

Efficiency and productivity of Brazilian banks: A new approach based on two-stage network DEA

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Abstract This article creates a conceptual model, called a network system, to represent the Brazilian banking production system, based on its internal operational processes. The first, called the intermediation process, measures a bank's efficiency in extending loans from its available resources. The second, called the revenue process, measures a bank's efficiency in earning profit, mainly from loans granted. We adopt a two-stage DEA model. In the first stage, a relational network DEA model measures both the network system efficiency scores and internal processes. This technique, associated with the Malmquist Index, assesses performance changes over time. In the second stage, these efficiency scores are considered dependent variables, such that Tobit models can determine how the Brazilian credit market's characteristics can explain the network system and internal processes' efficiency. Results show not only a growing trend toward greater efficiency in the revenue process, but also an increase in productivity accompanied by a decline in the intermediation process technology. Given the high banking spreads in Brazil, these results indicate deterioration in the quality of the credit portfolio and the prospect of future insolvency. We discuss implications of this scenario for domestic banks and collateral policy.

Keywords: Two-stage DEA; Relational model; Malmquist index; Tobit; Brazilian banks.

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1. Introduction

This paper studies the performance of the Brazilian banking industry, applying the data envelopment analysis (DEA) model to general multi-stage systems (GMSS) (Kao, 2014), with panel data from the largest commercial banks. Unlike other DEA models, GMSS permits simultaneous computation of the efficiency of a production system and its corresponding internal processes without prior identification of the weights of aggregated products and inputs, but rather by the conventional definition of a ratio between aggregated output and input. Furthermore, GMSS permits relaxation of the assumption that internal processes do not consume exogenous inputs or do not produce exogenous outputs. These properties are particularly useful in studying a complex production system, comprised of a network of internal processes, as financial institutions are classified.

The credit market is strategic for the economic growth of emerging countries, insofar as financial intermediation activity boosts the flow of scarce resources to the productive sector (Levine et al., 2000; McCaig and Stengos, 2005; Boháček, 2007). This importance is attenuated in emerging countries, which generally do not have a developed capital market, giving the banks a worrying responsibility for increased liquidity (Tecles and Tabak, 2010). In contrast, the impact of intermediation activities on economic growth depends on the degree of financial sector development (Eggoh and Villieu, 2014). Because of this, research on the performance of the banking sector is essential (Berger and Humphrey, 1997; Paradi and Zhu, 2013; Liu et al., 2013b; Shyu et al., 2015).

Although a considerable amount of research applies DEA models to analyze the performance of the banking industry (Liu et al., 2013b; Guo et al., 2017; da Silva Fernandes et al., 2018; Li et al., 2018), few have focused on Brazil (Wanke and Barros, 2014). Brazil has the largest and most complex financial system in Latin America, and its banking industry has experienced an accelerated process of deep restructuring since the first half of the 1990s (Tecles and Tabak, 2010).

This restructuring originated with changes in the Brazilian regulatory framework. The purpose was to adjust the credit market to the new macroeconomic context of inflation stabilization, combined with a fiscal adjustment, and additionally to overcome the critical situation of the financial sector. The sector was characterized by highly leveraged institutions, and credit portfolios lacking strict supervision of risk (Barros and Wanke, 2014).

Therefore, regulatory reforms focused on encouraging privatization, accompanied by the sector opening to greater participation by foreign banks.

What suffered were efforts to decrease the presence of public banks, hitherto focused on financing redistributive policy and fiscal policy. The whole process involved a large number of merger and acquisitions (M&A) (Wanke et al., 2017). The plan was that the banking system's recovery would favor development and minimize the country's systemic financial instability.

Almost twenty years since the start of the restructuring process, evidence indicates that the Brazilian banking industry is close to its consolidation. However, the intense use of M&A as a privatization and economic liberalization mechanism has resulted in an increase in the degree of banking concentration (Belaisch, 2003), characterized by an abrupt decrease in the number of institutions and the emergence of large banks and financial holding companies. Consequently, competition is decreasing (Yildirim and Philippatos, 2007).

Based on the foregoing, the motivation of this article can be described as follows. First, we evaluate at which efficiency levels the Brazilian credit market has been consolidating recently. Although similar studies exist, analysis of the Brazilian banking system, comprised of a financial intermediation process and a revenue process connected in a network, has not yet been explored. Second, using the Malmquist index, we carry out an intertemporal analysis of change in productivity and technology for each semester in a four-year period. This application extends the literature by combining the concepts of the Malmquist Index to network DEA-GMSS, measuring changes in the banking system and their internal processes in an integrated manner. Finally, we examine how the financial environment impacted the performance achieved by Brazilian banks through truncated Tobit models. This method is necessary to better understand the impact of the bank restructuring.

The remaining paper is organized as follows. Section 2 explains the conceptual framework of Brazil's banking industry. The purpose of Section 3 is to explain the proposed conceptual model used to analyze banking institutions. Section 4 intends to clarify the applied methodology. Section 5 details data analysis. Sections 6 and 7 present empirical results and discuss their policy implications, respectively. Section 8 concludes.

2. Overview of Brazilian banking industry

Brazil's credit market was largely made up of public banks until the early 1990s. It represented 50% of total bank assets and 55% of total credits granted. Most loans were characterized as long-term. Based on this, some authors argue that public banks were important in funding investment, and thus to the country's economic growth (Ness, 2000; Beck et al., 2005).

However, in that time, banks were characterized by high administrative

expenses, personnel expenses, and operational expenses. They depended chiefly on inflationary transfers to survive in the market. Brazil experienced double-digit inflation levels starting in the late 1950s, reaching a maximum level of about 2,708.17% per year in 1993. Thus, the inflationary transfers to the Brazilian banking system were about 3.4% of gross domestic product (GDP) in the corresponding period (Franco, 2000; Nakane and Weintraub, 2005).

