Global Risk Evolution and Diversification: A Copula-DCC-GARCH Model Approach

(Evolução e Diversificação do Risco Global: Uma Abordagem com Modelo Copula-DCC-GARCH)

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Abstract

In this paper we estimate a dynamic portfolio composed by the U.S., German, British, Brazilian, Hong Kong and Australian markets, the period considered started on September 2001 and finished in September 2011. We ran the Copula-DCC-GARCH model on the daily returns conditional covariance matrix. The results allow us to conclude that there were changes in portfolio composition, occasioned by modifications in volatility and dependence between markets. The dynamic approach significantly reduced the portfolio risk if compared to the traditional static approach, especially in turbulent periods. Furthermore, we verified that the estimated copula model outperformed the conventional DCC model for the sample studied.

***Keywords*:** dynamic portfolio; risk management; copulas; multivariate GARCH.

JEL codes: G11; F37; C32; C61.

Resumo

No presente artigo é estimado um portfólio dinâmico composto pelos mercados norte americano, alemão, britânico, brasileiro, honconguês e australiano, considerando o período de setembro de 2001 a setembro de 2011. Para tanto, é obtida por meio de um modelo Copula-DCC-GARCH a matriz de covariância condicional dos retornos diários. Os resultados permitem concluir que houve mudanças na composição do portfólio, ocasionadas por modificações na volatilidade e dependência entre os mercados. A abordagem dinâmica reduziu significativamente o risco do portfólio se comparada com a abordagem estática tradicional, especialmente em períodos de turbulência. Mais além, verificou-se que o modelo estimado com cópulas supera o modelo DCC convencional na amostra estudada.

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Palavras-chave**:** portfólio dinâmico; gestão do risco; cópulas; GARCH multivariado.

1. Introduction

Since the introduction of the mathematical theory of portfolio selection and the Capital Asset Pricing Model (CAPM), the dependence issue has played an important role in financial economics. Within this context, the behavior of correlations and covariances between returns is an essential part in asset pricing, portfolio selection and risk management.

In the context of international diversification, there is a necessity for minimizing risk of specific assets through optimal resources allocation. Events of global importance tend to have a significant impact on stock markets. Financial markets crisis can lead to dramatic changes in investment behavior and so it is important to study the stock markets dynamic interdependence before and after any significant economic shock.

It is necessary to understand the multivariate relationship between different markets. Thus, we need a statistical model able to measure the temporal dependence between shocks in different countries. The traditional approach to measurerisk of financial assets is based in the historical covariance matrix. The use of this unconditional static measure has as drawback the fact that it is constant. However, an inappropriate model for dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Hence, it is necessary knowing the dynamic behavior of the covariance among financial assets.

To deal with this question, it was developed the ARCH (autoregressive conditional heteroscedasticity) introduced by Engle (1982) and its generalization GARCH (generalized autoregressive conditional heteroscedasticity) proposed by Bollerslev (1986), which are widely applied to model financial series volatility. Since the proposal of these models isto account for variance heterogeneity in financial time series, some multivariate extensions of GARCH models have been introduced. The most consolidated models in literature are the Constant Conditional Correlation (CCC-GARCH) model of Bollerslev (1990), the BEKK model of Engle & Kroner (1995) and later the Dynamic Conditional Correlation (DCC-GARCH), developed by Engle & Sheppard (2001) and Tse & Tsui (2002).

Based in the covariance matrix estimation, it is possible to extend the traditional portfolio optimization to a dynamic one. To that, it is necessary to minimize the portfolio variance based in weights for each time point. So an investor can control the evolution of a particular asset participation in risk diversification, as well as make predictions about the future composition, in order to reduce portfolio volatility.

The dynamic portfolio literature is dominated by multivariate GARCH models. Campbell *et al.* (2001) maximizes the portfolio return subject to the VaR restriction. Polasek & Momtchill (2003) found superiority of BEKK model over an asymmetric univariate GARCH. De Goeij & Marquering (2004) estimate stocks and bonds covariances through diagonal and asymmetric VECH models. Specht & Winker (2008) use a principal component GARCH to estimate the conditional covariance matrix. Bauwens *et al.* (2010) indicate that multivariate GARCH models improve the optimal resources allocation for intraday data.

However, all of the models mentioned in the previous section are estimated under the assumption of multivariate normality or based some mixture of elliptical distributions. However, this assumption is unrealistic, as evidenced by numerous empirical studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent.These difficulties can be treated as a problem of copulas. The concept of copula was introduced by Sklar (1959). A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of dependence, which is the copula. Revisions of copula methods can be found Melo & Mendes (2009), Morettin *et al.* (2010) and Pereira & Santos (2011). Thus, the joint distribution of the asset returns can be specified with full flexibility, which is more realistic.

