Pricing Asian Interest Rate Options with a Three-Factor HJM Model

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Abstract

Pricing interest rate derivatives is a challenging task that has attracted the attention of many researchers in recent decades. Portfolio and risk managers, policymakers, traders and more generally all market participants are looking for valuable information from derivative instruments. We use a standard procedure to implement the HJM model and to price IDI options. We intend to assess the importance of the principal components of pricing and interest rate hedging derivatives in Brazil, one of the major emerging markets. Our results indicate that the HJM model consistently underprices IDI options traded in the over-the-counter market while it overprices those traded in the exchange studied. We also find a direct relationship between time to maturity and pricing error and a negative relation with moneyness.

Keywords: IDI options; term structure; HJM.

JEL codes: G12; G13.

Resumo

O apreçamento de instrumentos derivativos de taxa de juros tem atraído a atenção de muitos pesquisadores. Gerentes de risco, operadores e mais genericamente, todos os participantes do mercado, procuram informações nos derivativos. Neste trabalho, implementamos o modelo HJM para apreçar opções de IDI. O objetivo é demonstrar a importância dos componentes principais da estrutura de taxa de juros no apreçamento e no hedge dos derivativos no mercado brasileiro. Os resultados indicam que o modelo HJM consistentemente subapreça as opções de IDI no mercado de balcão e superapreça as opções de prazo maior negociadas na BM&F. Além disso, observa-se que o erro de apreçamento apresenta uma relação diretamente positiva com o tempo para vencimento e negativa com a proximidade do dinheiro.

Palavras-chave: opções de IDI; estrutura a termo; HJM.

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1. Introduction

Pricing interest rate derivatives is a challenging task that has attracted the attention of many researchers in recent decades. From a practical point of view many reasons can justify this interest. Portfolio and risk managers, policymakers, traders and more generally all market participants find valuable information in forward, swap and option contracts. This information plays an important role in their strategies and decision making process. On the other hand, the yield curve is undoubtedly the most important economic variable. In this paper we implement a version of the famous Heath-Jarrow-Morton (HJM) model (Heath et al., 1992) in order to analyze its ability to capture features of a very popular interest rate option offered in the Brazilian market.

The general methodology to evaluate an asset is through a general equilibrium model. However, from an empirical perspective, implementing such a tool may be cumbersome. A smart solution to this problem consists of using arbitrage-free conditions, a replication technique of asset payoffs that retains the core fundamentals of equilibrium models.¹ Interest rate arbitrage-free models can be divided into two classes. The first approach started with the seminal papers of Vasicek (1977), Cox et al. (1985) and Black et al. (1990). In this approach the short-rate dynamics are directly modeled. The main advantage of this method is the freedom to specify the evolution of interest rates. However, short-rate models have a hard time fitting the current term structure. Alternatively, the HJM model considers the forward-rate as the basic ingredient in modeling the interest rate evolution. The assumption of arbitrage-free conditions restrains the ability to set the drift of the forward-rate process, since it is completely determined by the diffusion coefficient.² Nevertheless, the initial term structure is, by construction, an input of the model and consequently any yield curve can be matched within of HJM framework.

In order to implement the HJM model, one has to specify the volatility structure of forward rates. There are many alternatives to make this choice (see Brigo and Mercurio, 2006). In this paper we use a standard procedure in which the volatility of forward rates is determined by principal components analysis (see for instance Bühler et al. (1999)). Factor models have been employed since the empirical works of Litterman and Scheinkman (1991) and Knez et al. (1994) pointed to the existence of three main movements (level, slope, and curvature) driving the volatility of interest rates.³

Our aim here is to assess the importance of the principal components to pricing and hedging interest rate derivatives in one of the major emerging markets. To this end, we use the HJM model with the volatilities of the instantaneous forward rates

¹See Harrison and Kreps (1979) and Harrison and Pliska (1981) for seminal works on this topic.

