the instrumental process is as follows,

$$\begin{aligned}
&\tilde{F}(u_{T+1},\ldots,u_{T+H}|u_T)\\
&= \tilde{l}(u_{T+H}|u_{T+H-1})\tilde{l}(u_{T+H-1}|u_{T+H-2})\ldots\tilde{l}(u_{T+1}|u_T) \quad (12)\\
&= f(u_{T+H} - \tilde{\rho}u_{T+H-1})f(u_{T+H-1} - \tilde{\rho}u_{T+H-2})\ldots f(u_{T+1} - \tilde{\rho}u_T),
\end{aligned}$$

where $\tilde{F}$ is the predictive conditional distribution of h future u_t's from the instrumental model and f the *pdf* of a normal distribution with mean zero and variance $\hat{\sigma}^2$. Even though this model is clearly misspecified, the resampling step does automatically correct for the induced misspecifications. See Hecq et al. (2022) for the algorithm for h-step ahead predictions.

Note that we will employ the closed-form estimator of GJ for 1-month ahead forecasts and the SIR algorithm for 3 and 6-month ahead predictions.

3. Empirical results

3.1 *Data and model*

We consider IPCA series from January 1997 to December 2022, namely, a sample size of $T = 312$ observations.[4] Since the Inflation-Targeting Regime operates for year-end inflation, checking whether or not it is within the tolerance bounds, we decided to model

[4]Source: IBGE, downloaded in January 2023 from the BCB website.

12-month inflation, or year-over-year (YoY) inflation. This is crucial for an early-warning system that could detect a high probability of being outside the bounds in any given month – not just by the end of the year. Thus, the Brazilian Central Bank (BCB) could operate a preempting of future shocks, bringing inflation back into the tolerance bounds. Figure 1 depicts the YoY inflation rate over the analyzed sample as well as the tolerance bounds of the BCB targeting policy.

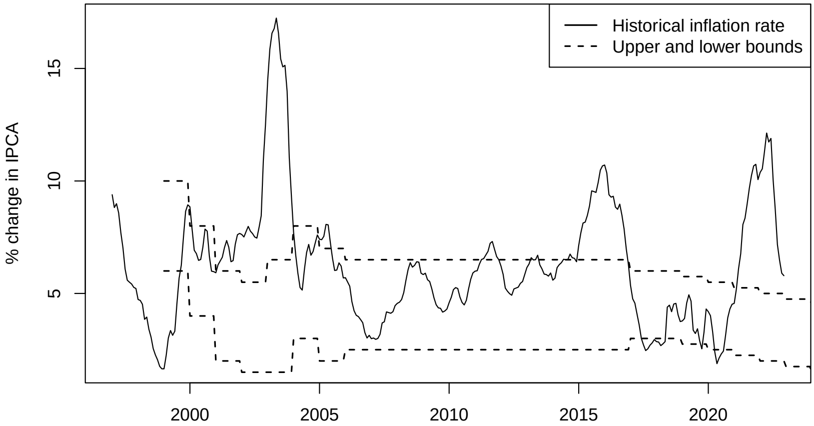

Figure 1. Monthly year-on-year inflation rate in Brazil and the target bounds.

Using the whole sample of $T = 312$ observations, the process of YoY inflation is identified as an MAR(1,1) with Student's-t distributed errors with 3.30 degrees of freedom. A complete description of the identification and the estimation of MAR models by maximum likelihood is provided, e.g., in Hecq et al. (2016) or Giancaterini and Hecq (2022). In short, information criteria and/or the correlogram are used to find by OLS an autoregressive model of order p. Testing for the null of no-autocorrelation can be used to double-check that the number of lags is adequate to whiten the residuals. We found $p = 2$ for the inflation rate and no evidence of autocorrelation using LM-tests.

Next, we test the null of normality in the previous estimated equation. If normality is rejected we choose a non-Gaussian likelihood, in our case the Student's-t distribution, and we estimate every potential MAR(r,s) for $p = r + s$. The model that provides the highest likelihood is chosen.