The critical situation of the banking sector was further intensified by macroeconomic changes introduced by the inflation stabilization plan, called the Real Plan. Inflation effectively converged at low levels. Thus, one of the main sources of public banks' income, and of the banking industry in general, was minimized. Inflationary transfers decreased to 1.8% of GDP in 1994, reaching 0.03% of GDP in 1995. Specifically in relation to banks, inflationary transfers followed the same movement, decreasing from an average of 35% of total revenues in 1993 to close to zero in 1995 (Franco, 2000; Nakane and Weintraub, 2005).

In addition, the Real Plan planned to maintain the basic interest rates at high levels. The purpose was to signal to the market the economic policy's commitment to stabilizing inflation. As a result, other real interest rates in the economy also increased, including Brazil's banking spreads Beck et al. (2005). Aiming to replace inflationary transfers, the banking industry greatly increased its financial intermediation activities. However, it did so without conducting a rigorous study of risk with borrowers, and without strict supervision by the Central Bank of Brazil (Ness, 2000; Nakane and Weintraub, 2005).

Accordingly, to contain the increase in financial intermediation and its corresponding impact on stimulating aggregate demand, the Central Bank increased the percentage of reserve requirements. In 1995, the rate of reserve requirement of new demand deposits increased from 40% to 100%, while the rate of reserve requirements for time deposits was set at 20%. As a result, many companies went bankrupt, either due to decreased demand, or from increased debt from the increased bank spreads. In addition, the banks' credit portfolio deteriorated significantly, with a simultaneous decrease in the financial sector's liquidity (Ness, 2000; Nakane and Weintraub, 2005).

Given the catastrophic situation in its banking industry, with high expenses and low revenues, rising defaults and decreased liquidity, Brazil's Central Bank implemented a broad and aggressive normative project to restructure the financial sector. Strictly speaking, this was divided in two phases (Tecles and Tabak, 2010; Staub et al., 2010). In the first phase, based on a 1987 law, the Temporary Special Administration Regime (RAET), the Cen-

tral Bank assumed the responsibility to extra-judicially liquidate banks or to place them under special temporary management in the event of problematic liabilities. It simultaneously provided funds to insure customers' deposits (Barros and Wanke, 2014; Ness, 2000). In the second phase, two regulatory measures were drafted: the Program of Incentives for the Restructuring and Strengthening of the National Financial System (PROER) in 1995, and the Program of Incentives for the Restructuring of the State Public Financial System (PROES) in 1996.

After these acts, the public sector's share in the Brazilian credit market has been minimized. Some small public banks were liquidated. In addition, the tool of M&A was used widely. Tax incentives were created and credit lines subsidized for both domestic and foreign banks to acquire public banks (Barros and Wanke, 2014; Ness, 2000). The Central Bank also passed Resolution 2,303 in 1996, allowing banks to increase revenues by charging fees for services.

The current situation of the Brazilian banking industry, since the regulatory changes implemented by the Central Bank, points to a process of bank consolidation, since no significant changes are observed in the number of banks and/or bank branches. Moreover, the sector composition related to bank ownership has shown some stabilization (Faria Júnior et al., 2006; Almeida and Jayme Jr., 2008).

In contrast, several studies show that, due to the mechanism of M&A, consolidation of the banking system occurred at the cost of high levels of bank concentration, with negative impacts on the competitive structure of the credit market (Barros and Wanke, 2014; Yildirim and Philippatos, 2007). Furthermore, according to Olivero et al. (2011), current evidence suggests that the combination of banking concentration and an uncompetitive credit market weakens the financial intermediation activities in emerging countries, including Brazil. This hurts these countries' economic growth.

More recent studies assess the level of efficiency and solvency of the Brazilian credit market with respect to contextual variables (Wanke et al., 2015). They see levels of efficiency of financial institutions and their relationship with corporate governance practices and times of crisis (Freitas et al., 2018). They make international comparisons based on the experience of countries with similar cultures, such as Mozambique and Angola (Barros et al., 2018), and countries with similar socioeconomic conditions, such as the BRICS (Wanke et al., 2018).

In general, results of these recent analyses indicate that the efficiency levels of the Brazilian credit market are associated with public banks, the corporate practices adopted, the levels of gross savings, income inequality, and

inflation in the country.

3. Proposed model

Financial intermediation refers to the process of banks organizing transfers of funds from surplus agents, savers, to deficit agents, investors and consumers. This is an ideal activity on which to evaluate bank performance, since these financial transactions can positively impact national economies. In addition, these activities are of proven importance to the banks themselves, representing up to two-thirds of total costs (Berger and Humphrey, 1997).

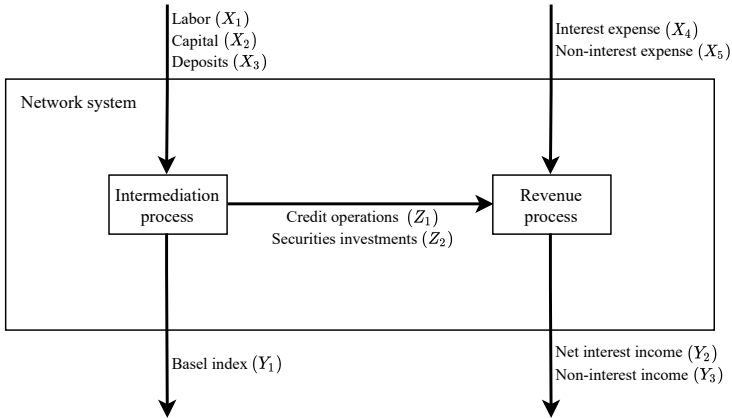
Seen from another angle, banks' solvency is closely related to the profitability of their operations. In this sense, granting problematic loans increases the probability of failure of these institutions (Seiford and Zhu, 1999; Luo, 2003; Avkiran, 2011). Along with showing this possible effect, analysis of the revenue process overlaps other aspects of financial performance, such as the cost perspective. This analysis provides substantial information on the competitive structure of credit markets, see Berger and Humphrey (1997).