²A major problem with the HJM model lies in the fact that the short-rate process may not be a Markov process. See Ritchken and Sankarasubramanian (1995) for a deeper discussion about this point.

³In the Brazilian market, Barcinski (2000) tests the hypothesis of three factors with data from nine different maturities and obtain similar results to those of the U.S. market.

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computed by the factor loadings and the volatilities of the independent factors. We analyze models with one, two and three factors.

Based on a dataset of interest rate Asian options traded in the Brazilian market (IDI options), we find that the most naive specification, that is, the one with information of only the first principal component of the interest rate, performs best. This could mean that the models used by market agents to price these options simplify the interest rate volatility structure to only one component or that the market price of IDI options⁴ may not be an appropriate measure to quantify the quality of the HJM model.

The IDI option has unusual characteristics that make its pricing different from plain vanilla interest rates options. IDI options are Asian options reflecting the behavior of interest rates between the trade date and the maturity of the option. Plain vanilla interest rates options depend on the short-term rate evaluated only at the maturity date. Thus traditional pricing models developed for other markets should be adjusted to evaluate them. Some recent studies have addressed this issue using different term structure models. Junior et al. (2003) fitted the spot rate term structure with the Hull-White model. Gluckstern (2001) adopted the Hull-White model (Hull and White, 1993) and found good performance. Almeida et al. (2003) also used the Hull-White model and identified that some parameters are unstable in times with high volatility or after crises. Vieira Neto and Valls Pereira (2000), assuming that short-term rates follow a Vasicek (1977) model, obtained a closedform formula to price IDI options. Barbachan and Ornelas (2003) adopted the Cox-Ingersol-Ross model (Cox et al., 1985) and Almeida and Vicente (2006) used affine models (see Duffie and Kan, 1996) to evaluate IDI options.

Notwithstanding the fact that the aim of all the above papers was to price IDI options, they differ from ours in that they worked only with data from the Brazilian exchange and only with short-rate models. Thus, this work has a second goal, which is to use the non-Markovian implementation of the HJM model for the first time to price this kind of Brazilian option. Chiarelli and Kwon (2007) pointed out that although the models of Vasicek, Cox-Ingersoll-Ross and Hull-White are the most popular to price interest rate derivatives, the HJM model is more consistent. Furthermore, they showed that the HJM model is a general model and the others are just special cases of it. We contribute to the financial literature in at least more two aspects: by comparing over-the-counter and exchange market prices and applying the model of Bühler et al. for an emerging market database.

Our results indicate that the HJM model consistently underprices IDI options traded in the over-the-counter market, while it apparently overprices these options when traded in the exchange market. In the first case, it happens because our data are composed through a call process that quote asks prices. In the exchange market, we verify this overpricing only with long-term options. We also test, by

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⁴IDI options are traded in the Brazilian exchange and over-the-counter market. The liquidity in the Brazilian exchange is very poor and the prices collected in the over-the-counter market are obtained by means of a call process. Therefore the prices can present some sample errors.

running linear regression, whether the time to maturity, moneyness or seasonal effects can affect the pricing error. We show there is a direct relationship between time to maturity and pricing error and a negative relation between moneyness and pricing error (the more at-the-money the option is, the less the pricing error is) for both the over-the-counter and exchange market database. The calendar dummy variable is not significant.

This paper is organized as follows. Section 2 presents the characteristics of the sample and treatment of the database. The Section 3 covers the methodology. The results are presented and commented in Section 4 and Section 5 concludes.

2. Sample

Our data consist of time series of the yields of the One-day Interbank Deposit Future Contract (ID-Future)⁵ for all different liquid maturities, and the values of IDI options for different strikes and maturities traded in the over-the-counter and exchange market. The data cover the period from January 12, 2004 to July 5, 2008.