The coefficients of the identified MAR(1,1), based on the maximum likelihood, are as follows, with standard errors (in parentheses) computed as in Hecq et al. (2016):

$$(1 - \underset{(0.033)}{0.60}\, L)(1 - \underset{(0.016)}{0.93}\, L^{-1})\pi_t = \varepsilon_t, \qquad \varepsilon_t \sim t(3.30).$$

We show in Hecq et al. (2022) that the estimation of the coefficients is stable in an expanding window, something we also verify here. The empirical results show that roots of the lag and the lead polynomials are outside the unit circle, typical of stationary series. We must stress, however, that forward persistence is much higher than backward persistence, which is interesting, since it highlights the importance of the noncausal component of inflation brought in by the MAR(r,s) specification.

3.2 *Probability forecasts*

Once it is clear that we have a proper MAR(1,1) model at hand, in any given month, we compute the probability that YoY inflation will lie within the tolerance bounds that must be obeyed by the BCB at given horizons. We employ an expanding-window approach, and estimate these probabilities for horizons one-, three- and six-months ahead. Results for the COVID-19 pandemic period, which starts in March 2020, and its aftermath are presented in Figure 2.

It is interesting to link the graphs of Figure 2, and more specifically graph (a), with the movements of inflation and the target bounds from Figure 1. Indeed, in the beginning of 2020 the probability to stay within bounds is close to 1. After a few months, that probability quickly dropped to 10% when inflation crossed the lower bound of the target for two months, with a minimum of 1.88% YoY in May 2020. Brazilian inflation stayed within bounds until February 2021. In March, YoY inflation was 6.10%, hence higher than the 5.25% upper bound. The peak was reached in April 2022 with 12.13% before inflation started to slowly decrease, indicating it should reach the tolerance bounds again at some point in the future. Indeed, convergence towards the target limits did not happen at the time of writing.

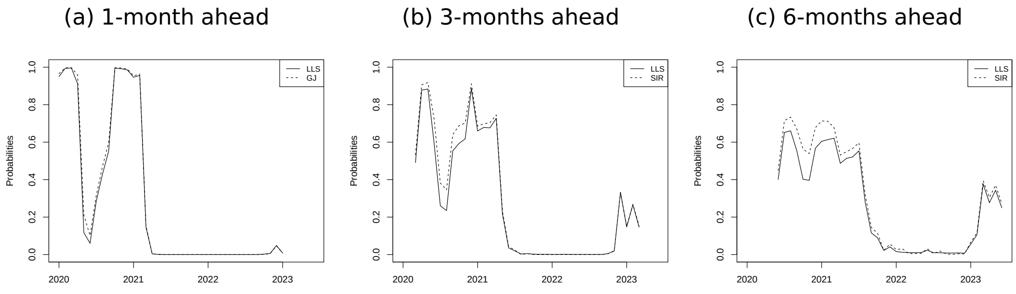

Figure 2. Probability forecasts for inflation to remain within the bounds.

Coming back to 1-month ahead probabilities, notice that the probability to stay within bounds sharply drops from 0.96 for February 2021 to 0.15 for March 2021 and then to 0.01 for May. So, the interesting feature is that, using information on inflation until February, our model predicts a low probability of YoY inflation to stay within the bounds in March. However, the 3-month and the 6-month ahead probabilities did not detect that. Indeed, as was shown in Hecq et al. (2022), longer-term probability forecasts carry increasingly more uncertainty, and are uninformative on several occasions.

Next, Table 1 presents detailed information on 2021, a critical year for the early-warning system to work, since inflation ended this year outside the tolerance bounds:

