# Returns Predictability and Stock Market Efficiency in Brazil

**(Previsibilidade de Retornos e Eficiência no Mercado Acionário Brasileiro)**

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***Abstract***

This paper searches for evidence of predictability in the Brazilian stock market using portfolios grouped by sector and firm size with data from 1999 to 2008. I conduct an automatic variance ratio test using wild bootstrap. This methodology eliminates the arbitrary choice of the holding period as well as improves small sample properties. The results suggest (i) stocks from the industrial sector are highly predictable, (ii) stocks from small firms tend to be more predictable than the ones from large firms, (iii) the Brazilian stock market, measured by the Ibovespa index from 1986 to 2008, shows an increase of efficiency since 1994.

***Keywords*:** predictability; efficient markets; random walk; variance ratio; bootstrap.

***JEL codes:*** G14; G15.

***Resumo***

Este artigo procura evidências de previsibilidade no mercado acionário brasileiro usando portfólios agrupados por setores e tamanho das firmas com dados de 1999 a 2008. É conduzido um teste de razão de variância automático usando *wild bootstrap* que elimina a escolha arbitrária dos valores de truncagem e melhora as propriedades de amostra finita. Os resultados sugerem que (i) ações de empresas do setor industrial são altamente previsíveis, (ii) ações de empresas menores são mais previsíveis do que de empresas maiores, (iii) o mercado acionário brasileiro, mensurado através do índice Ibovespa de 1986 a 2008, mostrou um aumento de eficiência desde 1994.

***Palavras-chave*:** previsibilidade; mercados eficientes; passeio aleatório; razão de variância; *bootstrap*.

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## 1. Introduction

The presence of inefficiencies in the stock market has important implications for practitioners and academics, because it allows the prediction of future returns. A lot of work has been done by researches on testing predictability of financial series, but few of them focus in the Brazilian stock market. Usually, these tests are conducted using well known indexes which represent the market in question. Since liquidity is commonly related with efficiency in stock markets, it is also natural to use portfolios grouped by firm size in these testings. As far as I can tell, there has been no work testing for predictable patterns in stocks from different sectors of the Brazilian stock market. Such study can lead to important contributions to the literature, since identifying patterns of predictability in specific groups of stocks can enable further research to develop prediction models which incorporate these inefficiencies.

The random walk hypothesis (RWH) provides a means to evaluate predictability of stock returns and weak-form efficiency of financial markets. Rejection of the RWH indicates that future returns can be predicted based on past prices. The variance ratio (VR) test has been extensively used for testing the RWH since the works of Lo & MacKinlay (1988) and Cochrane (1988). Improvements of the original test include the multiple variance ratio test of Chow & Denning (1993) and the Wald-type join tests of Richardson & Smith (1991) and Cecchetti & Lam (1994), both designed to control size distortions. A number of other methodologies have been developed to improve small sample properties. Among them, Wright (2000) proposed an exact test based on ranks and signs. Later, Chen & Deo (2006) developed a power-transformed statistic and Kim (2006) used the wild bootstrap method to derive the empirical distribution of the variance ratios.

In this paper, I use the automatic variance ratio (AVR) test, first proposed by Choi (1999) and later improved by Kim (2009). The test selects the optimal holding period by employing a data-dependent method of Andrews (1991) for spectral density at the zero frequency. Recently, Kim (2009) evaluated the small sample properties of Choi's AVR test and proposed the use of wild bootstrap, finding no size distortions as well as finding test powers that are substantially higher than powers of other tests such as Chen & Deo (2006) and Chow & Denning (1993). Here, I apply the wild bootstrapped AVR test proposed by Kim (2009) for daily and monthly returns of the Ibovespa index from the period of 1986 to 2008 in order to test the efficiency of the Brazilian stock market. I also apply the same test for portfolios grouped by firm size, intending to analyze the relation between efficiency and liquidity, and portfolios based on sectors, to search for predictable patterns driven by specific stocks.