The ID-Future database yields allow extracting forward rates by cubic spline interpolation to fixed maturities for all trading days. For each fixed time to maturity, a reference bond is a zero coupon bond with the same time to maturity. We fixed the times to maturity from 21 to 546 days, with increments of 21 business day. Cubic spline interpolation can cause a bias due to the incorporation of similar information at all vertices. However, as we obtain the same three factors verified in Brazilian finance literature as meaningful, according to Luna (2006) the bias is not so strong.

IDI options have as underlying assets the theoretical value of 100,000 points on am initial date defined by the BM&F,⁶ accumulated by the one-day interest rate computed every business day by the clearinghouse CETIP until the maturity date. The option is European.

Our initial database of exchange market options consisted of 4,928 call and 1,525 put options. We excluded away put options from our sample, first because their liquidity was low – only 10% of financial volume – and second, because on about 50% of the days, the number of trades was at most two.

The over-the-counter database is composed of trades, settled or not, registered through underlying asset volatility. The initial sample consisted of 63,654 individual call option volatility trades. We put this volatility, estimated by Black's model from market participants, into the original model to price the options and to allow comparison with HJM prices.

We performed two filtering procedures in both databases. The first filter aimed to reduce the problem that the data are not obtained by observing simultaneous

⁵The ID rate is the average one-day interbank borrowing/lending rate, calculated by CETIP (Center for Custody and Financial Settlement of Securities) every business day. The ID rate is expressed in effective rate per annum, based on 252 business days.

⁶The Brazilian Mercantile and Futures Exchange, which has now merged with the São Paulo Stock Exchange (Bovespa).

option and underlying asset prices during trading hours. We eliminated all trades whose implied volatility was not determined by Black (1976) and we also eliminated trades whose implied volatilities were 35% higher or lower than the last trading day's implied volatility. This maximum variation was estimated to avoid a substantial reduction of the sample and at the same time to allow a reasonable variation in volatility behavior. As our aim is just to check the relative performance of the presented models, we believe that this filtering does not cause bias in our sample. Besides this, we eliminated options with time to maturity lower than five days. The final database of traded exchange options consisted of 2,977 observations with all moneyness and maturity until 546 days.

The second filtering, applied to the over-the-counter database, eliminated options whose prices, estimated by Black's model, were equal to zero and whose volatilities were higher than 200%. We also eliminated options with time to maturity lower than five days. The final database of over-the-counter options consisted of 46,243 observations with all moneyness and maturity until 546 days.

3. Methodology

An IDI option is an interest rate derivative instrument traded in the BM&F used to hedge and to speculate on interest rates. Consequently, pricing this instrument means pricing the Brazilian yield curve. This study aims to identify the weight of principal components in the process of option pricing. We apply the model of Heath et al. (1992) considering one factor, two factors and three factors driving the IDI pricing.

As noted by Almeida and Vicente (2006), an IDI option is just an Asian option whose payoff is a function of the short-term rate through the path between the trading date t and the option maturity date T.

$$IDI_T = IDI_t \prod_{i=t}^{T-1} (1 + CDI_i)$$
(1)

where $CDI_i = (1 + CDI_i \%_{year})^{(\frac{1}{252})} - 1.$

Denote by c(t,T) the time t price of a call option on the IDI, with time to maturity T and strike price K. Then the payoff is:

$$c(t = T, T) = \max(0, IDI_T - K)$$
⁽²⁾

If the accumulated IDI rate between the trading date and the option maturity is higher than the implicit option interest rate, given by the ratio of the exercise price and the IDI spot price, the option will be exercised.

The class of term structure models chosen is a multi-factor model. A one-factor model assumes that all bonds are influenced by the same source of uncertainty. By incorporating multiple factors, we allow different types of shifts in the interest rate behavior, despite the great computation effort. Besides this, the term structure put

into the HJM model follows the market behavior, which avoids arbitrage transactions. The main feature of the HJM model is that it allows interest rate volatility to change across time, which gives flexibility to pricing derivatives. However, the demand for the volatility term structure complicates this model's use even in the international literature.

