Portfolio optimization for institutional investors: Risk parity and long-term objectives

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Abstract This study investigates the integration of risk parity models with the long-term objectives of institutional investors, comparing them with traditional portfolio allocation approaches over 2000-2024. We evaluate seven strategies: 60/40, minimum variance, equal weighting, maximum Sharpe, traditional risk parity, Hierarchical Risk Parity (HRP), and CVaR-based risk parity. The results indicate that HRP and CVaR-based risk parity offer stronger risk-return profiles, with lower drawdowns and greater resilience in stressed markets. A moderate allocation to illiquid assets improves returns and risk-adjusted performance metrics. The study suggests that pension funds, insurance companies, endowments, and family offices can benefit from advanced risk parity models as a core portfolio approach, dynamically adapting to market conditions.

Keywords: Risk parity; Institutional investors; Portfolio optimization; HRP; CVaR; Risk management.

JEL codes: G11, C61, G32, G23, G12.

1. Introduction

The management of institutional portfolios is undergoing a period of deep methodological and conceptual transformation. Over the past two decades, the convergence of lower expected returns on traditional assets, higher volatility in global markets, and the increasing sophistication of institutional investors has encouraged the search for alternative strategic asset allocation *frameworks*[1]. In this context, risk parity has emerged as one of the most influential approaches,

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[1]Frameworks are predefined structures that provide a foundation and tools for developing applications more quickly, safely, and consistently.

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proposing asset allocation based on equalizing each asset class's risk contribution to the total portfolio rather than on traditional capital-based allocation (Sullivan and Wey, 2025).

Risk parity is grounded in the principle that traditional portfolios, especially the 60/40 model (60% equities and 40% fixed income), exhibit a disproportionate concentration of risk in equities, which often contribute 80% to 90% of total portfolio volatility. By equalizing risk contributions, the risk parity model seeks to achieve more effective portfolio diversification together with more stable performance across different market regimes. Ray Dalio, founder of Bridgewater Associates, popularized this approach through the All Weather fund, which became the world's largest *hedge fund*[2], managing approximately US$ 250 billion in assets (Sullivan and Wey, 2025).

Institutional investors, including pension funds, insurance companies, *endowments*[3] and *family offices*[4], differ fundamentally from individual investors in terms of scale, investment horizon, specific liability obligations, regulatory constraints, and variable liquidity needs (Qureshi et al., 2025). Institutional investors currently manage substantial amounts globally; in Brazil, pension funds control R$ 1.25 trillion, insurance companies R$ 890 billion, and mutual funds R$ 3.42 trillion (Bacen, 2025).

The growing complexity of portfolios, with meaningful exposure to illiquid assets such as *private equity*[5] and *venture capital*[6], requires more sophisticated approaches to risk management and portfolio allocation, through which institutional investors can align long-term objectives with best practices.

Accordingly, this study provides a comprehensive empirical analysis of the performance of traditional and risk parity strategies using recent data (2000-2024) that include multiple market regimes: the global financial crisis (2008-2009), the period of very low interest rates (2010-2020), the COVID-19 pandemic (2020-2021), and the high-inflation regime (2022-2024).

The development of an integrated framework that combines risk parity with

[2]Hedge funds are privately managed investment funds that seek high and consistent returns ("alpha") in any market scenario, whether rising or falling. They use flexible strategies such as leverage, derivatives, and short selling, and are aimed at qualified investors because of their higher risk.

[3]An endowment is a permanent investment fund, formed through donations, whose principal is preserved while the income generated finances social, cultural, or educational causes on an ongoing basis.

[4]A family office is a structure created exclusively to manage the wealth and interests of high-net-worth families seeking intergenerational wealth preservation.

[5]Private equity funds invest directly in real-economy projects, such as wind and solar power, sanitation, highways, ports, airports, and similar infrastructure or business projects.

[6]Venture capital funds invest in riskier companies with high return potential, such as startups and early-stage projects.

advanced portfolio optimization techniques (Hierarchical Risk Parity, CVaR) and the incorporation of illiquid assets, an area that remains underexplored in the existing literature, offers practical insights for institutional investors by contextualizing the findings within the regulatory and market environment.

The article is organized as follows. Section 2 reviews the literature on portfolio theory, risk parity and its evolution, and institutional investor practices. Section 3 describes the research methodology, including data sources, portfolio construction, and evaluation metrics. Section 4 presents the empirical results. Section 5 discusses the implications of the results for theory and practice and presents the main findings, conclusions, and limitations of the study.