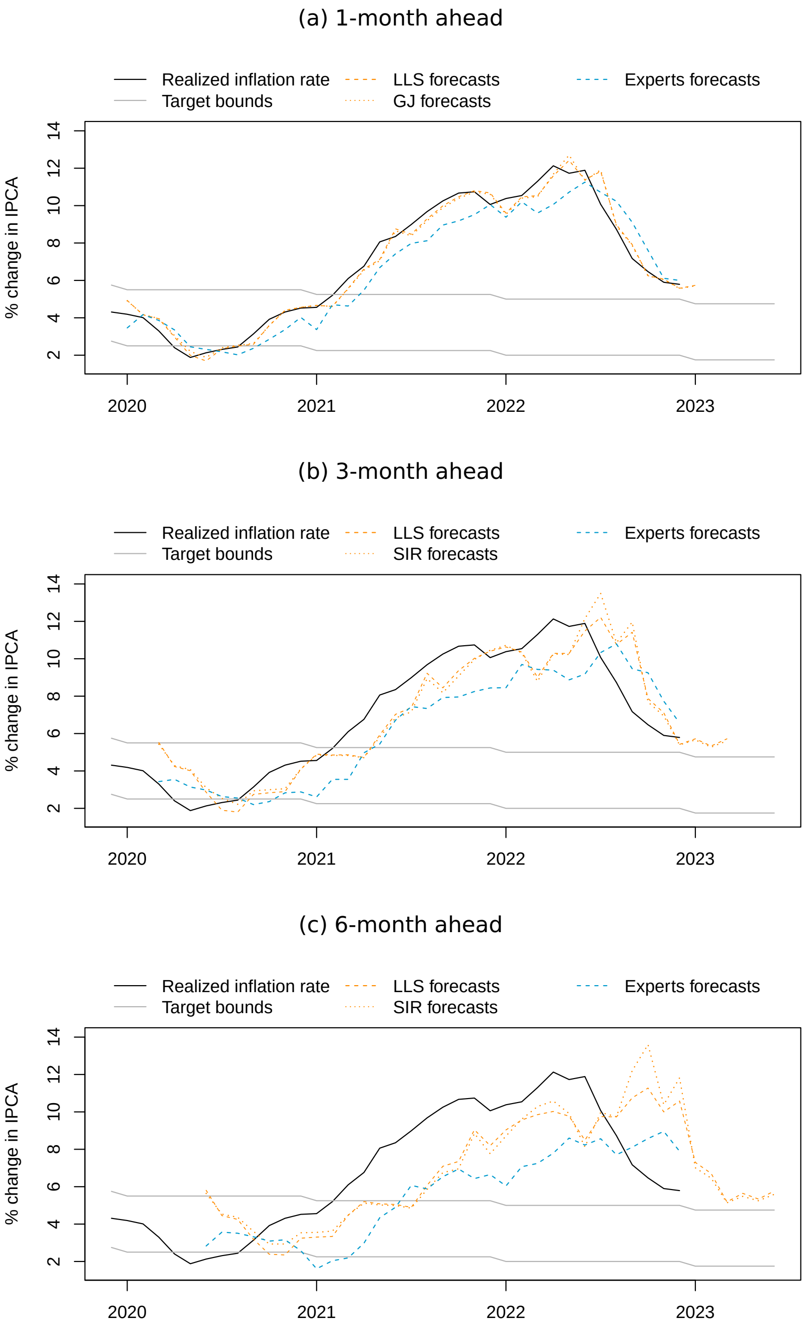

Figure 3. Point forecasts LLS, GJ/SIR and Experts forecasts.

10.06% versus 5.25% for the upper bound. February and March 2021 are critical months on that regard. YoY inflation was on the rise from January onward, but it was still below the upper bound in February: 5.20% versus 5.25%. Using information until February, our model predicted a huge drop on the probability to stay within the bounds: from 96.3% to 15.5%, dropping further to virtually zero until the end of the year. Indeed, in March, the BCB appropriately raised the Selic rate from 2% to 2.75% to start preempting the rise of inflation, playing catching-up with it until the end of the year.

Table 1. Zoom on 2021 bounds, forecasts, realizations, probabilities and Selic % rate

2021	YoY Inflation %	Upper bound %	Consensus %	MAR_f %	MAR_p	Selic Year %
Jan	4.56	5.25	3.37	4.66	0.953	2.00
Fev	5.20	5.25	4.70	4.61	0.963	2.00
Mar	6.10	5.25	4.63	5.55	0.155	2.75
Apr	6.76	5.25	5.48	6.55	0.004	2.75
May	8.06	5.25	6.69	7.05	0.001	3.50
Jun	8.35	5.25	7.42	8.65	0.000	4.25
Jul	8.99	5.25	7.98	8.39	0.000	4.25
Aug	9.68	5.25	8.12	9.19	0.000	5.25
Sep	10.25	5.25	8.96	9.89	0.000	6.25
Oct	10.67	5.25	9.18	10.39	0.000	7.75
Nov	10.74	5.25	9.52	10.73	0.000	7.75
Dec	10.06	5.25	10.06	10.61	0.000	9.25