Amin and Morton (1994) analyzed different specifications for the volatility term structure of the forward rates in a HJM framework for Eurodollar futures and options during the period from 1987 to 1992. They found that the single-factor HJM model fared well in valuing short-term options because it results in implied parameter estimates that are more stable.

Bühler et al. (1999) performed a comprehensive empirical study of one – and two-factor HJM type models. Principal components analysis was performed in order to determine the parameters for the one – and for the two-factor models. They found the surprising result that the one-factor HJM with proportional linear volatility outperformed the two-factor model for German interest rate warrants over 1989 to 1993. According to the authors, this could be due to the incorrect estimation of the factor loadings of the second factor. The volatility parameters were estimated directly from the volatilities of the two factors and the corresponding factor loadings. Here we adapt the Bühler et al. work for an Asian option and include the three-factor HJM model.

3.1 Principal components

Principal components analysis can be used to reduce the dimensionality of the data through an orthogonal linear transformation so that the greatest variance by any projection of the data comes to lie on the first coordinate of a new coordinate system, and so on. This technique helps to investigate data. The time window used by PCA comprehends all the period from January 02, 2003 to June 5, 2008.



Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9

Figure 1 Principal component analysis of the Brazilian yield curve from 01/02/2003 to 06/05/2008

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Figure 1 shows that the three-factor model is a good representation of the yield curve, for the entire period studied. This is the same model represented in international curves by Litterman and Scheinkman (1991).

The process of estimating variance for the next day requires calculating principal components on each day. So, we built a daily database of factor series for each vertex and subsequently estimated the volatility according to a definite methodology.

3.2 Volatility

We estimated the forward rate volatility structure through historical series of the yield curve for two reasons. First, the implied volatility demands simultaneousness between option price and underlying asset price. Second, pricing based on implied volatility means a local test, according Bühler et al. (1999), because in this case volatility is only used to price the option in the next period. As this work proposes a global, or an overall, test, comparing the model performance with one, two and three factors, we chose not to use information from the derivatives market. The process of estimating total variance follows Bühler et al. (1999) and the generalized formula is given by:

$$\sigma_p^2 = \sigma_{factor1}^2 \times L_1 + \sigma_{factor2}^2 \times L_2 + \sigma_{factor3}^2 \times L_3 \tag{3}$$

where σ_{factor}^2 is the factor variance and L is the factor loading. When testing only the one-factor model, we used only the first part of the equation's right side. When testing the two-factor model, we used the first and second part, and all parts for the three-factor model.

We used two methods to compute the factor variance. First, we estimated volatilities based on standard deviation of a 378 business day window. Second, we selected a GARCH (1,1) methodology. So, for each vertex we have six volatilities: volatilities according to the number of factors (one, two or three) and according to the volatility method (standard deviation or GARCH). The proportional forward rate volatility structure for the six volatilities is built by:

$$\sigma_{T_1,T_2}^2 = \frac{\sigma_{t_0,T_2}^2 \times D_{t_0,T_2} - \sigma_{t_0,T_1}^2 \times D_{t_0,T_1}}{D_{T_1,T_2}}$$
(4)

where D_{T_1,T_2} is the number of business day between T1 and T2 and the forward volatility, and σ_{T_1,T_2}^2 means the expected volatility between two dates.