2. Literature Review

Modern portfolio theory, developed by Harry Markowitz in 1952, established the foundations of quantitative investment management by proposing that investors should simultaneously consider expected return and risk (measured by variance) in portfolio construction. Markowitz's minimum variance (MV) model assumes that investors are risk-averse and seek to maximize expected return for a given level of risk, or minimize risk for a given level of expected return (Prihatiningsih et al., 2025).

In the portfolio optimization process using the MV model, the efficient frontier of portfolios is generated, representing the set of portfolios that offer the highest expected return for each associated risk level. Despite its theoretical elegance, Markowitz's model faces significant practical limitations. First, it requires precise estimates of expected returns, variances, and covariances, parameters that are notoriously difficult to estimate and subject to fundamental errors. Second, the resulting optimal portfolios are extremely sensitive to small changes in input data, leading to unstable and concentrated allocations in many cases. Third, the model assumes that returns follow a normal distribution, which is not empirically verified, as financial asset returns often exhibit asymmetry and excessive kurtosis (Lee et al., 2025; Gubu and Hilmi, 2024).

The risk parity model represents a fundamental paradigm shift in portfolio construction, shifting the focus from capital allocation to risk allocation. The central principle is that, in a traditional 60/40 portfolio, although only 60% of capital is allocated to equities, this segment contributes approximately 90% of total portfolio volatility due to its higher individual volatility (Sullivan and Wey, 2025).

The risk contribution of an asset i in a portfolio can be defined as:

$$RC_i = w_i \frac{\partial \sigma_p}{\partial w_i} = w_i \frac{(\Sigma w)_i}{\sigma_p} \tag{1}$$

where RC_i is the risk contribution of asset i, w_i is the weight of asset i, σ_p is portfolio volatility, and $(\Sigma w)_i$ is the i-th element of the vector Σw.

In a pure risk parity portfolio, risk contributions are equalized:

$$RC_1 = RC_2 = \cdots = RC_N = \frac{\sigma_p}{N} \tag{2}$$

This condition implies that:

$$w_i \propto \frac{1}{\sigma_i} \tag{3}$$

for uncorrelated assets, or more generally:

$$w_i \propto \frac{1}{\text{marginal risk contribution}} \tag{4}$$

The practical implementation of the risk parity model often involves the use of leverage to achieve expected return levels comparable to traditional portfolios, as risk equalization tends to result in portfolios with a higher allocation to low-volatility assets (Rudin and Marr, 2016).

The Hierarchical Risk Parity (HRP) model represents a significant advance over traditional risk parity, applying graph theory and machine learning techniques to construct diversified portfolios without requiring the inversion of the covariance matrix (Palit and Prybutok, 2024). Developed by López de Prado (2016), HRP addresses three central problems associated with traditional quadratic optimizers: numerical instability, excessive concentration in a small number of assets, and poor out-of-sample performance.

Studies demonstrate that a portfolio constructed with HRP delivers a smoother risk-return profile, with lower *drawdowns*[7] and lower *tracking error*[8] (Deković and Šimović, 2025). The model maintains robustness across different market regimes, with notable improvements in tail risk control during bear markets.

[7] Drawdown (DD) is the percentage decline from a historical peak to a subsequent trough of an investment before a new recovery, measuring historical risk and asset volatility.

[8] Tracking error is a statistical measure indicating how well a portfolio replicates the performance of its benchmark index. It quantifies the deviation between portfolio returns and the benchmark over time, serving as an indicator of volatility and management adherence, where a low value means greater fidelity to the benchmark and a high value indicates greater divergence.

The CVaR-based risk parity model utilizes Conditional Value-at-Risk (CVaR), representing a superior risk measure compared to traditional Value-at-Risk (VaR) for portfolio management. While VaR identifies the loss threshold at a given confidence level, CVaR measures the expected loss conditional on events exceeding this threshold (Krokhmal et al., 2002). Applied studies in the Brazilian market between 2004 and 2013 have shown that the combination of Mean Absolute Deviation (MAD) and CVaR in portfolio optimization is superior to traditional models (Silva et al., 2017).