Thus, emerges the Copula-DCC-GARCH model, which was proposed with a financial application by Jondeau & Rockinger (2006). Some posterior studies employed the Copula-DCC-GARCH model because of its advantages. Fantazzini (2009) performed Value at Risk simulations. Aas & Berg (2009), Ausin & Lopes (2010) and Hafner & Reznikova (2010) investigated dependence structures between financial assets. Righi & Ceretta (2011a) identified structural changes in European markets volatility. Righi & Ceretta (2011b) estimated value at risk and optimal hedge ratio in Latin markets. Righi & Ceretta (2012) performed daily risk predictions for a global portfolio.

In this sense, the aim of this paper is toanalyze evolution in global risk diversification. We used data of U.S., German, British, Brazilian, Hong

Kong and Australian markets.The main contribution of the present paper isthe use a Copula-DCC-GARCH in order to estimate the conditional covariance matrix. This is an advance in dynamic portfolio construction because this model is more flexible than other multivariate GARCH approaches. It is also an advance in Copula-DCC-GARCH financial applications, once was not performed studies with this type of analysis. Nonetheless, we compute the dynamic evolution of each market weight, identifying the turbulent period effects on optimal resources allocation. There is a lack of such economic analysis of weights changing in crisis literature, especially considering differences regarding to portfolio strategies.

2. Methodological Procedures

In order to analyze the evolution of the global risk diversification we used daily prices of U.S. (*Standard and Poor's 500* – S&P500), German (*Deutscher Aktien Index* – DAX), British (Financial Times Securities Exchange Index 100 – FTSE100), Brazilian (*Índice da Bolsa de Valores de São Paulo* – Ibovespa), Hong Kong (*Hang Seng Index* – HSI) and Australian (*Australian Securities Exchange* – ASX) markets, from September 2001 to September 2011, totalizing 2280 observations. These bourses were chosen because they have great liquidity and are frequently used to international risk diversification, especially during the last decade with growing globalization. Furthermore, we attempted to not use many markets of the same continent, due to the geographic dependence inherent to neighbor countries. This dependence could cause biases on the real participation because neighbor countries tend to have more association.

To eliminate problems of non-stationarity, we calculated the indices log-returns. We used a vector autoregressive model (VAR) to obtain the average estimate of series conditional mean. The mathematical form of the VAR(p) model used is represented by (1).

$$\boldsymbol{r}_t = \boldsymbol{\phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{r}_{t-1} + ...\boldsymbol{\Phi}_p \boldsymbol{r}_{t-p} + \boldsymbol{\alpha}_t \tag{1}$$

In (1), $\boldsymbol{r}_t$ is a k-dimensional vector of the log-returns at period t; $\boldsymbol{\phi}_0$ is a k-dimensional vector of constants; $\boldsymbol{\Phi}_1$, $i = 1, \ldots, p$ are kxk matrixes of parameters; $\{\boldsymbol{\alpha}_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix Σ. The VAR model was estimated by ML. Subsequently, using the residuals $\{\boldsymbol{\alpha}_t\}$ which were obtained through VAR, we estimated the conditional covariance matrix with acopula-DCC-

GARCH model, represented by the formulation (2), which is able to lead with the asymmetric leptokurtic behavior of financial assets returns.

$$\mathbf{H}_t = \mathbf{D}_t^{'}\mathbf{R}_t\mathbf{D}_t \tag{2}$$

where $\mathbf{D}_t = diag(h_{1,t}^{1/2} \dots h_{N,t}^{1/2})$;
$\mathbf{R}_t = diag(q_{11,t}^{-1/2} ... q_{NN,t}^{-1/2})\mathbf{Q}_t diag(q_{11,t}^{-1/2} ... q_{NN,t}^{-1/2})$;
$\mathbf{Q}_t = (1-\alpha-\beta)\hat{\mathbf{Q}} + \alpha u_{t-1}u_{t-1}^{'} + \beta\mathbf{Q}_{t-1}, u_{i,t} = \epsilon_{i,t}/\sqrt{h_{i,t}} \sim skew-t_v$;
$\hat{\mathbf{Q}}$ is the NxN matrix composed by unconditional covariance of u_t; α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$. The definition of residuals $u_{i,t}$ joint distribution extends the traditional DCC (based on multivariate normality or mixture of elliptical distributions) through copulas, which allow more flexibility indescribing the data once copulas are estimated with separately from marginal. We test the fit of six-dimensional copulas Normal, Student's t, Clayton, Gumbel and Frank. Cherubini *et al.* (2012) present definition of these copulas. We used the AIC in order to select the family which best fitted the data. The model parameters were estimated through Quasi Maximum Likelihood (QML) in two steps Jondeau *et al.* (2007). In the first stage, the conditional variance is estimated by means of an univariate GARCH model, respectively, for each asset. In the second step,the parameters for the conditional correlation, given the parameters of the first stage, are estimated. To validate the model, we use the Q statistic in linear and squared residuals in order to test the null hypothesis that the data is random against the alternative of non-randomness.