Note: Year-on-Year inflation, Consensus of the experts, MAR one-step ahead forecasts (MAR_f), MAR Probabilities to stay within bounds (MAR_p) on that month using the information up to the month before. The Selic rate in percentage per year. Selic Changes occurred within months at days 20/01, 17/03, 05/05, 16/06, 04/08, 22/09, 27/10, 08/12.

Therefore, the model performed well as a 1-month ahead early warning method, since it anticipated the increase in the Selic rate that happened in March 2021. Its performance is even better if one looks at the *Consensus* forecasts in the Focus database, which is the average of professional forecasters (or experts). In March, April and May 2021, the YoY inflation was respectively 6.10%, 6.76% and 8.06% whereas the 1-step ahead *Consensus* were respectively 4.63%, 5.47% and 6.68%. This means that a crossing of the upper bound is foreseen in April, using information up to March. But, then, inflation was already out of the tolerance bounds. Our model however, forecasts it in March using information up to February, a time at which inflation was still within the target bounds; see the next section for a more thorough comparison using point forecasts of both strategies.

3.3 *Comparisons of point forecasts*

The main goal of our paper is to focus, using the mixed causal/noncausal specification, on the evaluation of the probability for inflation to stay within BCB defined bounds during and in the aftermath of the Covid period. We have observed in the previous subsection that our approach was able to detect that the inflation rate would exceed the bounds one month before it happened. Our framework also allows us to investigate the year-on-year inflation point forecasts.

We compare the forecast performances of the MAR(1,1) model using both SIR and LLS algorithms at horizons 1, 3 and 6 months with forecasts from the Brazilian Focus experts. We extract the latter data from the market expectation database maintained by the BCB. We have considered in this paper the average forecasts for monthly inflation rates made by all participants on the last day of the month[5] for the next six months. Then we build the year-on-year inflation experts' forecasts by adding the realized monthly inflation rates to their forecast.

For example, for forecasting the year-on-year inflation for January 2020 we use the monthly forecasts made by experts on the 31/12/2019. Next, we accumulate the realizations of the monthly inflation from February 2019 to December 2019 with the monthly expert forecast for January. We shift the sequence between realizations and forecasts for obtaining the other horizons. This means that for $h=3$ we accumulate 9 realizations and 3 forecasts, and we accumulate 6 realizations and 6 monthly forecasts for $h=6$. We proceed in the same manner for each month of the sample. We finally obtain 36 point forecasts from January 2020 to December 2022.

Figure 3 depicts the LLS, the SIR (or GJ for 1-month ahead) and the average of the experts' point forecasts at one-, three- and six-months horizons. We compare the evolution of the forecasts to the realized YoY inflation rate as well as the corresponding tolerance bounds. We only show the experts' forecasts for months for which official inflation rates have been released at the time of writing. We can notice that for all horizons, the experts forecasts (in blue) tend to be lower than the MAR forecasts (dashed and dotted orange lines).

When comparing to the actual observed inflation rate, the RMSE (root mean square error) of expert forecasts are respectively 1.0505, 1.8267 and 2.7741 for horizons $h=1$, $h=3$ and $h=6$ months. The MAE (mean absolute error) for expert forecasts are respectively 0.9078, 1.6331 and 2.3879. Table 2 reports the ratio between SIR and LLS to the experts RMSE and MAE. It emerges, for instance, that the RMSE from the MAR model is about one half the RMSE of experts for $h=1$. This is a huge difference in favor of a simple linear model in leads and lags (that requires the computation of the nonlinear conditional distribution though). The gain is 10% to 16% for $h=3$. At horizon $h=6$ however, all ratios are close to 1 meaning that the performance of both methods (MAR and experts survey) are similar.