This paper shows that Ibovespa monthly returns follow a random walk, however there are evidences against the model for daily returns if the period before 1994 is included in the sample. Similar results can be found in the works of Tabak (2003) and Chang *et al.* (2004) for daily returns, and Karemera *et al.* (1999) and Torres *et al.* (2002) for monthly returns. This result suggests an increase of effi-

ciency in the Brazilian stock market since 1994. Also, I find that small firms tend to be more predictable than large ones, specially for monthly returns, supporting the results found in the American literature (Chordia *et al.*, 2008). When I conduct the test for five different sector portfolios, only the industrial sector rejected the RWH. This result has not yet been addressed in the literature, however it is possible that the return pattern of industrial firms might correlate with the business cycle of the Brazilian economy, which follows a more predictable pattern.

## 2. Brief Literature Review

### 2.1 Random walk hypothesis and VR tests

An efficient market is one where the market price reflects all the information available for investors and is an unbiased estimate of the true value of an investment. In weak-form efficient markets the information set contains only past prices, so the RWH is satisfied and future equity prices are not predictable based on past prices. This result has several implications to practitioners and academics since the RWH is not compatible with the existence of patterns in stock prices and the use of univariate forecasting models to predict future returns.

The RWH for a time series $\{p_t\}_{t=1}^{T}$ corresponds to the following model:

$$p_t = \mu + p_{t-1} + \varepsilon_t \tag{1}$$

where $\mu$ is an unknown drift parameter and the error term $\varepsilon_t$ is a white noise not necessarily normal, satisfying $E[\varepsilon_t] = 0$, $E[\varepsilon_t^2] = \sigma^2$ and $E[\varepsilon_t \varepsilon_\tau] = 0$ for all $t \neq \tau$.

Variance ratio statistics are commonly employed for testing the RWH since the works of Lo & MacKinlay (1988) and Cochrane (1988). If a stock price follows a random walk then the variance of its increments is linear in the observation interval. Therefore, the variance of $p_t - p_{t-k}$ must be k times the variance of $r_t = p_t - p_{t-1}$. In this case, we have

$$VR(k) = \frac{var(p_t - p_{t-k})/k}{var(r_t)} = 1. \tag{2}$$

We can construct a test using the unbiased estimators proposed by Lo & MacKinlay (1988),

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{T}(r_t - \hat{\mu})^2}{T-1} \tag{3}$$

$$\hat{\sigma}^2(k) = \frac{\sum_{t=k}^{T}(p_t - p_{t-k} - k\hat{\mu})^2}{k(T-k+1)(1-kT^{-1})} \tag{4}$$

$$\hat{VR}(k) = \frac{\hat{\sigma}^2(k)}{\hat{\sigma}^2} \tag{5}$$

where T is the sample size, $\hat{\mu} = T^{-1}\sum_{t=1}^{T} r_t$ is the estimated mean of $r_t$, $\hat{\sigma}^2$ is the sample variance of $r_t$ and $\hat{\sigma}^2(k)$ is the sample variance of $k$-period returns. The estimated $\hat{VR}(k)$ satisfies the relation

$$\hat{VR}(k) \cong 1 + 2\sum_{i=1}^{k-1} \frac{(k-i)}{k}\hat{\rho}_i \tag{6}$$

where $\hat{\rho}_i$ is the estimated $i$th order autocorrelation coefficient of returns. Note that if $p_t$ is a random walk, then $\hat{\rho}_i$ must equals zero for all $i$, and $\hat{VR}(k) = 1$ for all $k$.

Cochrane (1988) showed that $\hat{VR}(k)$ is asymptotically equivalent to $2\pi$ times the normalized spectral density estimator at the zero frequency, which uses the Bartlett kernel. Formally,

$$\hat{VR}(k) \sim 2\pi \frac{f_{\Delta y}(0)}{\hat{\sigma}^2} \tag{7}$$

where $f_{\Delta y}(0)$ represents the estimator of the spectrum evaluated at frequency zero.