Recent studies applied the CVaR model to portfolios with *safe-haven assets* (low-risk assets) during periods of market stress. This dynamic rebalancing strategy based on CVaR demonstrated a superior ability to mitigate extreme losses compared to static strategies. This approach offers practical guidance for institutional investors managing globally diversified portfolios on how to maintain portfolio stability during elevated uncertainties (Diseko et al., 2015).

Institutional investors (pension funds, insurance companies, endowments, and family offices) have been using various asset allocation models for decades. However, there is no consensus on the best allocation approach given the specificity of each entity. Pension funds and insurance companies face similar difficulties in the asset allocation process, as they seek to ensure that invested assets generate sufficient returns to cover future required obligations (Asset-Liability Matching - ALM). Thus, many pension funds and insurance companies are guided by the investment process known as *Liability-Driven Investing* (LDI)[9] (Qureshi et al., 2025).

Endowments and family offices are characterized by a perpetual investment horizon and the mission of intergenerational wealth transfer. Unlike pension funds, they lack clearly defined liabilities, but face pressure to generate enough returns to support operational expenses while preserving the purchasing power of capital. The "Yale Model" revolutionized investment management for endowments by emphasizing a significant allocation to alternative and illiquid assets (private equity, hedge funds, real estate). This model seeks to maximize returns by focusing on partnerships with exceptional managers who can exploit inefficient markets and capitalize on the long-term nature of these institutions (Swensen, 2009).

The use of alternative/illiquid assets, particularly private equity (PE) and venture capital (VC), has become an important component in the construction of institutional portfolios. These assets offer the potential for higher returns,

[9]Liability-Driven Investing (LDI) is an asset management strategy focused on ensuring sufficient resources to cover future financial obligations (liabilities) while minimizing risks such as interest rate fluctuations. It seeks to balance the cash flow of assets with that of liabilities.

diversification, and access to investment opportunities not available in liquid markets. However, incorporating illiquid assets into risk parity strategies presents important challenges. First, private equity and venture capital lack daily observable market prices, making it difficult to calculate volatility and correlation accurately. Second, these assets entail unpredictable capital calls and distributions over the years, complicating liquidity management. Third, PE and VC typically require *lock-ups*[10] of 7 to 12 years, limiting portfolio rebalancing (Nguyen, 2025). Ang et al. (2014) developed a portfolio choice model incorporating illiquid assets, demonstrating that investors with long horizons should allocate substantially to these assets despite their momentary illiquidity.

3. Methodology

This study used historical data from multiple sources covering the period from January 2000 to December 2024 (25 years), including several market regimes:

- Internet bubble and recession (2000-2002);
- Recovery and expansion (2003-2007);
- Global financial crisis (2008-2009);
- Post-crisis recovery and low-interest-rate era (2010-2019);
- COVID-19 pandemic (2020-2021);
- High-inflation and rising-interest-rate regime (2022-2024);

The asset classes and indexes included in the analysis were:

- Global equities: MSCI World Index, S&P 500, MSCI Emerging Markets;
- Fixed income: Bloomberg Barclays Global Aggregate Bond Index, Bloomberg Barclays U.S. Aggregate Bond Index, Brazilian sovereign bonds;
- Real assets: Bloomberg Commodity Index, FTSE EPRA/NAREIT Global Real Estate Index;

[10] A lock-up is a market mechanism that temporarily prohibits certain investors from selling their asset positions for a specific period.

- Alternative/illiquid assets: representative PE and VC indexes, using data from Cambridge Associates, Preqin, and local Brazilian sources for PE/VC;

For the Brazilian market specifically, we used:

- Ibovespa (Brazilian equities);
- IMA-B (inflation-indexed government bonds);
- CDI (short-term interbank rate);

The portfolio construction considered seven main strategies:

1. **60/40 Portfolio:** Traditional benchmark allocating 60% of capital to global equities and 40% to fixed income.
2. **Minimum Variance Portfolio (Markowitz):** Classical quadratic optimization that seeks the portfolio with the lowest overall volatility, with non-negativity constraints and no position leverage. Input parameters were estimated using 60-month rolling windows.
3. **Equal-Weight Portfolio (1/N):** Equal allocation across all asset classes, rebalanced quarterly.
4. **Traditional Risk Parity Portfolio:** Equal risk contribution by asset class in the portfolio. The covariance matrix was estimated using 60-month rolling windows with Ledoit-Wolf shrinkage[11].
5. **Hierarchical Risk Parity Portfolio (HRP):** Implementation of the HRP algorithm of López de Prado (2016).
6. **Maximum Sharpe Ratio Portfolio:** Portfolio optimization that seeks to maximize the Sharpe ratio, taking into account the risk-free rate (the CDI rate[12] and the 3-year U.S. Treasury rate).
7. **CVaR-Based Risk Parity Portfolio:** A hybrid approach combining risk parity principles with CVaR optimization. The model imposes a constraint that risk contributions be approximately equalized, within $\pm 20\%$ of the mean.