With estimated covariances, we constructed a dynamic portfolio formed bymarket indices in order to analyze the diversification structure existing in global investment risk. We made an extension of classic static approach introduced in the seminal paper of Markowitz (1952). Such extension isrepresented by formulation (4). This expression is optimized at each period t, based on weights vector subject to restrictions (5) and (6).

$$var_t = min_t \mathbf{w}_t^{'}\mathbf{H}_t\mathbf{w}_t \tag{3}$$

$$w_{i,t} \geq 0,0, \forall i = 1, ..., n \tag{4}$$

$$\sum_{i=1}^{n} w_{i,t} = 1 \tag{5}$$

In (4), var_t is the variance of the portfolio in instant t; $\mathbf{w}_t$ is the weights vector in the instant t; $\mathbf{H}_t$ is the estimated covariance matrix in instant t; In (5) and (6) $w_{i,t}$ is the weight of (the) asset i in the composition of the portfolio at instant t; n is the number of assets. Formulation (5) assures the non-negativity of the weights, while (6) states that is necessary to use all available capital in the portfolio.

In the next step, we compared the dynamic and static risk minimization in-sample and out-sample. For out-sample, we removed the last 220 estimated covariance matrices, which correspond to one year, and replaced them for Copula-DCC-GARCH model forecasts, estimating again optimal weights. For static portfolio we used the Copula-DCC-GARCH model estimates and forecasts, but weights were fixed, being those obtained through unconditional covariance matrix.

We compared the volatility mean of both portfolios, i.e., we test the null hypothesis $H_0 : Volatility_{dynamic} \geq Volatility_{static}$, against $H1 : Volatility_{dynamic} < Volatility_{static}$. The rejection of this null hypothesis implies in Copula-DCC-GARCH based portfolio having less risk than static one. Moreover, we verified if the mean proportional reduction in risk with the dynamic approach is significantly larger than zero, i.e., $H_0 : RiskReduction \geq 0$ against $H_0 : RiskReduction > 0$. The rejection of this null hypothesis implies in Copula-DCC-GARCH based portfolio significantly reduces risk in relation with static approach.

These hypothesis tests were calculatedwith two approaches. One is the non-parametric Wilcoxon rank-sum test, which was chosen because does not requires the normality assumption, is calculated summing the ranks of the absolute differences between two samples. The Wilcoxon rank-sum test is asymptotically normally distributed. The other is the Superior Predictive Ability (SPA), proposed by Hansen (2005). This testcompares favorably to the reality check for data snooping, because it is more powerful and less sensitive to poor and irrelevant alternatives. Thus, this procedure can properly verify if strategy outperforms a benchmarkin past data, or even if some variable is significant, using bootstrapping to construct a confidence interval.

Furthermore, in order to give robustness to the results, we applied similar procedure for comparison between Copula-DCC-GARCH approach and traditional DCC-GARCH, with Normal and Student's t multivariate distributions. Finally, we are able to validate both dynamic optimizations as its realization by Copula-DCC-Model.

3. Results and Discussion

We initially calculated the indices log-returns for the period of September 2001 to September 2011. The daily prices are exposed in Figure 1, while log-returns in Figure 2.