Figure 1 illustrates the production system we propose to represent banks. In this system, two internal processes make up the network structure. In the first case, we selected variables to assess a bank's efficiency in its lending activities, i.e. the intermediation process. In the second case, the variables must provide evidence of the bank's efficiency in producing income, also primarily from intermediation activities it performs, i.e. the revenue process.

As Figure 1 shows, the evaluation designed for the intermediation process measures bank efficiency in performing activities of financial intermediation, represented by outputs credit operations (Z_1) and securities investments (Z_2); using its available resources, represented by the inputs labor (X_1), capital (X_2), and deposits (X_3). In addition, we add a third output, the Basel index (Y_1), because a bank is not considered efficient when it increases the volume of loans without ensuring minimally adequate equity conditions. This design takes into account the fundamentals of Berger and Humphrey (1997), Sturm and Williams (2010) and Faria Júnior et al. (2006).

More specifically, the outputs credit operations (Z_1) and securities investments (Z_2) represent the total loans granted and total investments in stocks and bonds held by a bank, respectively. The output Basel index (Y_1) corresponds to the net equity requirements imposed by the Basel Agreement. On the other side, the labor input (X_1) is the sum of personnel and other administrative expenses, capital (X_2) is the sum of fixed components of assets, and deposits (X_3) equal the total deposits available at the bank.

Figure 1
Bank system composed by intermediation and revenue processes



Continuing with Figure 1, we evaluate the revenue process with measures of a bank’s efficiency in making profit, represented by outputs net interest income (Y_2) and non-interest income (Y_3) after incurring expenses represented by inputs interest expense (X_4) and non-interest expense (X_5). We consider the fundamental ideas of Berger and Humphrey (1997), Seiford and Zhu (1999), Luo (2003), Sturm and Williams (2010), Faria Júnior et al. (2006), and Avkiran (2011) in this design.

The outputs net interest income (Y_2) and non-interest income (Y_3) are the total income from financial intermediation and the sum of service incomes and other operating incomes, respectively. The inputs interest expense (X_4) and non-interest expense (X_5) correspond to the total expense of financial intermediation and the sum of taxes and other operating expenses, respectively.

In the proposed banking production system, we see that the revenue and intermediation processes are connected in a series network structure, from the endogenous variables credit operations (Z_1) and securities investments (Z_2). The idea is that efficiency results from both the volume of intermediation activity realized and the expenses incurred. The other variables are exogenous, forming a parallel network structure. This allows a full system scan. This complex network structure, wherein the production system variables are linked in series and in parallel simultaneously, supports the application of GMSS.

4. Methodological fundamentals

4.1 DEA-GMSS model background

Many studies analyze the performance of the banking industry. Most of these studies use DEA models (Paradi and Zhu, 2013; Li et al., 2018). According to Liu et al. (2013a), bank efficiency analysis using DEA models corresponds to a search field trend, since much of this method's development has been tailored to the specific needs of this type of study.

The DEA model employs a non-parametric technique to assess the relative efficiency of decision making units (DMUs). The DMUs perform the same operations, and therefore use many similar inputs to generate outputs that are also similar (Banker et al., 2011; liang Yang et al., 2018). Based on concepts of productivity and efficiency introduced by Farrell (1957), the first DEA model was developed by Charnes et al. (1978).

Liu et al. (2013a) and Lampe and Hilgers (2015) explain that from that initial DEA model, a number of other, more sophisticated, DEA models have been developed. These newer models meet different objectives of relative efficiency analysis. Among the various types of models currently available, one relevant approach involves examining a DMU's performance from the perspective of network structures, i.e. network DEA models.

Fukuyama and Mirdehghan (2012) define network structure as a system that processes inputs into outputs. It consists of the internal production processes linked together, whereby the composition of these processes produces the final output of the DMU. Rho and An (2007), Ebrahimnejad et al. (2014), and liang Yang et al. (2018) argue that developing network DEA models is necessary, since traditional DEA models, called "black box" models, may measure efficiency inaccurately. Black box measures do not incorporate the performance of internal processes of DMUs. Thus, they may consider a DMU to be efficient, despite it having inefficient internal processes.

As Kao (2014), Liu et al. (2013a), Lampe and Hilgers (2015), and Mariz et al. (2018) explain, early work on network DEA models is credited to Färe (1991), Färe and Whittaker (1995), and Färe and Grosskopf (1996). However, it was only Kao and Hwang (2008) who began to compute system efficiencies and their respective internal processes simultaneously in the same model. Since then, a number of researchers have incorporated internal processes to compute DMU performance in various ways, as shown by Yu and Lin (2008), Avkiran (2009), Tone and Tsutsui (2009), Kao and Hwang (2008), Kao (2009a,b), and Chen et al. (2009).

According to Mariz et al. (2018), of the different methods to assess production systems' performance using network DEA, Kao (2014) proposes an

improved model, called general multi-stage systems (GMSS). GMSS is based on the decomposition of multiple processes in multiple system stages. From that decomposition, the system is transformed into a network structure. Its efficiency is expressed as a function of the efficiency of each process simultaneously. The approach of decomposition of system efficiency has important properties: 1. no limitation on the type of network structure, allowing the whole complex system to be modeled; 2. no need to pre-set the weights of the internal processes to compose system efficiency, reducing the risk of inaccuracy in estimating efficiency; 3. possibility of relaxing the constraints of non-use and/or no production, of exogenous variables in the system's internal processes.