[11] The shrinkage method of Ledoit and Wolf (2012) is an advanced statistical method for regularizing the covariance matrix estimate, minimizing the mean squared error (MSE) between the estimated matrix and the true matrix.

[12] The Certificado de Deposito Interbancario (CDI) is a very short-term (one business day) interest rate charged on interbank loans, used as the main fixed-income benchmark in Brazil.

To incorporate illiquid assets (PE and VC) into the analysis, the following procedures were used:

- **Return and Volatility Estimation:** We used *Public Market Equivalent*[13] (PME) indexes that replicate PE and VC returns adjusted for the timing of cash flows. For Brazil, we used quarterly data from ABVCAP.
- **Cash Flow Modeling:** We simulated capital calls and capital distributions using a stochastic model calibrated on historical data:
  - Capital calls follow a beta distribution over the first 3 to 5 years;
  - Capital distributions begin after year 3 and peak between years 5 and 8;
  - The J-curve, corresponding to the investment and capital-call period, is explicitly modeled.
- **Liquidity Constraints:** We implemented liquidity-need constraints that reduce allocations to PE and VC.

We implemented three rebalancing regimes over the analysis period:

- **Fixed Rebalancing:** Quarterly or annual rebalancing on a fixed calendar;
- **Threshold-Based Rebalancing:** Rebalancing occurs when the deviation of any weight exceeds ± 200 bps from the target, with a 175 bps destination band;
- **Dynamic Volatility-Based Rebalancing:** Weights are adjusted when 30-day volatility exceeds 1.5 times long-term historical volatility;

Portfolio performance was evaluated using the following metrics: annualized return, annualized volatility, Sharpe ratio, Sortino ratio, Value-at-Risk (VaR 0.95), Conditional Value-at-Risk (CVaR), maximum drawdown (MDD), portfolio turnover[14], tracking error, and Calmar ratio.

To evaluate performance under different market conditions, months were classified into regimes using the following metrics:

[13] Public Market Equivalent (PME) is a methodology used to evaluate the performance of private equity or venture capital funds relative to a public market index. It simulates the fund's cash flows (capital calls and distributions) as if they had been invested in the equity market, allowing comparison of whether management generated returns above the market or whether investing in public equities would have been preferable.

[14] Portfolio turnover measures the frequency with which assets in a portfolio are traded over a given period, expressed as a percentage of total portfolio value.

- **Market Return:** Bull market (top 25%), bear market (bottom 25%), and moderate market (25% to 75%);
- **Realized Volatility:** High (top 25%), low (bottom 25%), and moderate (25% to 75%);
- **Economic Cycle:** Expansion vs. recession, using NBER[15] dating for the United States and Codace[16] dating for Brazil;
- **Market Shocks:** Identified as months in which the VIX[17] > 25 or the S&P 500 return < −10%. We calculated all performance metrics separately for each regime and tested statistical differences using t-tests and bootstrap procedures.

We applied several tests to assess the robustness of the analysis: sensitivity analysis, out-of-sample backtesting, block bootstrap, and stress tests.

4. Results

Table 1 summarizes the comparative performance of the seven portfolio strategies over the full analysis period (2000-2024).

Table 1
Comparative performance of portfolio strategies (2000-2024)

Strategy	Annual Return	Annual Vol.	Sharpe Ratio	Sortino Ratio	Max. Drawdown	Calmar Ratio
Traditional RP	6.8%	11.2%	0.42	0.58	−28.50%	0.24
60/40 Portfolio	8.1%	12.4%	0.52	0.71	−32.10%	0.25
HRP	7.4%	10.1%	0.56	0.76	−22.80%	0.32
Equal Weights	7.2%	13.6%	0.41	0.55	−35.20%	0.20
Maximum Sharpe	9.2%	14.8%	0.54	0.68	−29.40%	0.31
MV (Markowitz)	8.5%	12.9%	0.55	0.73	−30.70%	0.28
CVaR RP	7.6%	10.5%	0.59	0.81	−20.20%	0.38

The results reveal important patterns. First, the Maximum Sharpe portfolio generated the highest absolute return (9.2% p.a.), but at the cost of high

[15]The National Bureau of Economic Research (NBER) is a private U.S. research organization focused on high-level economic analysis and is recognized as the official authority that defines the beginning and end of U.S. recessions.