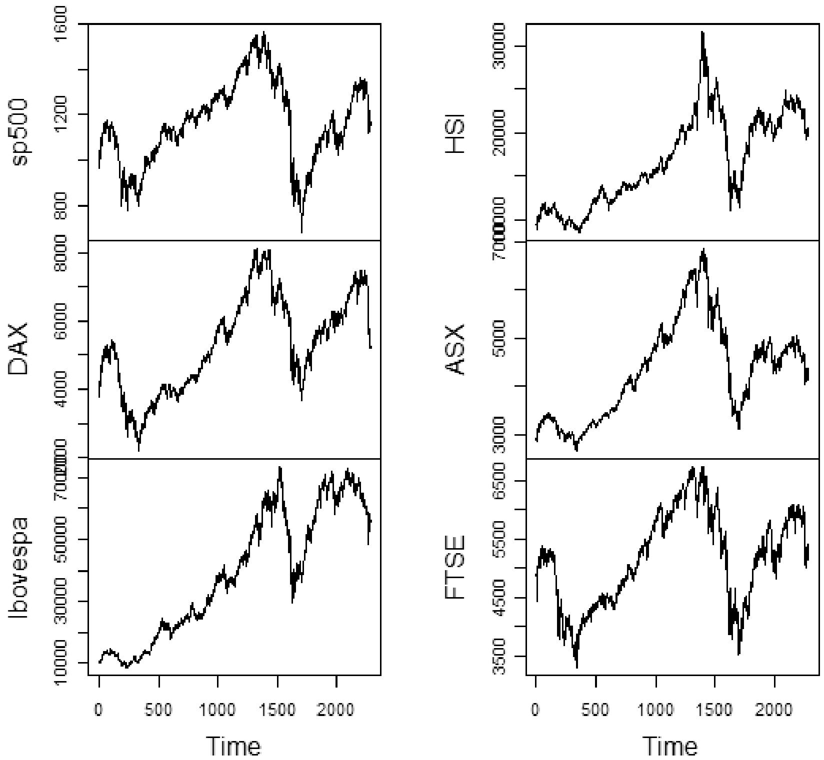

Figure 1
Daily prices of the U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011

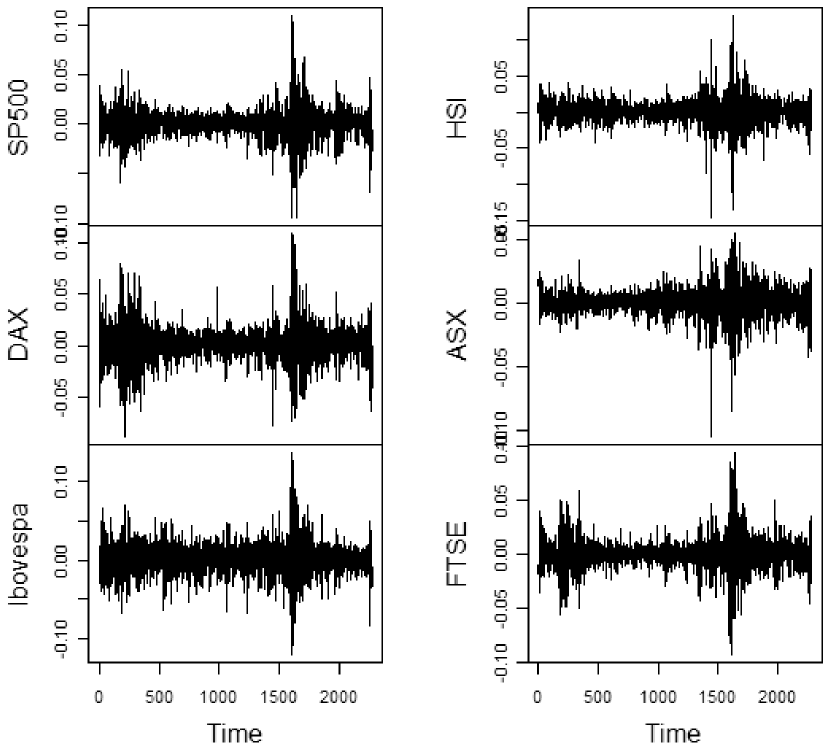

Figure 2
Daily log-returns of the U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011

Figure 1 indicates that markets daily prices do not have the stationarity propriety due to the presence of trends occasioned by economic shocks. It should be noted that there was ahuge fall in all markets during the observations 100-300, 1600-1800 and 2200. These periods of strong turbulence are, respectively, the consequences of the 9/11 terrorist attacks, the subprime crisis of 2007-2008 and, the Greek crisis that began in 2010. Figure 2 evidences that all markets presented three notable volatility clusters, around the previous cited turbulences. There were magnitude differences in markets volatilities. These discrepancies are supposedly occasioned by distinct economic maturity and liquidity levels.

In order to complement the visual analysis of Figure 1, Table 1 presents the log-returns descriptive statistics, while Table 2 exhibits the correlation matrix of them. Table 1 emphasizes that the studied markets had central trend measures very close to zero, as indicated by values for means and medians. Standard deviation was similar for all markets, with bigger value for Ibovespa. It confirms the visual analysis in Figure 2 and can be explained by the fact that Brazil is an emerging country, being relatively riskier than developed ones. All markets, with exception to the Brazilian, had more probability on the left tail, leading to a negative skewness. Kurtosis evidences that markets are leptokurtic. These behaviorsare quite common in financial assets, as documented in previous researches (Longin & Solnik, 2001, Ang & Chen, 2002, Patton, 2006).

Table 1
Descriptive statistics of the daily log-returns of U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011

Statistic	SP500	DAX	IBOV	HSI	ASX	FTSE
Minimum	-0.0947	-0.9883	-0.1210	-0.1469	-0.1049	-0.1265
Maximum	0.1096	0.1079	0.1368	0.1341	0.0540	0.0938
Mean	0.0000	0.0001	0.0007	0.0003	0.0002	0.0000
Median	0.0007	0.0008	0.0013	0.0005	0.0006	0.0005
St. Deviation	0.0142	0.0173	0.0201	0.0168	0.0111	0.0139
Skewness	-0.2026	0.0195	-0.0784	-0.2118	-0.7669	-0.1336
Kurtosis	79.843	44.108	38.490	103.624	81.562	61.427

Table 2 indicates that there were differences between the analyzed markets linear dependences. The stronger linear dependence was between DAX and FTSE, which is normal because they belong to the same continent. The weaker correlation is for the relationship of S&P500 and ASX. In general, the U.S. market has bigger correlations with others markets. This happens because U.S. market is the most influent due to the North American economic power.