According to Kao (2014), the system efficiency estimation (θ_k), of a DMU k , from a total of $n \in N$ DMUs, $j = 1, 2, \dots, n \mid k \in N$, given q internal processes, $p = 1, 2, \dots, q$, is defined in the GMSS as

$$\max \theta_k = \sum_{r=1}^s u_r Y_{rk}$$

subject to

$$\left\{ \begin{array}{l} \sum_{i=1}^m v_i X_{ik} = 1 \\ \sum_{r=1}^s u_r Y_{rj} - \sum_{i=1}^m v_i X_{ij} \leq 0, \\ \sum_{r \in O^{(p)}} u_r Y_{rj}^{(p)} + \sum_{f \in M^{(p)}} w_f Z_{fj}^{(p)} \leq \sum_{i \in I^{(p)}} v_i X_{ij}^{(p)} + \sum_{f \in M^{(p-1)}} w_f Z_{fj}^{(p-1)}, \\ u_r, v_i, w_f \geq \varepsilon, \end{array} \right.$$

for all $j = 1, \dots, n; p = 2, \dots, q - 1; r = 1, \dots, s; i = 1, \dots, m; \text{ and } f = 1, \dots, g$.

Still in reference to the GMSS model proposed by Kao (2014), to denote the set (u_r^*, v_i^*, w_f^*) as an optimal solution, and s_k^* and $s_k^{(p)*}$ as the slacks added to the constraints of the system and of the process p , respectively, the system efficiency (θ_k) and its decomposition to their internal processes ($\theta_k^{(p)}$), are defined as

$$\theta_k = \frac{\sum_{r=1}^s u_r^* Y_{rk}}{\sum_{i=1}^m v_i^* X_{ik}} = \sum_{r=1}^s u_r^* Y_{rk} = 1 - s_k^*$$

and

$$\theta_k^{(p)} = \frac{\sum_{r \in O^{(p)}} u_r^* Y_{rk}^{(p)} + \sum_{f \in M^{(p)}} w_f^* Z_{fk}^{(p)}}{\sum_{i \in I^{(p)}} v_i^* X_{ik}^{(p)} + \sum_{f \in M^{(p-1)}} w_f^* Z_{fk}^{(p-1)}}$$

for $p = 2, \dots, q - 1$.

In this way, the system's slack is defined as the sum of the slacks computed for each internal process $s_k^* = \sum_{p=1}^q s_k^{(p)*}$. This is the main property of GMSS. A corollary is that the system will only be efficient if all internal processes are also efficient, thus eliminating the errors and efficiency estimation inaccuracies of the black box DEA models.

Using these definitions, this paper applies the DEA-GMSS model to estimate the efficiency of banks that operated in the Brazilian credit market in each semester during a four-year period. This application allows an estimation of a bank's production system efficiency score, and of its intermediation and revenue processes. It provides a fairly complete picture of bank performance in each time period.

4.2 Decomposition of the Malmquist productivity index for GMSS

The Malmquist productivity index (MPI), originally proposed by [Malmquist \(1953\)](#) and [Caves et al. \(1982\)](#), aims to measure the change in the total factor productivity incurred by a DMU in adjacent periods of time. The MPI can be estimated with DEA models. This index is used in different approaches to study the performance of the financial sector; see [Berg et al. \(1992\)](#), [Berg et al. \(1993\)](#), [Portela and Thanassoulis \(2010\)](#), and [Wijesiri and Meoli \(2015\)](#). Furthermore, due to its precision, the MPI is one of the most frequently used indexes to estimate performance variations ([Pastor et al., 2011](#)).

From the proposal of [Färe et al. \(1994\)](#), the change in the total productivity factor can be computed through its disaggregation into two mutually exclusive and exhaustive components: change in technical efficiency and change in technology. These components are defined as representations of a DMU's recovery and innovation, respectively. Therefore, under the DEA, MPI equals the change in technical efficiency weighted by the change in technology. The latter is interpreted as a change in the efficiency frontier, which is calculated from a Fischer index.

[Färe et al. \(1994\)](#) present the adjacent periods as $t, t' = 1, 2 | t \neq t'; t, t' \in T$. Also, $(\theta_k^T | \phi^T(\mathbf{x}, \mathbf{y}))$ is the efficiency achieved by the DMU k considering their productivity in the period T designed in efficiency frontier observed in the period T , given a set of $n \in N$ DMUs, $j = 1, 2, \dots, n | k \in N$. Thus, the MPI is calculated as

$$MPI_k = \frac{(\theta_k^2 | \phi^2(\mathbf{x}, \mathbf{y}))}{(\theta_k^1 | \phi^1(\mathbf{x}, \mathbf{y}))} \left(\frac{(\theta_k^2 | \phi^1(\mathbf{x}, \mathbf{y}))}{(\theta_k^2 | \phi^2(\mathbf{x}, \mathbf{y}))} \frac{(\theta_k^1 | \phi^1(\mathbf{x}, \mathbf{y}))}{(\theta_k^1 | \phi^2(\mathbf{x}, \mathbf{y}))} \right)^{\frac{1}{2}}$$

The first term of the MPI corresponds to the estimated change in technical

efficiency, while the second term refers to the estimated change in technology, considering the fixed reference base in the period. In addition, each efficiency $(\theta_k^{\bar{T}} | \phi^{\bar{T}}(\mathbf{x}, \mathbf{y}))$ achieved by a DMU k , from a combination of its productivity in period \bar{T} , $\forall \bar{T} \in T$, projected on the frontier of period \bar{T}' , $\forall \bar{T}' \in T$ is obtained recursively through the DEA model, as follows:

$$\max \left(\theta_k^{\bar{T}} | \phi^{\bar{T}}(\mathbf{x}, \mathbf{y}) \right) = \sum_{r=1}^s u_r Y_{rk}^{\bar{T}}$$

subject to

$$\begin{cases} \sum_{i=1}^m v_i Y_{ik}^{\bar{T}} \\ \sum_{r=1}^s u_r Y_{rk}^{\bar{T}} \leq \sum_{i=1}^m v_i Y_{ik}^{\bar{T}} \\ \sum_{r=1}^s u_r Y_{rj}^{\bar{T}'} \leq \sum_{i=1}^m v_i Y_{ij}^{\bar{T}'} \\ u_r, v_i \geq \varepsilon \end{cases}$$

for all $r = 1, \dots, s$; $i = 1, \dots, m$; and $j \neq k$.