The models used to obtain forecasts (i.e., survey and statistical models) are non-nested. Hence, Table 2 also reports the computed Diebold-Mariano tests using both the differences in squared errors and in absolute errors. The null hypothesis is that there are no differences between forecasting methods used. One can see in Table 2 that the null hypothesis of the equivalence in accuracy between MAR models and experts survey is rejected (in favor of the MAR model) for $h=1$. For $h=3$, the DM absolute error test rejects the null at 10% for the SIR and at 5% for the LLS. At horizon $h=6$ there are no significant differences between methods. Note that LLS performs marginally better than SIR (results not reported to save space).

3.4 *A comment on real time forecasts and data releases*

An informed observer might correctly argue that the information set we use to make our forecasts with the MAR and the experts consensus is slightly different, favoring MAR

[5] See next subsection for a different date

Table 2. Forecast-accuracy Ratios and Diebold-Mariano tests

	SIR/Experts				LLS/Experts			
	RMSE	MAE	DM t-test		RMSE	MAE	DM t-test	
			sqr error	abs error			sqr error	abs error
$h=1$	0.54	0.49	3.83***	4.68***	0.52	0.47	4.07***	4.95***
$h=3$	0.90	0.80	0.79	1.65*	0.84	0.77	1.77*	2.32**
$h=6$	1.02	0.96	0.15	0.30	0.96	0.98	1.10	1.17

Note: Ratios between MAR and experts survey RMSE and MAE on 2020:01 to 2022:12, $T = 36$ observations. A value below one means that MAR models are more accurate than consensus forecasts. *, ** and *** denote rejections of the two sided DM test at 10%, 5% and 1%.

forecasts. Indeed, take as an example an 1-month ahead forecast for March 2021. Using the econometric framework of our paper we consider data up to February. However, the release of inflation figures is on average around the 10th of the next month, depending on whether it is a weekend or not. Also, Gaglianone et al. (2022) report evidence that an incentive mechanism present in the Focus database to post *nowcasts* of inflation plays an important role for agent's decision to post forecasts.

Obviously, we cannot take into account all human decisions to post forecasts in our forecasting exercise. However, to alleviate this problem, we consider as a proxy the forecasts made on the 15th of the current month or the closest day if the 15th turns out to be a day off. So, we believe that we give the *Consensus* a small advantage. Using that approach we recomputed the consensus forecasts for robustness. To focus on the most important months for our analysis, we obtain for the YoY inflation respectively 4.82%, 5.00%, 5.45% and 6.80% for the period from February to May 2021. There are consequently no major differences from the figures obtained in Table 1 where we considered the last day of the previous month. The strongest difference is for March (5.00% instead of 4.63%) but with a forecast that is still within target bounds.

As far as forecast evaluations are concerned, the RMSE ratio with SIR is now 0.600 (instead of 0.543), the MAE ratio is 0.554 (instead of 0.491), DM tests reject the null that both forecasts are equivalent at a 1% significance level using both the squared errors and absolute errors loss function. Lastly, it emerges that, using the DM test, the forecast made on the 15th are significantly different from those made 15 days earlier. This potentially favors the news interpretation of the revision process.

4. Conclusion

In this paper we employ a mixed causal noncausal model MAR(1,1) for Brazilian inflation year-over-year (YoY) and ask whether it could serve as an early-warning system for the Brazilian Central Bank during the COVID-19 pandemic era. We focus on forecasting the Covid-19 pandemic and its aftermath, namely, the sample from January 2020 to December 2022. We believe that this challenging period is an important test to any early-warning system, given the difficulty that central banks around the globe had with erratic and increasing inflation.

Our estimate of the probability of Brazilian inflation to leave the tolerance bounds of the Inflation-Targeting System in March 2021, using information up to February 2021, beats by one month in advance the consensus forecast of the professional forecasters in the Focus database.

Using Diebold-Mariano tests to compare the accuracy of point forecasts of the MAR(1,1) model and those of the consensus forecast of the Focus database shows a significant improvement for 1 and 3-months ahead horizons. As stressed above, our model only requires the estimation of a linear model with leads and lags under non-Gaussian disturbances to be implemented.

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