Table 2
Correlation matrix of the daily log-returns of U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011

	SP500	DAX	IBOV	HSI	ASX	FTSE
S&P500	10.000	0.6423	0.6601	0.2579	0.1497	0.5850
DAX		10.000	0.4859	0.3740	0.3233	0.8232
IBOV			10.000	0.3374	0.2165	0.5039
HSI				10.000	0.6454	0.4167
ASX					10.000	0.3942
FTSE						10.000

Subsequently, we estimated the VAR (12) model, as specified in (1). The number of lags was chosen based on the traditional AIC. Due to lack of space we omitted the results about this model. With the residuals of this model we estimated the Copula-DCC-GARCH, as formulated in (2), in order to obtain the dynamic covariance matrix of the assets. The results of this model are presented in Table 3. The results contained in Table 3 indicate that markets volatility is conditional to past information, as evidenced by the statistically significance of b_i parameter for all assets. The volatility of Ibovespa, HSI and DAX was affected by past squared shocks during the studied period, as noted by their respective α_i parameter. Therefore, it would be a mistake to use unconditional covariance matrix as a risk measure in the portfolio construction.

Table 3
Results of the estimated copula-DCC-GARCH model for daily log-returnsof U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011

Parameter	Coefficient	p-value
SP500	0.0000	0.8413
SP500	0.0840	0.2329
SP500	**0.9126**	0.0000
DAX	0.0000	0.8363
DAX	0.0916	0.3878
DAX	**0.9044**	0.0000
Ibovespa	0.0000	0.3678
Ibovespa	**0.0698**	0.0000
Ibovespa	**0.9161**	0.0000
HSI	0.0000	0.6719
HSI	**0.0593**	0.0396
HSI	**0.9353**	0.0000
ASX	0.0000	0.6420
ASX	**0.0809**	0.0001
ASX	**0.9172**	0.0000
FTSE	0.0000	0.8945
FTSE	0.1156	0.5454
FTSE	**0.8780**	0.0000
	AIC	-38.923
Copula	Parameter	p-value
Student'st	78.459	0.0000
	Log-Likelihood	34.565
	AIC	-8.345

*Bold values are significant at 5% level.

c_i, a_i and b_i represent the constant, the impact of past squared shocks and the impact of the lagged conditional volatilities, respectively.

Q statistics did not reject the null hypothesis of randomness.

The results in Table 3 show the best fit of the copula-DCC-GARCH based on student t copula, i.e., the best fit for both marginal distributions and joint distribution. This result corroborate with previous researches, like Marshal & Zeevi (2003) among others, which have shown that the fit of this copula family is generally superior to that of other copulas for financial data. It is worth to note that these copulas assign, in certain degree, most importance to the tails of the joint probability distribution than the Gaussian one. This factindicates higher dependence among sectors in extreme events than the normally expected.

Complementing, theestimated volatilities and dynamic correlations are shown, respectively, in Figures 3 and 4. Figure 3 reinforces the previous results exhibiting peaks in conditional volatility during the observations that represent the consequences of the terrorist attacks in 2001, the sub-prime crisis of 2007/2008 and, the Greek crisis that begin in 2010. It is worth to note that Australian market had the lower risk level during the period. Further, the horizontal lines in the plots represent the unconditional counterparts. Again, it is evidenced that unconditional measures are unable to match risk dynamics in financial markets. Figure 4 visually indicates that unconditional covariance matrix do not seems to follow financial data evolution. Just like conditional volatilities, there are horizontal lines that represent unconditional correlation between each pair of markets. It is evident that there were many oscilations in the linear dependence of the analyzed assets. There was a similar behavior of initial decrease followed by a growth in the correlation with the series evolution, indicating a possible vestige of globalization. The peak of linear dependence is associated with the turbulent period of the 2007/2008 sub-prime crisis, which affected all markets.

With the log-returns conditional covariance matrix we constructed a portfolio for minimizing risk, as explained in (4). Participation of each asset evolution in portfolio composition is presented in Figure 5. Plots in Figure 5 emphasizes that there was a large amount of variation in global risk diversification. Horizontal lines represent the static weights obtained by minimizing an expression similar to (4) for the unconditional covariance matrix. In the first sample part, ASX had greater participation, followed by S&P500 and FTSE. This result emergesthe possibilitythat after the terrorist attack of 2001 investors could have changed their applications to other markets. The chosen markets were those with high liquidity and less risk at that time. After this period, ASX lost space. FTSE kept a regular weight in the composition. S&P500 recovered its participation after the initial period, but had a fall during the sub-prime crisis. DAX and HSI had small weights along the period, exhibiting peaks during sub-prime and the Greek crisis, respectively. Ibovespa presented more participation especially after the sub-prime crisis, where Brazilian market conquered international respectability.