However, as already explained, the using black box DEA models can generate efficiency inference errors, since efficiency can be imputed to the system of a DMU operating with inefficient internal processes. Thus the subsequent measurement of change in productivity and change in technology also incur error, to the extent that the inaccurate efficiency estimation of the DEA model is propagated in the calculation of the MPI (Kao, 2017).

Some recent investigators seek to overcome this limitation; see Kao and Liu (2014) and Kao (2017). In this study, we propose an association between the DEA-GMSS model of Kao (2014) and MPI (Färe et al., 1994). On the one hand, adopting the DEA-GMSS model is justified to eliminate inaccuracy in estimating efficiency, and to simultaneously incorporate information about internal processes. DEA-GMSS can be applied to any complex network structure, broadening the scope of our proposal to other research. Adopting MPI is also justified, because it is possible to decompose the performance change in its components of productivity and technology. This information is crucial, to provide a complete view of the banking sector's degree of development, a factor that impacts countries' economic growth.

Based on the foregoing, our proposal is to calculate the MPI for the system, and in an integrated manner, calculate the MPI for its internal processes in a relational network DEA model. Thus, defining MPI_k as the Malmquist

index calculated for the system, and $MPI_k^{(p)}$ as the Malmquist index calculated for each process $p = 1, 2, \dots, q$, we have

$$MPI_k = \frac{(\theta_k^2 | \phi^2(\mathbf{x}, \mathbf{y}))}{(\theta_k^1 | \phi^1(\mathbf{x}, \mathbf{y}))} \left(\frac{(\theta_k^2 | \phi^1(\mathbf{x}, \mathbf{y}))}{(\theta_k^2 | \phi^2(\mathbf{x}, \mathbf{y}))} \frac{(\theta_k^1 | \phi^1(\mathbf{x}, \mathbf{y}))}{(\theta_k^1 | \phi^2(\mathbf{x}, \mathbf{y}))} \right)^{\frac{1}{2}}$$

and

$$MPI_k^{(p)} = \frac{(\theta_k^{2,(p)} | \phi^{2,(p)}(\mathbf{x}, \mathbf{y}))}{(\theta_k^{1,(p)} | \phi^{1,(p)}(\mathbf{x}, \mathbf{y}))} \left(\frac{(\theta_k^{2,(p)} | \phi^{1,(p)}(\mathbf{x}, \mathbf{y}))}{(\theta_k^{2,(p)} | \phi^{2,(p)}(\mathbf{x}, \mathbf{y}))} \frac{(\theta_k^{1,(p)} | \phi^{1,(p)}(\mathbf{x}, \mathbf{y}))}{(\theta_k^{1,(p)} | \phi^{2,(p)}(\mathbf{x}, \mathbf{y}))} \right)^{\frac{1}{2}}$$

The system efficiencies $(\theta_k^{\bar{T}} | \phi_k^{\bar{T}}(\mathbf{x}, \mathbf{y}))$, arising from productivity combinations projected in the frontier, when considering adjacent periods, are initially calculated by the DEA-GMSS model:

$$\max \left(\theta_k^{\bar{T}} | \phi_k^{\bar{T}}(\mathbf{x}, \mathbf{y}) \right) = \sum_{r=1}^s u_r Y_{rk}^{\bar{T}}$$

subject to

$$\left\{ \begin{array}{l} \sum_{i=1}^m v_i X_{ik}^{\bar{T}} = 1 \\ \sum_{r=1}^s u_r Y_{rk}^{\bar{T}} \leq \sum_{i=1}^m v_i X_{ik}^{\bar{T}} \\ \sum_{r=1}^s u_r Y_{rj}^{\bar{T}'} \leq \sum_{i=1}^m v_i X_{ij}^{\bar{T}'} \\ \sum_{r \in O^{(p)}} u_r Y_{rk}^{\bar{T},(p)} + \sum_{f \in M^{(p)}} w_f Z_{fk}^{\bar{T},(p)} \leq \sum_{i \in I^{(p)}} v_i X_{ik}^{\bar{T},(p)} + \sum_{f \in M^{(p-1)}} w_f Z_{fk}^{\bar{T},(p-1)} \\ u_r, v_i, w_f \geq \varepsilon, \end{array} \right.$$

for all $p = 2, \dots, q - 1$; $r = 1, \dots, s$; $i = 1, \dots, m$; $f = 1, \dots, g$; and $j \neq k$.

For each system efficiency, we decompose efficiencies for q internal processes, where $p = 1, 2, \dots, q$. If the set $(\tilde{u}_r^*, \tilde{v}_i^*, \tilde{w}_f^*)$ is the optimal solution for a given combination of productivity and efficiency frontier, and \tilde{s}_k^* and $\tilde{s}_k^{(p)*}$ are the slacks added to the system and process p constraints, respectively, the system efficiency $(\theta_k^{\bar{T}} | \phi_k^{\bar{T}}(\mathbf{x}, \mathbf{y}))$ and its decomposition to their internal processes $(\theta_k^{\bar{T},(p)} | \phi_k^{\bar{T},(p)}(\mathbf{x}, \mathbf{y}))$ are defined as

$$(\theta_k^{\bar{T}} | \phi_k^{\bar{T}}(\mathbf{x}, \mathbf{y})) = \frac{\sum_{r=1}^s \tilde{u}_r^* Y_{rk}^{\bar{T}}}{\sum_{i=1}^m \tilde{v}_i^* X_{ik}^{\bar{T}}} = \sum_{r=1}^s \tilde{u}_r^* Y_{rk}^{\bar{T}} = 1 - \tilde{s}_k^*$$

and

$$(\theta_k^{\bar{T},(p)} | \phi^{\bar{T}',(p)}(\mathbf{x}, \mathbf{y})) = \frac{\sum_{r \in O(p)} \tilde{u}_r^* Y_{rk}^{\bar{T},(p)} + \sum_{f \in M(p)} \tilde{w}_f^* Z_{fk}^{\bar{T},(p)}}{\sum_{i \in I(p)} \tilde{v}_i^* X_{ik}^{\bar{T},(p)} + \sum_{f \in M(p-1)} \tilde{w}_f^* Z_{fk}^{\bar{T},(p-1)}}$$

for $p = 2, \dots, q - 1$.