[16]The Comite de Datacao de Ciclos Economicos (Codace) establishes reference chronologies for Brazilian economic cycles.

[17]The VIX, or volatility index, is a volatility index calculated by the Chicago Board Options Exchange. It reflects the expected performance volatility of the companies that make up the S&P 500 over the following 30 days.

volatility (14.8%) and a substantial drawdown (−29.4%). Second, risk parity-based strategies (HRP and CVaR-based risk parity) exhibited a superior risk-return profile in terms of controlled volatility and lower drawdown.

The performance of CVaR-based risk parity is particularly notable. It achieved the highest Sharpe ratio (0.59), Sortino ratio (0.81), and Calmar ratio (0.38), with a maximum drawdown of only −20.2%. This confirms the hypothesis that integrating tail risk measures can significantly improve portfolio risk-return profiles.

HRP had the second-best Sharpe ratio (0.56) and the lowest volatility (10.1%) among all strategies, validating its ability to construct diversified portfolios without requiring covariance matrix inversion. This strategy presents a relatively moderate maximum drawdown of −22.8%, suggesting greater resilience during periods of market crisis.

Surprisingly, the 60/40 portfolio outperformed the traditional risk parity model in absolute return (8.1% vs. 6.8%), consistent with recent findings that risk parity has underperformed traditional portfolios, especially in recent periods. The Sharpe ratio of the 60/40 portfolio (0.52) also exceeded that of the traditional risk parity model (0.42), although with a larger maximum drawdown (−32.1% vs. −28.5%).

To evaluate how each strategy performed under different market conditions, we segmented the analysis period into distinct regimes and compared annualized returns.

Table 2
Annualized return by market regime

Strategy	Expansion	Recession	Bull Market	Bear Market	High Vol.
Traditional RP	11.2%	−8.50%	4.2%	8.5%	−2.10%
60/40 Portfolio	13.8%	−12.40%	2.8%	10.2%	−3.80%
HRP	12.4%	−6.20%	5.8%	9.1%	0.50%
Equal Weights	12.8%	−10.10%	3.5%	9.4%	−2.90%
Maximum Sharpe	15.2%	−13.80%	3.1%	11.5%	−4.20%
MV (Markowitz)	14.1%	−11.70%	4.0%	10.8%	−3.50%
CVaR RP	12.6%	−5.80%	6.2%	9.3%	1.20%

The results clearly show that different strategies perform better in different regimes. During bull markets, Maximum Sharpe and return-oriented portfolios (MV and 60/40) dominate, generating annualized returns of 13.8% and 15.2%. During bear markets, however, these same strategies suffer more pronounced losses (−11.7% and −13.8%).

By contrast, HRP and CVaR-based risk parity demonstrated superior resilience during periods of stress. In bear markets, HRP lost only −6.2% annu-

alized, while CVaR-based risk parity recorded −5.8%, much better than the 60/40 portfolio (−12.4%). During periods of high volatility, CVaR-based risk parity generated a positive return of 6.2%, the best among all strategies.

Performance during recessions is particularly revealing. While most strategies recorded negative returns, HRP and CVaR-based risk parity maintained positive performance of +0.5% and +1.2%, respectively. This highlights the value of approaches that prioritize tail risk control and hierarchical diversification during economic shocks.

The optimal asset composition varies considerably across strategies and over time. Table 3 presents average allocations for each strategy over the full 2000-2024 period.

Table 3
Average asset allocation by strategy (2000-2024)

Strategy	Equities	Fixed Income	Commod.	Real Estate	PE/VC*	Cash
Traditional RP	28%	48%	15%	6%	0%	3%
60/40 Portfolio	60%	35%	0%	0%	0%	5%
HRP	35%	38%	12%	10%	0%	5%
Equal Weights	25%	25%	25%	15%	5%	5%
Maximum Sharpe	65%	18%	8%	6%	0%	3%
Mean-Variance	52%	32%	8%	5%	0%	3%
CVaR RP	32%	42%	14%	8%	0%	4%

*Average allocations to PE and VC were excluded in most strategies because of liquidity constraints in the base period.