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3.3 The tree of the HJM model

The HJM model starts with a fixed number of unspecified factors that drive the dynamics of the forward rates:

$$df(t,T) = \alpha(t,T,f)dt + \sum_{i=1}^{3} \sigma_i(t,T,f)dz_i(t)$$
(5)

where df(t,T) denotes the instantaneous forward interest rate on date t for borrowing or lending on date $T, z_i(t)$ is independent one-dimensional Wiener process, $\alpha(t,T,f)dt$ is the drift and $\sigma_i(t,T,f)$ the volatility coefficients of the forward rate of maturity T. As the original HJM paper shows, the drift of the forward rates under the risk-neutral measure is determined by the volatility functions as:

$$\alpha(t,T,f)dt = \sum_{i=1}^{3} \sigma_i(t,T,f) \int_{t}^{T} \sigma_i(t,s,f)ds$$
(6)

The equation above denotes the main result in that paper. It shows that when a number of regularity conditions and a standard no-arbitrage condition are satisfied, $\alpha(t, T, f)dt$ is uniquely determined by the forward volatility functions. In this work, we adopt six volatility specifications: two volatilities for each factor model.

From the forward rate volatility structure and the interest rate term structure, we created the HJM forward rate tree. This tree represents the evolution of the IDI, based on a HJM statistical process. The payoff in the last step is given by Equation 2.

When the HJM process is non-Markovian, the tree becomes bushy, the number of branches increases exponentially and they never recombine. However, Heath et al. (1992) showed that, assuming the twelve-step tree as a benchmark, the error beyond five steps is always within 0.5%. The trees in this work have a minimum of three and a maximum of twelve steps until maturity and they were driven by the time to maturity of each option.

3.4 Pricing errors

We compared the performance of the HJM model with one, two and three factors by the difference between the model price and the market price, using the root mean square error as the metric.

Our final step was to check for any systematic pricing errors by regressing the root mean square error on time to expiration, moneyness and the semester of valuation, along with dummy variables that specify the model used. We used the regression equation below to evaluate the errors associated with the call option pricing:

$$Error_{t} = \alpha + \beta_{1}(T - t) + \beta_{2}M_{t} + \beta_{3}S_{t} + \sum_{i=1}^{3}\delta_{i}D_{i}$$
(7)

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where T - t is the time to maturity; M_t is the moneyness, calculated by dividing the underlying asset price by the present value of the strike price; S_t is the semester that the option was traded and D is the dummy variable used to classify if the error was caused by the HJM model with one, two or three factors. Our null hypothesis was that the dummy coefficients are statistically different from zero, so that we could check for a relationship between pricing errors and the chosen method. We also checked if the results of the three models are significantly different from each other through the nonparametric Kruskal-Wallis test.

4. Results

Table 1 shows the root mean square error statistics considering the model with one, two and three factors for the trades in the exchange market.

Table 1

Error measure statistics from HJM model with one, two and three factors and standard deviation or GARCH volatility. The database is composed of options traded in the exchange market

Volatility	Factors	Average error	Standard deviation error	1stQ (errors)	3 rd Q (errors)	
Historical	Ι	39.82%	31.65%	16.41%	73.77%	
standard	II	41.62%	32.22%	15.67%	74.24%	
deviation	III	45.86%	33.57%	23.11%	85.75%	
Kruskal Wallis: p-value 0.0000						
	Ι	38.16%	33.12%	17.72%	78.17%	
GARCH	II	38.58%	33.31%	13.82%	75.95%	
	III	42.56%	34.59%	20.98%	85.54%	
Kruskal Wallis: p-value 0.0000						

The Kruskal-Wallis test's null hypothesis of similar distribution functions for the three models is rejected at 10% significance level. This means that we really are changing the results when we include new factors in the HJM model. The onefactor models present the lowest RMSE and standard deviation and the one with GARCH volatility performs best.

These results can mean that the principal component factors are not enough to explain the movements of derivative prices. In fact, Collin and Goldstein (2002) and Heidari and Wu (2003) also suggested that term structure factors are not sufficient to explain the dynamics of fixed-income derivatives.

Since the GARCH volatility performed relatively better for each factor model, we chose this methodology to price the options traded in the over-the-counter market.