The VR test is based on the fact that if $p_t$ is a random walk, then $\hat{VR}(k)$ must equal unity in all horizons $k$. The time series $\{p_t\}_{t=1}^{T}$ is mean reverting (averting) if the variance ratio is significantly lower (higher) than unity. Lo & MacKinlay (1988) claim that[1]

$$z(k) = \frac{\sqrt{T}(\hat{VR}(k) - 1)}{\sqrt{\hat{\theta}(k)}} \sim N(0,1) \tag{8}$$

where $\hat{\theta}(k)$ is the asymptotic variance estimator of $\hat{VR}(k)$, expressed by

$$\hat{\theta}(k) = \sum_{i=1}^{k-1} \left[\frac{2(k-i)}{k}\right]^2 \hat{\delta}(i) \tag{9}$$

and $\hat{\delta}(i)$ is the asymptotic variance estimator of $\hat{\rho}_i$, calculated by the following equation:

$$\hat{\delta}(i) = \frac{\sum_{t=i+1}^{T}(p_t - p_{t-1} - \hat{\mu})^2 (p_{t-i} - p_{t-i-1} - \hat{\mu})^2}{\left[\sum_{t=1}^{T}(p_t - p_{t-1} - \hat{\mu})^2\right]^2}. \tag{10}$$

Note that expression (9) is derived from equation (6).

The test proposed in (8) allows for quite general forms of conditional heteroscedasticity and non-normality of the stochastic disturbance term. If the time

[1]They assumed that $k$ is fixed and $T \to \infty$. This relation is valid under their null hypothesis, where $p_t$ has uncorrelated increments, some restrictions on the maximum degree of dependency and heterogeneity are applied and the sample autocorrelations of $\varepsilon_t$ are asymptotic uncorrelated. See Lo & MacKinlay (1988) for further details.

series is a random walk, the null hypothesis holds true for all values of $k$ and $\hat{VR}(k)$ must equal unity for all $k$. Frequently, the values of $k$ are chosen rather arbitrarily and the same statistic is conducted for all horizons selected. Chow & Denning (1993) showed that failing to control test size for multiple comparisons causes large probability of type 1 error, leading to an over-rejection of the null hypothesis.

Chow & Denning (1993) proposed a procedure for the multiple comparison of the set of VR estimates with unity. To test the joint null hypothesis, they defined the following statistic:

$$MVR = \sqrt{T} \max_{1 \leq i \leq m} |z(k_i)| \tag{11}$$

where $m$ is the number of horizons $k$ used for testing the null and $z(k_i)$ is defined in (8). The statistic follows the studentized maximum modulus (SMM) distribution with $m$ and $T$ degrees of freedom. When $T \to \infty$, at the $\alpha$ level of significance, $\pm SMM(\alpha, m, \infty) = Z_{\alpha^*/2}$, where $\alpha^*/2 = 1 - (1-\alpha)^{1/m}$.

Cecchetti & Lam (1994), in order to control the joint size of the VR test, proposed a Wald statistic that incorporates the correlations between the variance ratios at different horizons and weights them according to their variances

$$S(m) = \{\mathbf{VR}(m) - E[\mathbf{VR}(m)]\}' \mathbf{\Sigma}^{-1}(m) \{\mathbf{VR}(m) - E[\mathbf{VR}(m)]\} \tag{12}$$

where $\mathbf{VR}$ is the $(m \times 1)$ vector of VR statistics, $\mathbf{VR}(m) = [\hat{VR}(2), ..., \hat{VR}(m)]'$, with $\hat{VR}(k)$ defined in (5), $E$ is the expectation operator and $\mathbf{\Sigma}(m)$ is a measure of the covariance matrix of $\mathbf{VR}(m)$. $\mathbf{VR}(m)$ is asymptotically distributed as a multivariate normal and $S(m)$ is distributed as a chi-squared with $(m-1)$ degrees of freedom. In finite samples, as pointed by Cecchetti & Lam (1994), this asymptotic approximation can be misleading.

Tests of variance ratios based on asymptotic approximations may have significant size distortions and low power in finite samples. Wright (2000) proposed non-parametric tests based on ranks and signs which have exact sampling distribution. The critical values of these tests can be obtained by simulating their exact distributions. Since there is no need to appeal to any asymptotic approximation, these tests do not present size distortions and they may be more powerful then other tests if the data are highly non-normal. Belaire-Franch & Contreras (2004) suggested multiple versions of Wright's tests.

To overcome the low accuracy of VR tests based on asymptotic approximation, many researches have employed resampling methods to derive the empirical distribution of these statistics. Whang & Kim (2003) used a subsampling technique of Politis *et al.* (1997). Malliaropulos & Priestley (1999) employed a weighted bootstrap method of Wu (1986) and Kim (2006) applied a wild bootstrap method of Mammen (1993).