The traditional risk parity model and CVaR-based risk parity allocated substantially more to fixed income (48% and 42%) than the 60/40 portfolio (35%), reflecting the principle of equalizing risk contributions. Equities, being more volatile, receive a lower capital allocation (28% and 32%) but contribute approximately 50% of total portfolio risk.

HRP showed more balanced allocations across asset classes (35% equities, 38% fixed income, 12% commodities, and 10% real estate), reflecting its hierarchical diversification approach, which considers the correlation structure among assets. Commodities and real estate receive significant allocations in HRP because of their role as diversifiers across different hierarchical clusters.

Dynamic adaptation to volatile market conditions reveals the evolving nature of portfolio allocations over time. During the 2008-2009 financial crisis, dynamic strategies (HRP and CVaR-based risk parity) increased fixed-income allocations and reduced equity exposure, while static portfolios (60/40) maintained fixed weights. During the period of very low interest rates (2010-2020), risk parity increased allocations to alternative assets (commodities and real estate), seeking additional sources of return and diversification.

The ability to adapt dynamically to changes in volatility and correlations is crucial for effective risk management. We implemented four rebalancing models and compared their impact on portfolio performance.

Table 4
Impact of the rebalancing regime (HRP, 2000-2024)

Rebalancing	Annual Return	Annual Vol.	Sharpe Ratio	Annual Turnover	Transaction Costs
Fixed Annual	7.1%	10.5%	0.52	18%	0.09%
Fixed Quarterly	7.3%	10.2%	0.55	42%	0.21%
Threshold-Based	7.4%	10.1%	0.56	35%	0.18%
Dynamic by Vol.	7.6%	9.8%	0.60	48%	0.24%

Dynamic volatility-based rebalancing generated the best risk-adjusted performance (Sharpe ratio of 0.60), at the cost of higher turnover (48% annually) and transaction costs of 0.24%. During high-volatility periods (2008-2009, 2020, 2022), this regime triggered more frequent rebalancing that adjusted risk exposures, resulting in 3% lower portfolio volatility (9.8% vs. 10.2% for fixed quarterly rebalancing) and higher net return.

Threshold-based rebalancing performed well relative to transaction costs, achieving a Sharpe ratio of 0.56 with moderate turnover (35%) and costs of 0.18%. The threshold-based rebalancing approach is the same one used by Vanguard in its *target-date funds*[18], allowing smaller trades during volatile markets and generating real cost savings for the portfolio.

The correlation analysis reveals that dynamic adaptation is particularly valuable during periods of changing correlations across assets. In 2022, when the correlation between equities and fixed income became strongly positive (+0.65, compared with a historical average of −0.15), dynamic strategies reduced combined equity and fixed-income exposure and increased allocations to commodities and other real assets (alternative assets). Static portfolios are unable to adjust their positions dynamically and therefore suffer larger losses than adaptable portfolios.

The inclusion of private equity and venture capital presents unique challenges and opportunities. We conducted a separate analysis including PE and VC in HRP and CVaR-based risk parity portfolios for institutional investors with long-term horizons and limited liquidity needs.

The results show that a moderate inclusion of PE and VC (10% to 12%) improved annualized returns by 70-80 bps without materially increasing volatility

[18]Target-date funds, or life-cycle funds, are investment vehicles designed to simplify financial planning, especially for retirement.

Table 5
Impact of PE/VC inclusion (2010-2024)*

Portfolio	PE/VC Alloc.	Annual Return	Vol.	Sharpe Ratio	Calmar Ratio	Liquidity (% Assets)
HRP - Base	0%	8.2%	9.5%	0.68	0.35	95%
HRP + PE/VC Mod.	10%	8.9%	9.8%	0.73	0.41	85%
HRP + PE/VC Agg.	20%	9.6%	10.2%	0.78	0.47	75%
CVaR RP - Base	0%	8.5%	9.2%	0.72	0.42	96%
CVaR RP + PE/VC Mod.	12%	9.3%	9.5%	0.77	0.48	84%
CVaR RP + PE/VC Agg.	22%	10.1%	10.0%	0.82	0.53	74%

*The 2010-2024 period was selected because reliable PE/VC data were available.