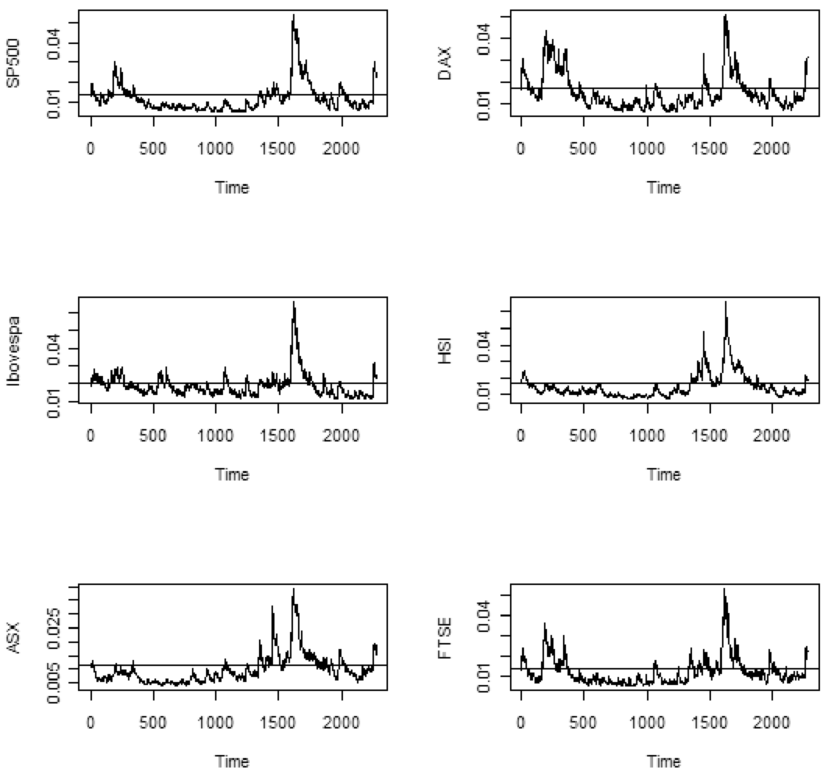

Figure 3
Estimated conditional volatilities of daily log-returns of the U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011. The horizontal lines represent the standard deviation of log-returns in each market during the analyzed period

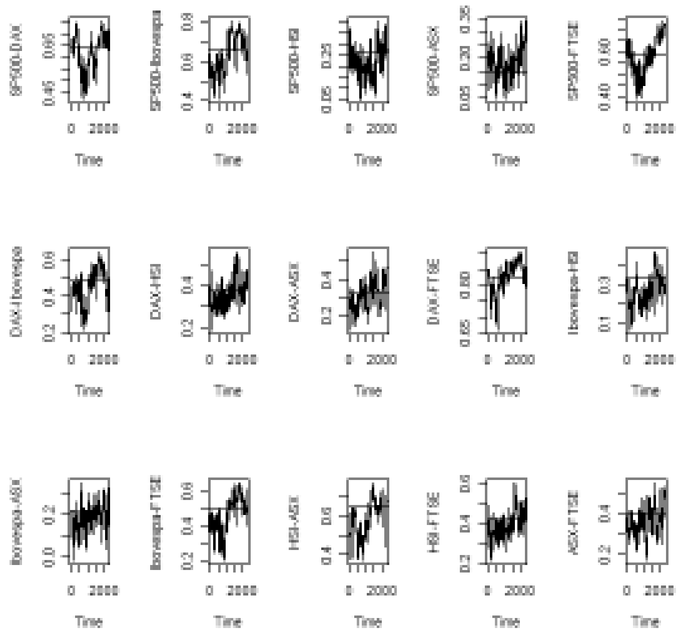

Figure 4
Estimated conditional correlations among daily log-returns of the U.S., German, British, Brazilian, Hong Kong and Australian markets, in the period from September 2001 to September 2011. The horizontal lines represent the unconditional correlation of log-returns for each bivariate relationship between the markets during the analyzed period

After, we compared the risk reduction proportioned by this dynamic portfolio over the traditional static approach. Figures 6 and 7 exhibit, respectively, the absolute and proportional reduction in-sample, while Figures 8 and 9 present the same on out-sample, which is composed by 220 points. Moreover, Table 4 presents the results of the Wilcoxon rank-sum test for the analyzed data and simulated samples.