Kunsch (1989) and Liu & Singh (1992) developed an overlapping block bootstrap known as moving blocks bootstrap (MBB) which was used by Tabak & Lima

(2009) in the context of VR tests. The non-stationarity of the resampled series generated by the MBB methodology was criticized by Liu & Singh (1992). Politis & Romano (1992) proposed the circular block bootstrap (CBB) and Politis & Romano (1994) suggested the stationary bootstrap (SB) in order to circumvent this problem.

Lima & Tabak (2009) compared the power and size of different bootstrapped variance ratio tests. They concluded that the MBB methodology presents better performance for the construction of empirical distributions, but their results also suggest that the power of those tests are substantially affected by the choice of the maximum holding period $k$.

To eliminate the arbitrary choice of the holding period and improve small sample properties, this paper uses a wild bootstrapped variance ratio test which determines the holding period optimally using a data-dependent method. The sample properties of this test were recently studied by Kim (2009), who concluded that the test shows no size distortion and it has substantially higher power than other commonly used tests. The methodology is presented in section 3.1.

### 2.2 Empirical results

The study of efficiency in the Brazilian stock market suggests that daily returns of the Ibovespa index do not follow a random walk. Tabak (2003) rejected the RWH for Ibovespa daily returns in US dollars using data from 1986 to 1998. However, when the same test was conducted for the period of 1994 to 1998 the hypothesis could not be rejected. Chang *et al.* (2004) also rejected the RWH using a wild bootstrapped version of the test of Cecchetti & Lam (1994), with data from 1991 to 2004.

While Ibovespa daily returns seem not to follow a random walk, the RWH cannot be rejected for monthly returns. Karemera *et al.* (1999) did not reject the hypothesis for nominal and US-based monthly returns from 1987 to 1997, despite the evidence against the model for nominal returns. Torres *et al.* (2002) also concluded that the RWH cannot be rejected for monthly real returns from the period of 1970 to 1998.

These results contradict a well established fact that long horizons returns are more predictable than short ones. Since the period before 1994 affects the result obtained for daily returns, the rejection of the RWH could reflect a particular characteristic of the data or the market prior that date.

Variance ratio tests are commonly applied to the Ibovespa index, but few works focus on portfolios based on firm size or sector. Torres *et al.* (2002) tested the RWH for five different sized portfolios using real returns and concluded that small firms tend to be more predictable than large ones, supporting the results found in the American literature.

## 3. Methodology

### 3.1 Automatic variance ratio test

The random walk model described in equation (1) is equivalent to the following equation:

$$r_t = \mu + \varepsilon_t \tag{13}$$

where $r_t$ is the continuously compound return of the asset, $\mu$ is the constant expected return and $\varepsilon$ is a white noise not necessarily normal.

The null and alternative hypothesis considered are

$$H_0: \quad r_t \; is \; not \; serially \; correlated \; (or \; 2\pi f_r(0) = 1) \tag{14}$$

$$H_1: \quad r_t \; is \; serially \; correlated \; (or \; 2\pi f_r(0) \neq 1) \tag{15}$$

where $f_r(0)$ is the normalized spectral density of $r_t$ at the zero frequency. It is important to notice that if $r_t$ is not serially correlated, then $2\pi f_r(0) = 1$, but the conversely is not necessarily true. Obviously, if $2\pi f_r(0) \neq 1$, then $r_t$ is serially correlated. With this relation in mind, we can build our test using a consistent estimator of $2\pi f_r(0)$, which is given by equation (7), as pointed by Cochrane (1988). Instead of the Bartlett kernel, I employ the Quadratic Spectral kernel, following Choi (1999), since this kernel is optimal in estimating the spectral density at the zero frequency (Andrews, 1991). So we have the following relation:

$$\hat{VR}(k) = 1 + 2\sum_{i=1}^{T-1} m(i/k)\hat{\rho}(i) \tag{16}$$

where $\hat{\rho}(i) = \frac{\sum_{t=1}^{T-i}(r_t - \hat{\mu})(r_{t+i} - \hat{\mu})}{\sum_{t=1}^{T}(r_t - \hat{\mu})^2}$ is the estimated $i$th order autocorrelation coefficient of $r_t$, $\hat{\mu} = T^{-1}\sum_{t=1}^{T} r_t$ is the estimated mean of $r_t$, and $m(x) = \frac{25}{12\pi^2 x^2}\left[\frac{\sin(6\pi/5)}{6\pi x/5} - \cos(6\pi x/5)\right]$ is the Quadractic Spectral kernel.