Thus, it becomes possible to measure the change in productivity and technology, both for the system network and for each of the internal processes.

We use the proposed integration of the DEA-GMSS model and MPI to measure the change in productivity and technology experienced by each DMU between semesters for four years. In addition, we measure the respective changes between the extremes of the period. This application is important for allowing the changes to be estimated for the production system of the bank, and for intermediation and revenue processes. Furthermore, the analysis of each semester and between the extreme periods has the potential to provide important information about a possible influence of the activity cycle on banks' performance (Berg et al., 1993).

4.3 Two-stage analysis

The second-stage analysis corresponds to an evaluation of the effect of contextual or environmental variables on production efficiency through a two-stage process. In the first stage, the DEA model provides the DMU's efficiency scores. In the second phase, these scores are correlated with some contextual variables relevant to the study (Liu et al., 2013a).

The second-stage analyses are essential to broaden our knowledge of the transformation process. They thus have the potential to improve the quality of management strategies defined by market operators and/or to guide policy makers towards best practices (Johnson and Kuosmanen, 2012). This perspective takes on special importance for studies aimed at analyzing credit market performance, as contextual variables significantly influence the efficiencies achieved by banks (Berger and Humphrey, 1997).

A wide range of statistical procedures exist for second-stage analysis; see Simar and Wilson (2007) and Banker and Natarajan (2008). Among these, the Tobit regression truncated at 1 has been widely used (Turner et al., 2004; Hoff, 2007). Simar and Wilson (2007) describe limitations of the truncated Tobit regression due to the possibility of inconsistent estimators. The second-stage variables may be correlated with the efficiency scores measured by the DEA model in the first stage. However, there is no guarantee that other estimates, such as the bootstrap, are more reliable, since they require a series of initial assumptions about the samples of variables (Çelen, 2013).

Moreover, [Banker and Natarajan \(2008\)](#) show that the truncated Tobit model presents satisfactory results in predicting the levels of efficiency. In addition, they sidestep sampling problems by selecting context variables to reduce their correlation with the efficiency scores. Moreover, as we see in the next section, the authors use a significant representation of the banking industry in Brazil.

The relative efficiency scores of a DMU's sample, calculated in the first stage via DEA programming, clearly correspond to a sample with strictures, since the respective scores can take any real value between 0 and 1 ([Johnson and Kuosmanen, 2012](#)). Thus, to set this efficiency score of DMU k , denoted by θ_{kh} , in terms of h contextual variables δ_{kh} , with $l = 1, \dots, h$, the latent variable $\hat{\theta}_{kh}$, which is dependent on the same contextual variables, is defined by

$$\hat{\theta}_{kh} = \delta'_{kh}\beta + \varepsilon_{kh}$$

where δ_{kh} is a vector ($h \times 1$) of contextual variables; β is a vector ($h \times 1$) of parameters to be estimated; and $\varepsilon_{kh} \approx N(0; \sigma^2)$.

Using the aforementioned latent variable, we may obtain the observed efficiency score from the stricture, below 0 and above 1, such that

$$\theta_{kh} = \begin{cases} 0, & \text{if } \hat{\theta}_{kh} \leq 0 \\ \hat{\theta}_{kh}, & \text{if } 0 < \hat{\theta}_{kh} < 1 \\ 1, & \text{if } \hat{\theta}_{kh} \geq 1. \end{cases}$$

Considering this structuring of the truncated Tobit model, the maximum likelihood method makes it possible to estimate the values of β parameter vector. The non-application of this formulation structure for Tobit regression by maximum likelihood method ultimately can cause problems of estimation inconsistency ([Greene, 2018](#)).

Taking this systematization into account, we perform a second-stage analysis from the integration of efficiency scores measured by DEA-GMSS, in the first stage, and the Tobit model truncated to 1, in the second stage. We define the contextual variables by the domains ownership, size, and type of institution. With this application, we aim to capture how the current structure of the Brazilian credit market, a consequence of the regulatory restructuring implemented by the Central Bank, impacts the performance of banks.

5. Data analysis

We collected data from sectorial technical reports organized, audited, and provided quarterly by the Central Bank of Brazil. More specifically, we used

the report named Selected Information On Supervised Institutions.¹ This report consists of the financial statements of financial holding companies and of banking institutions that are not part of a financial holding company, but that are in normal operation. This proves to be the best database available for analysis of the Brazilian banking sector.

We selected 30 banks operating in the Brazilian credit market. The criterion for selecting banks was the highest value of total assets, considering values from the year 2015. This sample accounts for 92.60% of total bank assets and 92.03% of total bank deposits. We consider the year 2015 to be representative of the recent conditions of the banking industry in Brazil.

We avoided analyzing a larger number of banks to decrease the sample dispersion. Given the level of concentration of the Brazilian banking sector, increasing the number of banks would bring only a small marginal contribution in total bank assets and total bank deposits. Our procedure aims to minimize the effects of scale heteroskedasticity in measured technical efficiency levels (Berger and Humphrey, 1997; Yuengert, 1993).

Regarding the defining criterion of the time interval, we sought to distance the performance analysis from periods near the subprime crisis of 2008. We decided this because of evidence that, in the short term, some financial instability is expected due to the banks adjusting to a significant crisis. This instability of adaptation can be confused with technical inefficiency in banking performance analysis (Siriopoulos and Tziogkidis, 2010).

Table 1 presents descriptive statistics for each variable of our proposed banking system model. With the exception of the Basel Agreement, because it is a financial ratio, the variables are expressed in terms of 1,000,000.00 Brazilian Real Rates (BRL).