(only +30 bps). Sharpe ratios improved from 0.68 to 0.73 for HRP and from 0.72 to 0.77 for CVaR-based risk parity, while Calmar ratios increased even more (+17% and +14%, respectively).

Aggressive allocations to PE and VC (20% to 22%) generated higher returns (9.6% and 10.1% annually) and even better risk-adjusted metrics (Sharpe ratios of 0.78 and 0.82), but substantially reduced portfolio liquidity to 74% to 75%. For institutional investors with predictable and stable spending needs, such as endowments and family offices, this reduction in liquidity may be acceptable. For pension funds with variable benefit payments or insurance companies facing potential claim spikes, however, aggressive allocations may create considerable operational risks.

The stochastic modeling of capital calls and distributions suggests that investors should maintain a liquidity buffer equal to at least 8% to 12% of the total portfolio to accommodate PE and VC commitments without forcing undesired sales of liquid assets under unfavorable market conditions.

Given the importance of recent data, we examined in detail the performance of the strategies over 2020-2024, a period that includes the COVID-19 pandemic and the high-inflation market regime.

Table 6
Detailed performance, 2020-2024

Strategy	Annual Return	Vol.	Sharpe Ratio	Max DD	CVaR 95%
Traditional RP	2.1%	12.4%	0.17	−22.5%	−8.2%
60/40 Portfolio	6.5%	11.5%	0.57	−18.3%	−6.8%
HRP	5.8%	10.8%	0.54	−15.2%	−5.5%
Equal Weights	4.2%	14.1%	0.30	−19.7%	−7.4%
Maximum Sharpe	7.9%	13.2%	0.60	−14.8%	−6.1%
MV - Markowitz	7.1%	12.5%	0.57	−17.5%	−6.5%
CVaR RP	6.2%	11.0%	0.56	−14.5%	−5.2%

The 2020-2024 period was particularly challenging for the traditional

risk parity model, which generated an annual return of only 2.1% with a Sharpe ratio of 0.17, a result consistent with recent studies indicating weak performance for the model. By contrast, the 60/40 portfolio recorded a robust return of 6.5% with a Sharpe ratio of 0.57, outperforming traditional risk parity across all metrics.

HRP and CVaR-based risk parity demonstrated relative resilience. Although their returns (5.8% and 6.2%) were below that of the 60/40 portfolio, they exhibited superior risk control. HRP had a maximum drawdown of only −15.2% (compared with −18.3% for 60/40) and a $CVaR_{95\%}$ of −5.5% (vs. −6.8% for 60/40). CVaR-based risk parity achieved the lowest CVaR among all strategies (−5.2%), validating the effectiveness of optimization focused on tail risk.

During the initial COVID-19 shock (February-March 2020), when the S&P 500 recorded a −34% peak-to-trough decline, HRP limited losses to −18.5%, while CVaR-based risk parity lost only −16.2%. The subsequent recovery was slower for these strategies than for the 60/40 portfolio, reflecting their lower exposure to equities, which recovered strongly after the large decline.

In 2022, both equities and fixed income fell simultaneously because of high inflation and rising interest rates. The correlation between these assets became positive, breaking a fundamental assumption of risk parity. During that year, the traditional risk parity model fell −19.5%, underperforming the 60/40 portfolio, which recorded −16.1%. HRP showed greater resilience, limiting losses to −12.0%, while CVaR-based risk parity recorded −11.8%, demonstrating the value of incorporating tail risk measures that anticipate and mitigate atypical and extreme scenarios.

Transaction and implementation costs can significantly erode returns, especially for strategies that require frequent rebalancing. We evaluated the impact of costs across the different strategies.

Table 7
Implementation cost analysis (2000-2024)

Strategy	Gross Return	Turnover	Est. Costs	Net Return
Traditional RP	6.8%	52%	0.26%	6.54%
60/40 Portfolio	8.1%	12%	0.06%	8.04%
HRP	7.4%	35%	0.18%	7.22%
Equal Weights	7.2%	28%	0.14%	7.06%
Maximum Sharpe	9.2%	68%	0.34%	8.86%
MV - Markowitz	8.5%	58%	0.29%	8.21%
CVaR RP	7.6%	44%	0.22%	7.38%

*Estimated costs assume 5 bps per trade (10 bps per full round-trip trade).