The plots of Figures 6 and 7 highlighted the superiority of the dynamic portfolio over the static approach regarding to risk minimization. The peaks in the reduction occasioned by the dynamic rebalance of weights were just during the turbulent periods of the 2001 terrorist attack and the 2007/2008 sub-prime crisis. In these periods risk reduction overcame 30%.

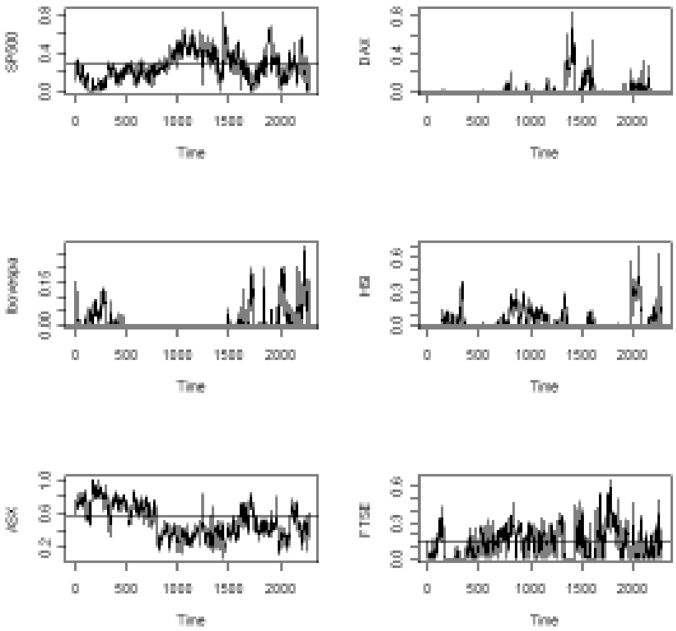

Figure 5
Temporal evolution of the U.S., German, British, Brazilian, Hong Kong and Australian marketsin the participation at the international diversification during the period from September 2001 to September 2011. The horizontal lines represent weights estimated through the unconditional covariance matrix of log-returns for each market during the analyzed period

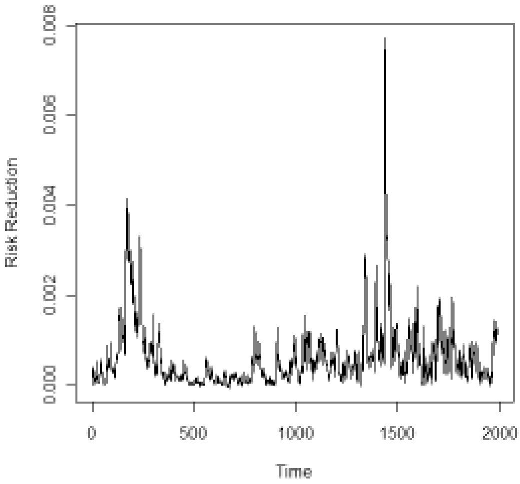

Figure 6
Absolute difference of the volatilities of the dynamic and static portfolios composed by the daily returns of the U.S., German, British, Brazilian, Hong Kong and Australian marketsin the period from September 2001 to September 2011

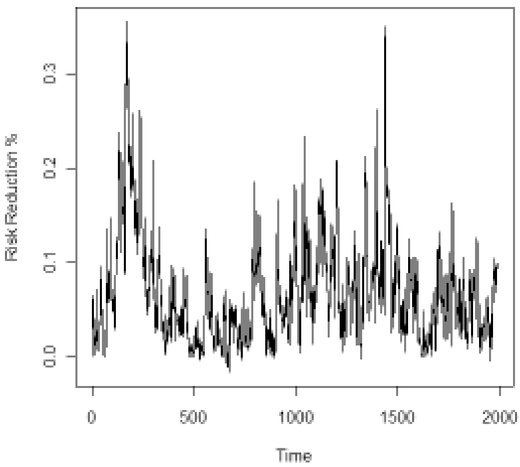

Figure 7
Proportional risk reduction obtained by the dynamic portfolio over the static one composed by the daily returns of the U.S., German, British, Brazilian, Hong Kong and Australian marketsin the period from September 2001 to September 2011

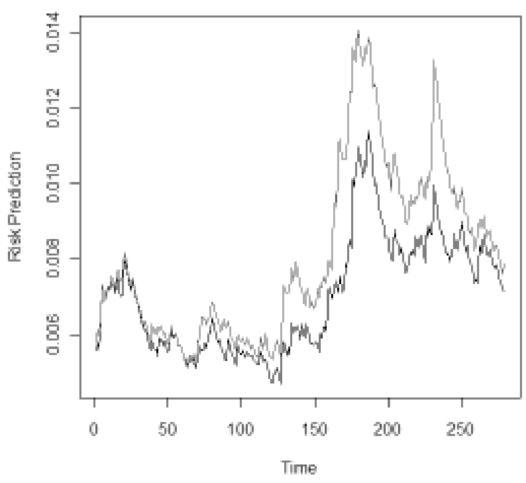

Figure 8
Risk predictions of the dynamic (black) and static (red) portfolios for the 220 observations on the out-sample period. These predictions were obtained through the copula-GARCH model