Regarding MPI, in order to project values on the efficiency frontier built in another period, we deflated the monetary values of the DMU variables under analysis to the respective period of the efficiency frontier. To do this, we relied on the evolution of the general price index (IGP-DI). We implemented this mechanism to minimize the effects of relative prices changes in bank productivity and technology analysis.

Regarding the structure of the second-stage analysis, the three areas: ownership, size, and type of institution, are qualitative. Ownership has three categories: public, domestic, and foreign. For each category, we created a dummy variable by assigning the value 1 if true and 0 otherwise. Size has two categories: large and small. We created another dummy variable with the value 1 for large and 0 for small. The type of institution has two categories: financial holding companies and banking institutions that are not part of a financial

¹This report is available at <https://www3.bcb.gov.br/informes/relatorios>

Table 1
Descriptive statistics for bank system variables

year	sem.	X_1	X_2	X_3	X_4	X_5	Z_1	Z_2	Y_1	Y_2	Y_3	
1	1 st	mean	53,269	2,421	11,727	7,632	1,487	86,258	52,326	15,85	9,780	2,036
		sd	108,478	4,343	24,811	12,691	2,725	148,273	86,204	2,45	16,543	3,599
	2 nd	mean	54,462	2,656	11,661	6,532	1,683	93,320	61,554	15,36	9,329	1,913
		sd	112,064	4,667	25,050	11,169	3,230	163,128	105,451	2,24	15,878	3,505
2	1 st	mean	55,450	2,632	10,901	7,165	1,585	101,636	64,357	16,08	9,480	2,121
		sd	114,981	4,723	23,563	11,980	2,948	178,075	106,264	2,65	15,772	3,885
	2 nd	mean	58,264	2,839	10,798	7,905	2,040	109,587	61,407	15,42	10,344	2,614
		sd	120,606	5,033	23,672	13,315	3,864	195,332	103,960	2,41	17,315	4,708
3	1 st	mean	59,240	2,795	10,427	7,709	1,938	112,076	66,192	15,27	10,756	2,666
		sd	122,322	5,020	22,693	13,639	3,657	202,817	113,368	1,98	18,856	4,937
	2 nd	mean	60,735	3,043	10,263	11,101	2,086	123,375	75,757	16,61	12,957	2,826
		sd	125,234	5,382	21,918	19,070	4,159	220,677	127,674	2,61	22,161	5,370
4	1 st	mean	59,666	2,975	11,034	12,161	2,353	128,186	79,193	15,84	14,099	3,236
		sd	122,533	5,338	23,896	20,750	4,792	228,549	131,255	2,04	23,962	6,450
	2 nd	mean	64,142	3,312	10,864	16,041	2,531	137,869	80,217	16,11	16,153	3,640
		sd	128,825	5,885	23,310	27,211	4,954	242,792	138,323	2,56	27,460	7,109
total	mean	58,154	2,834	10,959	9,531	1,963	111,538	67,625	15,82	11,612	2,631	
	sd	117,840	5,001	23,290	17,086	3,826	197,583	113,874	2,38	19,995	5,054	

sem.: semester. sd: standard deviation.

Table 2
Descriptive statistics for efficiency estimates

year	sem.	network system		intermediation process		revenue process	
		mean	sd	mean	sd	mean	sd
1	1 st	0.769	0.208	0.617	0.322	0.711	0.203
	2 nd	0.747	0.214	0.632	0.306	0.715	0.200
2	1 st	0.802	0.207	0.659	0.316	0.751	0.199
	2 nd	0.786	0.145	0.587	0.299	0.784	0.157
3	1 st	0.819	0.168	0.615	0.309	0.804	0.170
	2 nd	0.815	0.163	0.609	0.287	0.786	0.166
4	1 st	0.829	0.162	0.642	0.295	0.805	0.167
	2 nd	0.798	0.171	0.637	0.297	0.777	0.161
total		0.796	0.180	0.625	0.304	0.767	0.1780

sem.: semester. sd: standard deviation.

holding company. Therefore, we created a final dummy variable, assigning the value 1 for holding and 0 otherwise. In order to highlight the marginal contributions of each variable included in the regression, and simultaneously eliminate the situation of perfect multicollinearity, we made estimates without intercept (Greene, 2018).

Over the four-year period, the analysis domains did not change for any bank. Moreover, the ownership information and type of institution are explicitly provided in the Central Bank of Brazil's report. We categorized institution size based on the number of bank branches. We denoted as large those banks with a number of branches higher than three digits, in line with the specificities of the Brazilian credit market, as used by (Barros and Wanke, 2014).

6. Results and discussion

6.1 DEA network system results

Table 2 shows the descriptive statistics of efficiency scores achieved by the Brazilian banking system, and by their internal processes over each period studied. At the end, descriptive statistics calculated for the total data panel are added.

Table 2 shows that the Brazilian credit market operated with high levels of inefficiency throughout all periods studied. The banking system in total panel obtained a mean score of 0.7958 in efficiency. Both internal processes considered in the model propitiated these bad performances, as can be attested by the average efficiency scores of 0.6248 and 0.7667 in intermediation and

Table 3
Mann-Whitney χ^2 tests for
intermediation and revenue processes

year	sem.	<i>p</i> -value
1	1 st	0.157
	2 nd	0.230
2	1 st	0.415
	2 nd	0.008***
3	1 st	0.021**
	2 nd	0.013**
4	1 st	0.040**
	2 nd	0.092*

sem.: semester. *, **, ***: Statistical significance at 10%, 5%, and 1%, respectively. *p*-value adjusted for ties.

revenue processes, respectively. When comparing the average results of efficiency obtained in the system of internal processes, it appears that the banking industry has tended to be more efficient in obtaining revenue from loans at the expense of proper supply credit from available resources.

Table 3 shows the results of the Mann-Whitney test, designed to analyze the null hypothesis that there is no significant difference in performance between the internal processes in each time period. It is understood that the trend of better revenue process performance gradually became significant over the semesters.