Under the null hypothesis of no serial correlation, we have

$$AVR(k) = \frac{\sqrt{T/k}[\hat{VR}(k) - 1]}{\sqrt{2}} \sim N(0,1) \text{ as } T \to \infty, k \to \infty \text{ and } T/k \to \infty. \tag{17}$$

This result, as stated by Choi (1999), is valid if $r_t$ is a martingale difference process with proper moment restrictions.

The value of the holding period $k$ is chosen by an optimal method of selecting the truncation point for the spectral density at the zero frequency proposed by Andrews (1991):

$$\hat{k} = 1.3221(\hat{\alpha}(2)T)^{1/5} \tag{18}$$

where $\hat{\alpha}(2)$ under the null hypothesis is reduced to:[2]

$$\hat{\alpha}(2) = \frac{4\hat{\rho}(1)^2}{(1-\hat{\rho}(1))^4} \tag{19}$$

and $\hat{\rho}(1)$ is the first order autocorrelation coefficient of $r_t$.

Applying the optimal value of $k$ in the statistic (18) we have the Automatic Variance Ratio test $AVR(\hat{k})$, and the relation in (18) continues to be satisfied. This asymptotic test may show deficient small sample properties, as pointed by Kim (2009). To improve the performance of the test, the wild bootstrap method of Mammen (1993) is conducted in three stages:

1. With the original series, form a bootstrap sample of T observations $X_t^* = \eta_t X_t$, where $\eta_t$ is a random sequence with $E(\eta_t) = 0$ and $E(\eta_t^2) = 1$;

2. Calculate $AVR(\hat{k})$ for each pseudo-series $X_t^*$;

3. Repeat steps 1. and 2. 1000 times and compare the quantiles of the distribution of these statistics with the $AVR(\hat{k})$ calculated from the original data.

The p-value of the test is obtained from the proportion of the absolute values of $AVR(\hat{k})$ calculated with the pseudo-series greater than the absolute value of $AVR(\hat{k})$ obtained from the original data.

### 3.2 Data

To test the RWH for the Brazilian stock market I use the log of daily and monthly returns of the Ibovespa index from 1986 to 2008.[3] The $AVR(\hat{k})$ test is conducted for real returns, deflated by the General Price Index (IGP-DI). Two subperiods are analyzed, 1986 to 1994 and 1995 to 2008. This subdivision is intended to assess if the reduction of inflation after the price stabilization plan known as Plano Real has influenced the weak form efficiency of the Brazilian stock market. Table 1 shows the annual inflation rate (IGP-DI) from 1986 to 2008.

[2] See equation 6.4 of Andrews (1991) for further details.

[3] The use of daily and monthly returns intends to divide the analysis of predictability in short and long run. Weekly returns were omitted since there were no gains in the analysis, specially because the results were very close to the daily returns.

**Table 1**
Annual inflation rate measured by IGP-DI

| Date | IGP-DI (%) | Date | IGP-DI (%) | Date | IGP-DI (%) | Date | IGP-DI (%) |
|---|---|---|---|---|---|---|---|
| 1986 | 65.03 | 1992 | 1,157.83 | 1998 | 1.70 | 2004 | 12.14 |
| 1987 | 415.83 | 1993 | 2,708.17 | 1999 | 19.98 | 2005 | 1.22 |
| 1988 | 1,037.56 | 1994 | 1,093.89 | 2000 | 9.81 | 2006 | 3.79 |
| 1989 | 1,782.89 | 1995 | 14.78 | 2001 | 10.40 | 2007 | 7.89 |
| 1990 | 1,476.71 | 1996 | 9.34 | 2002 | 26.41 | 2008 | 9.10 |
| 1991 | 480.23 | 1997 | 7.48 | 2003 | 7.67 | | |

Data are from Fundação Getúlio Vargas (FGV).