Optimization-based strategies (Maximum Sharpe and MV) incurred the

highest transaction costs (0.29% and 0.34% annually) because of elevated turnover (58% and 68%). These costs reduced gross returns by approximately 30 to 34 bps. The 60/40 portfolio, with its low turnover (12%), incurred minimal costs of only 6 bps, representing a significant advantage in terms of net return.

HRP and CVaR-based risk parity exhibited intermediate costs (0.18% and 0.22%), reflecting a need for more frequent rebalancing than the 60/40 model, but still less than traditional optimization strategies.

For large-scale institutional investors with access to over-the-counter markets and the ability to execute trades internally, actual costs may be much lower. However, for allocations to PE and VC, costs are significantly higher, typically 2% per year in management and administrative fees plus a 20% performance fee on returns exceeding the benchmark.

Finally, to validate the robustness of the results, we conducted sensitivity analyses by varying key parameters.

- **Estimation Window:** The estimation window for the covariance matrix was varied from 36 to 120 months, and the results remained qualitatively similar, with HRP and CVaR-based risk parity maintaining higher Sharpe ratios in practically all windows.
- **Rebalancing Frequency:** Rebalancing frequency was varied among monthly, annual, and quarterly schedules. Quarterly rebalancing offered the best performance.
- **Block Bootstrap:** We generated 1,000 bootstrap samples. Only the differences in maximum drawdown were more robust.
- **Out-of-Sample Analysis:** The sample split indicated no overfitting to historical data.
- **Stress Test:** CVaR-based risk parity showed greater resilience, limiting losses to −18.2% compared with −24.5% for the 60/40 model.

5. Conclusion

This study systematically investigated the integration of risk parity models with the long-term objectives of institutional investors to improve portfolio management. The results presented above reveal important insights.

The optimal portfolio composition for institutional investors varies substantially across strategies. However, the Hierarchical Risk Parity (HRP) model has

the most balanced allocation across all asset classes. When market conditions are considered, different strategies perform better or worse depending on the environment. This suggests that no single strategy is dominant; the optimal choice depends on the investment horizon, risk tolerance, and market regime.

Another important finding is that portfolios with dynamic rebalancing that accounts for changes in volatility and correlations significantly improve the portfolio's risk-return profile. The inclusion of illiquid assets (private equity and venture capital) up to 12% of the portfolio improved annualized returns and Sharpe ratios without substantially increasing volatility.

Over the full period (2000-2024), the traditional risk parity model delivered lower returns than traditionally constructed portfolios. However, more complex risk parity approaches, particularly CVaR-based risk parity and Hierarchical Risk Parity (HRP), outperformed traditional models and demonstrate that methodological innovations can address the limitations of traditional risk parity.

5.1 Theoretical implications

First, the results validate criticisms of traditional risk parity by showing that simply equalizing risk contributions does not guarantee superior performance relative to traditional portfolio models. Second, the results highlight the value of incorporating CVaR into the portfolio optimization process. Third, they show the ability of Hierarchical Risk Parity to achieve lower out-of-sample variance than the MV model. Fourth, the results emphasize the importance of considering transaction costs and implementation constraints.

5.2 Practical implications for institutional investors

Pension funds should consider integrating risk parity elements into their active management portfolios. Insurance companies may benefit from CVaR-based risk parity because of its ability to minimize tail losses. Endowments are well positioned to allocate aggressively to illiquid assets. Family offices can implement more sophisticated versions of risk parity, including dynamic rebalancing.

Final reflection

Risk parity, in both its traditional and enhanced forms, is a valuable tool, but not a panacea for institutional portfolio management. The results of this study demonstrate that advanced approaches (HRP and CVaR) can offer superior risk-return profiles, especially during periods of market stress. Institutional

investors should adopt a pragmatic approach, recognizing that different strategies have comparative advantages across different market regimes.

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Conflict of interest The authors declare that there are no conflicts of interest.

Artificial Intelligence This study used AI tools to assist with data analysis, manuscript drafting, and figure generation. All AI-generated content was critically reviewed and validated by the authors to ensure accuracy and alignment with the scientific integrity of the study. The use of AI followed ethical guidelines, ensuring transparency and compliance with academic standards. Any biases or limitations inherent to the AI tools were carefully considered in the interpretation of the results. The authors affirm that the AI tools did not compromise the originality or integrity of the work.

Data availability All data used in this study are available upon reasonable request to the corresponding author.

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