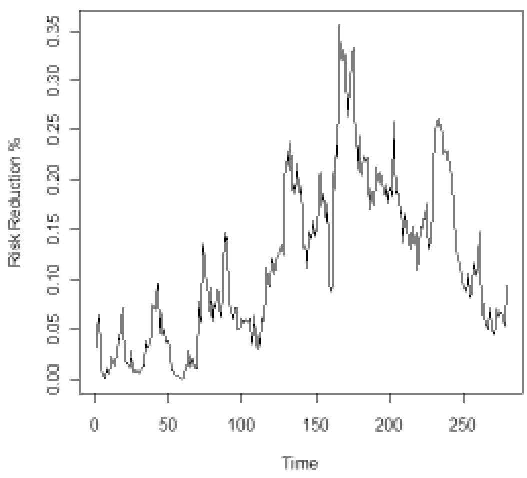

Figure 9
Proportional risk reduction obtained by the dynamic portfolio over the static one composed by the daily returns of the U.S., German, British, Brazilian, Hong Kong and Australian marketsin the out-sample period

Further, on out-sample, represented by Figures 8 and 9, the dynamic portfolio advantage remained at the same level. It is worth to frizz that the estimated Copula-DCC-GARCH model was able to predict the volatility cluster occurred during Greek crisis. Again, especially during this turbulent period, the dynamic portfolio reduced risk in more than 30%. In order to validate the advantage obtained by the dynamic approach, we compared its volatility with Static and conventional DCC approaches.We further verified if the volatility reduction in relation withStatic and conventional DCC approaches was significantly different from zero. These steps were made with the non-parametric Wilcoxon rank-sum test and the SPA test. Results of these tests are presented in Table 4.

Table 4
Wilcoxon rank-sum and SPA tests for daily returns of the U.S., German, British, Brazilian, Hong Kong and Australian marketsin the period from September 2001 to September 2011 obtained by the dynamic portfolio over the static and usual DCC ones

	Variance Difference		Relative Reduction	
In-sample	Wilcoxon	SPA	Wilcoxon	SPA
Unconditional Static	0.0000	0.0000	0.0000	0.0000
Normal DCC	0.0123	0.0099	0.0086	0.0067
Student DCC	0.0214	0.0156	0.0198	0.0076
Out-sample				
Unconditional Static	0.0000	0.0000	0.0000	0.0000
Normal DCC	0.0022	0.0109	0.0000	0.0000
Student DCC	0.0105	0.0010	0.0075	0.0090

* All values are significant at 1% level.

Results in Table 4 corroborate with plots in Figures 6 to 9, emphasizing that the estimated dynamic portfolio variance was significantly smaller than the static approach as both inside and outside the sample. This conclusion is supported by the Wilcoxon rank-sum and SPA testsnull hypothesis rejection. Furthermore, same tests for Normal and Student DCC indicate clear advantage for the dynamic composition obtained with a copula based model.

4. Concluding Remarks

With a copula-based GARCH model, we estimated the dynamic covariance matrix of the studied markets. The estimated covariance matrix resultspoints out the need of risk management able to fit the dynamic behavior of the volatility, as wellas the dependence of international relationships.In this sense we constructed a portfolio composed by the studied markets, minimizing risk at each instant, with base on the estimated dynamic covariance matrix.

The results indicated a large amount of variation in weights, associated with turbulent periods. After the terrorist attacks of 2001, markets with high liquidity and less risk,i.e., Australian and British, obtained more participation. After this period, S&P500 recovered its participation, but had a fall during the sub-prime crisis. In this crisis, DAX and Ibovespa were used for risk diversification. The Brazilian market kept its participation. The German market lost participation due to the Greek crisis, which spread to all Europe. HSI, which had some participation during the whole period, reached its peak during the Greek crisis, emphasizing the important role of emerging countries in the actual diversification context.

Subsequently, we compared the constructed dynamic portfolio with the static approach regarding to risk reduction inside and outside sample. The results indicated that there was significant risk reduction, especially during periods of turbulence. In these occasions, the dynamic portfolio overcame the static one in more than 30%. Moreover, on out-sample, the dynamic approach has the same superiority. Finally, to give robustness for the results found, we replicate the same comparison to Normal and Student DCC usual models. Even in this case, the dynamic portfolio constructed with Copula-DCC-GARCH was able to reduce risksignificantly if compared with to the benchmark composition.

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