The relation between the firm size and predictability of returns is a well established fact in the literature (Chordia *et al.*, 2008, Yakov, 2002, Torres *et al.*, 2002). To address this question, the database is divided into large and small firms. The data are from 1999 to 2008 in daily and monthly frequencies. The large firms portfolio is composed by ten equally weighted stocks from firms with the largest market capitalization during the sample period. The small firms portfolio is composed by ten equally weighted stocks from firms with the smallest market capitalization during the sample period. If the firm has both ordinary and preferential stocks, only the more liquid one is included in the portfolio. To circumvent the problem of infrequency of trading in daily returns, I excluded stocks which had no trades in more than 5% of the days included in the sample.

To investigate predictability in different sectors, five equally weighted portfolios are constructed based on the Sao Paulo Stock Exchange (BM&FBovespa) sector classification, representing consumer, energy, financial, industrial and telecommunications sectors. Also, if the firm has both ordinary and preferential stocks, only the more liquid one is included in the portfolio. The number of firms on each portfolio varies according to the sector. The data are from 1999 to 2008 in daily and monthly frequencies. Stocks which had no trades in more than 5% of the days included in the sample were also excluded.

The test is conducted for the log of real returns. All the series of real returns are calculated using the General Price Index (IGP-DI) obtained from Fundação Getúlio Vargas (FGV) and the data available from BM&FBovespa. The financial series are already adjusted for splits, subscriptions, dividend and bonuses.

## 4. Results

Table 2 presents the $AVR(\hat{k})$ test for the Ibovespa index. Using daily frequency, we can reject the RWH in the period of 1986 to 2008. This rejection might be due to the period of 1986 to 1994, since the hypothesis is not rejected using data from 1995 to 2008. Using monthly frequency, the index follows a random walk for both sub-periods.

**Table 2**
AVR tests of the Ibovespa index

| Sample | Number of base observations | | Daily returns | Monthly returns |
|---|---|---|---|---|
| | Days | Months | | |
| *Ibovespa* | | | | |
| 1986:01:01-2008:12:31 | 5659 | 276 | 6.969 | -1.269 |
| | | | (0.000)* | (0.238) |
| 1986:01:01-1994:12:31 | 2195 | 108 | 7.107 | -0.970 |
| | | | (0.000)* | (0.238) |
| 1995:01:01-2008:12:31 | 3464 | 168 | 1.164 | 0.003 |
| | | | (0.368) | (0.977) |

The values refer to the $AVR(\hat{k})$ statistic and the number in parenthesis is the p-value of the null hypothesis in (14). The *, ** and *** denote statistical significance at 1%, 5% and 10% level, respectively.

These results are rather counter-intuitive since the evidence of predictability is usually associated with economic cycles and long run returns, however they seem to illustrate a particular fact of the Brazilian stock market and are well documented in the literature. Karemera *et al.* (1999) and Torres *et al.* (2002) could not reject the RWH for monthly returns of the Ibovespa index. Chang *et al.* (2004) and Tabak (2003) rejected the hypothesis for Ibovespa daily returns.

This unappealing result, as we can see, might be a consequence of the data prior 1994. Possible explanations are the presence of nonsynchronous trading effects on daily data of the Ibovespa index prior that year or even the presence of inefficiencies in the stock market, since high inflation can potentially affect stock markets liquidity. These questions were not yet addressed in the literature.

Table 3 shows the result of the $AVR(\hat{k})$ test for two portfolios based on firm size. The data are from 1999 to 2008. The test did not find substantially evidence of predictability for daily returns for both large and small firms, but monthly returns are more predictable, specially for the small firms portfolio, which presents lower p-values in all cases.

**Table 3**
AVR tests of the portfolios based on firm size

| Sample | Number of base observations | | Daily returns | Monthly returns |
|---|---|---|---|---|
| | Days | Months | | |
| *Large firms* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 1.527 | 1.244 |
| | | | (0.200) | (0.085)*** |
| *Small firms* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 1.683 | 1.976 |
| | | | (0.111) | (0.011)** |

The values refer to the $AVR(\hat{k})$ statistic and the number in parenthesis is the p-value of the null hypothesis in (14). The *, ** and *** denote statistical significance at 1%, 5% and 10% level, respectively.