6.2 Malmquist productivity index results

Table 4 shows the Malmquist Productivity Index (MPI) results. The results show that the average of financial technologies in Brazil's credit market deteriorated in almost all short-term intervals, thus causing an average technological decrease of 0.66% in the banking system over the entire reporting period. The observed decrease originated in great extent from the deterioration of revenue process, since the average of technologies related to revenue support worsened by 10.99% over an average increase of 3.95% in financial intermediation technologies, also to be considered the total period.

In the opposite direction, the average productivity of the Brazil credit market increased in most short-term intervals, resulting in an average increase of 5.29% in the productivity factor, analyzed over the entire period. This evolution of total bank system productivity was driven by both internal processes, with average growth over the period of 50.96% and 3.34% for intermediation and revenue processes, respectively.

Table 4
Descriptive statistics for productivity and technology changes

technology base	productivity comparison period	network system			intermediation process			revenue process					
		tech. component	mean	sd	MPI	tech. component	mean	sd	MPI	tech. component	mean	sd	MPI
1/1 st	1/2 nd	1.058	0.1113	1.024	0.108	1.039	0.477	1.084	0.457	1.096	0.289	1.096	0.217
1/2 nd	2/1 st	0.990	0.308	1.056	0.240	1.000	0.261	1.072	0.296	0.961	0.355	1.003	0.302
2/1 st	2/2 nd	0.964	0.267	0.980	0.326	1.078	0.312	1.027	0.606	0.905	0.271	0.921	0.277
2/2 nd	3/1 st	1.001	0.077	1.053	0.200	0.985	0.051	1.174	0.694	1.013	0.068	1.051	0.196
3/1 st	3/2 nd	0.950	0.064	0.957	0.162	1.002	0.113	1.026	0.142	0.950	0.062	0.938	0.167
3/2 nd	4/1 st	0.995	0.056	1.019	0.122	1.014	0.112	1.062	0.118	0.994	0.057	1.027	0.134
4/1 st	4/2 nd	1.014	0.070	1.004	0.262	0.998	0.042	1.328	0.959	1.017	0.089	1.006	0.247
1/1 st	4/2 nd	0.993	0.450	1.053	0.352	1.040	0.202	1.510	1.209	0.890	0.119	1.033	0.356

sem.: semester. tech.: technology. sd: standard deviation.



Table 5
Truncated Tobit regression results

covariates	network system		intermediation process		revenue process	
	coef.	p-value	coef.	p-value	coef.	p-value
public	0.912	<0.001***	0.697	<0.001***	0.910	<0.001***
domestic	0.830	<0.001***	0.915	<0.001***	0.805	<0.001***
foreign	0.924	<0.001***	0.910	<0.001***	0.899	<0.001***
large	-0.162	<0.001***	-0.201	0.042**	-0.137	0.003***
holding	-0.106	0.021**	-0.290	0.003***	-0.127	0.005***

coef.: coefficients. *, **, ***: Statistical significance at 10%, 5%, and 1%, respectively.

Regarding changes in the technology component and productivity index between semesters of the same year, no pattern was identified over the four years. This result suggests that the banking cycle has little influence on the performance of banks operating in Brazil.

6.3 Truncated Tobit regression results

The estimated coefficients of contextual financial variables for the network system and for intermediation and revenue processes are shown in Table 5. Based on truncated Tobit regression results, we observe that most of the efficiency of banks in the Brazilian credit market can be explained by management strategies related to bank ownership. Foreign banks showed the best performance, followed by public banks and, finally, by domestic banks, when looking from the perspective of the network system, as can be attested by the marginal contributions of 0.9239, 0.9124 and 0.8298 by the corresponding proxies. This fact is related to the balance between the efficiency of intermediation and revenue processes shown by foreign banks, and not observed in other banks due to a high level of inefficiency, 0.6972 and 0.8046, of intermediation process of public banks and revenue process of domestic banks, respectively.

Table 5 also shows that the coefficients of proxies' largest banks and financial holding companies were significantly negative for network system efficiency, -0.1617 and -0.1062, respectively. We see a similar negative effect in banking processes. Thus, if the bank is large, and if it belongs to a financial holding company, its efficiency is impaired.

7. Conclusions

This study measured and evaluated the comprehensive framework of Brazilian banking industry performance over four years, bringing contributions to literature in different addressed perspectives.

From a methodological perspective, we can emphasize the application of an analysis that integrates two fundamental dimensions: financial intermediation activities, which are relevant to the economic growth of countries, and the operational activities responsible for profitability, which are relevant to banks' solvency, using a DEA-GMSS model. Among the studies reviewed, we find none with this elaboration for the financial sector in Brazil. In addition, we highlight the structuring of a temporal analysis of changes in productivity and technology in the Brazilian financial sector, based on the integration of the MPI and the DEA-GMSS models.

Our main results point to a tendency towards greater efficiency in the revenue process, in relation to the intermediation process. We can explain this, in part, by the high bank spreads still practiced in the country. These lead, on the one hand, to low levels of improvement of financial technologies and, on the other hand, to a significant increase in the productivity of banks in granting credit.

In practice, the combination of an increase in the supply of credit not accompanied by an improvement of financial technologies can lead to deteriorating controls over credit portfolios. We can also conclude that banks that operate in the Brazilian credit market may incur difficulties in long-term solvency, especially for domestic banks, as their spreads are relatively higher. Additionally, we find that the Brazilian credit market operates under high levels of diseconomies of scale and diseconomies of scope. These diseconomies can be indicated by an intense negative impact on the efficiency of large banks and financial holding companies, in both the network of systems and in the internal processes.

In general, we consider that the abrupt restructuring of Brazil's financial sector was unable to entirely correct its structural problems. Having used mergers and acquisitions as a mechanism of privatization and liberalization of the economy, the restructuring generated a non-competitive environment among banks. This circumstance, in turn, delayed the development of the Brazilian credit market, leading to the prospect of future financial instability.

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Compliance with ethical standards

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