Table 2

Error measure statistics from HJM model with one, two and three factors and standard deviation or GARCH volatility. The database is composed of options traded in the over-the-counter market

Volatility	Factors	Average error	Standard deviation error	1 st Q (errors)	3 rd Q (errors)
GARCH	Ι	86.29%	24.09%	50.79%	97.96%
	II	88.47%	25.57%	57.25%	97.78%
	III	94.85%	106.87%	61.06%	98.36%

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Table 2 shows that the RMSEs of the over-the-counter data are considerably higher. This can be explained by the high volatilities registered in over-the-counter trading. For instance, as the average annual volatility of this database is around 60%, the average annual short-rate volatility estimated by the first principal component is around 15%. Another explanation would be in the IDI market microstructure, with market makers usually taking short positions on call options and clients buying call options. Thus, market makers have an incentive to overprice quotes. A last possible explanation is a bias in the database, i.e., the database is composed only by quoted ask prices, instead of mid prices.

Besides the comparison with market prices, we evaluated the models' performances through mispricing patterns. We expected the calibrated models to be balanced and the results to be neither underpriced nor overpriced most of the time. Considering all the sample, we verify a bias of overpricing for the one and twofactor model. To investigate this pattern, we divide our sample according to moneyness and time to maturity. Table 3 presents the results for the exchange market data by volatility. The metric in this case is given by the model price minus the market price. In this case, the three-factor model presented the steadiest results, i.e., a lower bias. We verify that HJM model consistently overprices options with time to maturity higher than one year. It can be explained because of the lack of liquidity. These options represent only 20% of the trading volume of the market.

Table 3

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Overpricing and underpricing from HJM model with one, two and three factors and standard deviation or GARCH volatility. The database is composed of options traded in the exchange market. We divide by time to maturity

Time to Maturity	Volatility	Factors	Overpricing	Underpricing
	Historical	Ι	56.50%	44.50%
Until	Standard	П	61.40%	38.60%
one	Deviation	III	53.13%	46.87%
year		Ι	56.28%	43.72%
	GARCH	II	62.14%	37.86%
		III	52.83%	47.17%
From	Historical	Ι	83.07%	16.93%
one year	Standard	П	78.39%	21.61%
to 546	Deviation	III	44.98%	55.02%
business		Ι	81.42%	18.58%
day	GARCH	П	82.34%	17.66%
		III	40.76%	59.24%

For the over-the-counter data, all the factor models showed underpricing. This reinforces the finding of higher volatilities of these operations when compared to the historical interest rate volatilities. For the models with one, two and three factors, the underpricing measured was 99%, 98.2% and 97.8%, respectively. This can be explained by the IDI market microstructure, with market makers usually taking short positions on call options and clients buying call options, so the call process to create this database is composed by quoted ask prices.

We also analyzed pricing errors for the different models. First, for the exchange market data, we regressed the price RSME as the dependent variable of each model and respective volatility. This gave six regressions on the variables in Section 3.4. The moneyness and the time to maturity were significant in all regressions. We consistently found a significantly positive relationship between time to maturity and pricing errors and a negative relationship between moneyness and pricing errors. This means that long maturity and out-of-the-money options are the hardest options for pricing following the HJM model. In-the-money and atthe-money options performed well according to this model. The calendar dummy variables were not significant.

To support these results and to find a more robust result, we ran a panel data analysis with pricing errors from all models. Table 4 presents the results for the exchange market data.

Table 4

Error measure regression considering the HJM model with one, two and three factors and standard deviation (SD) or GARCH volatility. The database is composed of options traded in the exchange market

Variable	Coefficient (p-value)		
	SD	GARCH	
Time to maturity	0.0009	0.0003	
	(0.0000)	(0.0236)	
Moneyness	-23.632	-25.071	
	(0.0000)	(0.0000)	
2-factor dummy	0.0784	0.0406	
	(0.0000)	(0.0003)	
3-factor dummy	-0.0675	-0.0310	
	(0.0000)	(0.0085)	
Intercept	30.226	32.517	
	(0.0000)	(0.0000)	
Adjusted R^2	0.0276	0.0354	
F statistic P-value	0.0000	0.0000	