The higher predictability of small firms is well documented in the American literature (Chordia *et al.*, 2008). For the Brazilian stock market, Torres *et al.* (2002) found similar results using data from 1986 to 1998. The test also reports lower p-

values for monthly returns, supporting the evidence of higher predictability for long horizon returns.

Table 4 shows the test for portfolios based on five different sectors, using data from 1999 to 2008. Using both frequencies, the RWH cannot be rejected, except for the industrial sector, which strongly rejected the null. The p-values are lower for monthly returns, except for the financial and telecommunications sectors.

**Table 4**
AVR tests of the portfolios based on sectors

| Sample | Number of base observations | | Daily returns | Monthly returns |
|---|---|---|---|---|
| | Days | Months | | |
| *Consumer* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 0.411 | 0.694 |
| | | | (0.681) | (0.303) |
| *Energy* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 0.279 | 0.929 |
| | | | (0.751) | (0.103) |
| *Financial* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 1.725 | -0.389 |
| | | | (0.143) | (0.517) |
| *Industrial* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 3.140 | 3.270 |
| | | | (0.009)* | (0.001)* |
| *Telecommunications* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 0.084 | -0.001 |
| | | | (0.794) | (0.998) |

The values refer to the $AVR(\hat{k})$ statistic and the number in parenthesis is the p-value of the null hypothesis in (14). The *, ** and *** denote statistical significance at 1%, 5% and 10% level, respectively.

The strong evidence of predictability for the industrial sector might reflect a closer relation of this firms with economic cycles, since this sector seems to be more affected by fluctuations in GDP, which itself has been documented to follow a predictable pattern. This result is new in the literature for Brazil and such questions are not yet addressed. I conducted other tests to confirm the robustness of this result. The industrial sector portfolio had no outliers in the sample and the evidence of predictability is also present using nominal returns.

Since the industrial sector portfolio strongly rejected the RWH, I excluded these firms from the size based portfolios to assess if the evidence of predictability found for monthly returns of both small and large firms is affected by the exclusion of industrial firms. Table 5 shows the results for size based portfolios excluding industrial firms. All p-values are higher, but small firms still rejected the RWH for monthly returns at 10% level of significance. The evidence of predictability for monthly returns of the large firms portfolio disappeared. Instead, large firms seem to follow a random walk. The influence of industrial firms in the small-sized portfolio was not significant.

**Table 5**
AVR tests of the portfolios based on firm size excluding industrial firms

| Sample | Number of base observations | | Daily returns | Monthly returns |
|---|---|---|---|---|
| | Days | Months | | |
| *Large firms* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 0.908 | 0.209 |
| | | | (0.402) | (0.615) |
| *Small firms* | | | | |
| 1999:01:01-2008:12:31 | 2477 | 120 | 0.906 | 1.225 |
| | | | (0.346) | (0.054)*** |

The values refer to the $AVR(\hat{k})$ statistic and the number in parenthesis is the p-value of the null hypothesis in (14). The *, ** and *** denote statistical significance at 1%, 5% and 10% level, respectively.

This result suggests that a great part of the evidence of predictable returns in the Brazilian stock market might be from stocks of the industrial sector, as well as from stocks of small firms. This fact can have important impact on portfolio selection and the study of patterns of predictability in equity returns.

## 5. Summary

This paper searches for evidence of predictability in daily and monthly returns of the Brazilian stock market using a wild bootstrapped automatic variance ratio test. Some undesired features of VR tests are eliminated using this methodology, like the arbitrary choice of the holding period and the small sample deficiencies present in asymptotic tests.

The findings suggest that market efficiency was affected positively by the price stability after 1994. Small firms presented more evidence of predictability than large ones, supporting the results found in many other international equity markets. However, this evidence is present only in monthly returns.

A particularly interesting result is that returns from the industrial sector are highly predictable in both daily and monthly frequencies and the evidence against the RWH found in the sized based portfolios is weakened when the industrial firms are excluded from the sample. This result is new in the literature and might reflect some particular characteristics of those firms still unknown.

Despite the increased efficiency of equity markets, predictable patterns still exist that can be exploited, especially in emerging markets. One example of that is the strong evidence found from returns of industrial firms in the Brazilian stock market.

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