The same relationship in the prior regression between moneyness and the time to maturity was verified. The dummy variables included to differentiate the number of factors were statistically significant at 5%, indicating pricing differences among the models. Besides this, the coefficients' sign indicates that the third factor reduces the pricing error while the second increases this error. For the GARCH volatility, the third factor reduces the pricing error less. The second factor sign is similar to the finding of Bühler et al. (1999), who claimed that the outperformance could be due to the incorrect estimation of the loadings of this factor. The second factor is closely related to the spread between the long and the short rate and appears to be important, as the highest is the period studied. The regression considering only over-the-counter data has the same sign for the variables moneyness and time to maturity. However, the sign of the third factor dummy shows that this term increases the pricing errors. Table 5 reports these conclusions.

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Table 5

Error measure regression considering the HJM model with one, two and three factors and standard deviation or GARCH volatility. The database is composed of options traded in over-the-counter market

Variable	Coefficient (p-value)
Time to maturity	0.0003
	(0.0049)
Moneyness	-1.2402
	(0.0000)
2-factor dummy	0.0217
	(0.0154)
3-factor dummy	0.0855
	(0.0000)
Intercept	1.9314
	(0.0000)
Adjusted R^2	0.0124
F statistic P-value	0.0000

Table 6 presents the correlation between pricing errors across models for the exchange database. Our results are quite different from those of Bühler et al. (1999). In that work, the authors found correlations close to 1. Table 6 shows that correlations of the first factor with the other models are very low. This result is closer to those of Amin and Morton (1994) and can mean that the simplicity of the first factor model is closer to the Brazilian market empirical models.

Table 6

Correlation between pricing errors across HJM models with standard deviation (SD) and GARCH volatility. The database is composed of options traded in the exchange market

Model	1-factor	2-factor	3-factor	1-factor	2-factor	3-factor
	GARCH	GARCH	GARCH	SD	SD	SD
1-factor GARCH	1					
2-factor GARCH	0.55	1				
3-factor GARCH	0.47	0.74	1			
1-factor SD	0.52	0.86	0.55	1		
2-factor SD	0.29	0.87	0.53	0.88	1	
3-factor SD	0.32	0.79	0.91	0.70	0.73	1

5. Conclusion

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The aim of this study was to assess the importance of the principal components to pricing and hedging IDI options in the Brazilian market. We analyzed the HJM model with one, two and three factors with data covering the period from January 12, 2004 to July 5, 2008.

We found that the one-factor model, with information from only the first principal component of the interest rate, performs better, i.e., has the lowest error measure and the lowest standard deviation. This could mean that the models used by agents of the market to price these options simplify the interest rate volatility structure to only one component or even that the market price of IDI options may not be an appropriate measure to quantify the quality of the HJM model. We also showed that the second factor raises the error measure and the third factor increases or decreases it in accordance with the database. For the IDI options traded in the

over-the-counter market, the third factor increases the error measure and for the IDI options traded in the exchange market, it decreases the error.

Regarding the huge percentage of underpricing of the OTC data, one explanation would be in the IDI market microstructure, with market makers usually taking short positions on call options and clients buying call options. Another possible explanation is that the call process to create this database is composed by quoted ask prices, instead of mid prices.

We also tested whether the time to maturity, moneyness or seasonal effects can affect the pricing error. We showed that there is a direct relationship between time to maturity and pricing error and a negative relation between moneyness and pricing error for both over-the-counter and exchange market databases.

To our knowledge, this is the first paper to work with exchange and over-thecounter market data and to price Asian options with a three-factor model. We suggest complementing this work with the implementation of delta hedging strategies to verify arbitrage opportunities. We recognize that we limit our study by the fact that data are not obtained by observing simultaneous option and underlying asset prices during trading hours. However we believe that our results are representativeness.

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